PHY 742 Quantum Mechanics II
1-1:50 AM MWF via video link:
https://wakeforest-university.zoom.us/my/natalie.holzwarth

Extra notes for Lecture 21

Quantization of the Electromagnetic fields

Review the "raising" and "lowering" operators presented in Professor Carlson's textbook in V. The Harmonic Oscillators. Start reading XVII. Quantizing Electromagnetic Fields.

1. **Review of the harmonic oscillator**
2. **Particle creation and annihilation operator formalism**
3. **Hamiltonian for the electromagnetic fields**

Welcome to the new format of PHY 742. The same motivations and requirements mentioned in the PHY 712 slides apply to PHY 742. In the next few lectures, we will consider the "quantization" of the electromagnetic field, inspired by the detailed results of the analysis of the one dimensional harmonic oscillator.

Topics for Quantum Mechanics II

Single particle analysis

- **Single particle interacting with electromagnetic fields – EC Chap. 9**
- **Scattering of a particle from a spherical potential – EC Chap. 14**
- **More time independent perturbation methods – EC Chap. 12, 13**
- **Single electron states of a multi-well potential → molecules and solids – EC Chap. 2,6**
- **Time dependent perturbation methods – EC Chap. 15**
- **Relativistic effects and the Dirac Equation – EC Chap. 16**
- **Path integral formalism (Feynman) – EC Chap. 11.C**

Multiple particle analysis

- **Quantization of the electromagnetic fields – EC Chap. 17 also Chap. 5**
- **Photons and atoms – EC Chap. 18**
- **Multi particle systems; Bose and Fermi particles – EC Chap. 10**
- **Multi electron atoms and materials**
  - **Hartree-Fock approximation**
  - **Density functional approximation**

This is the schedule that we have used from the beginning. Comments/suggestions are welcome.

	Mon: 03/16/2020	No class	*Classes Cancelled*		
	Wed: 03/18/2020	No class	*Classes Cancelled*		
	Fri: 03/20/2020	No class	*Classes Cancelled*		
21	Mon: 03/23/2020	Chap. 17	Quantization of the Electromagnetic Field	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 17	Quantization of the Electromagnetic Field		
23	Fri: 03/27/2020				
24	Mon: 03/30/2020				
25	Wed: 04/01/2020				
26	Fri: 04/03/2020				
27	Mon: 04/06/2020				
28	Wed: 04/08/2020				
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020				
30	Wed: 04/15/2020				
31	Fri: 04/17/2020				
32	Mon: 04/20/2020				
33	Wed: 04/22/2020				
34	Fri: 04/24/2020				
35	Mon: 04/27/2020				
36	Wed: 04/29/2020		Review		

This is the altered schedule. Note that there is one homework problem which hopefully you will be able to complete before the next lecture.

Questions:

From Laxman –

1. I do not see why Potential is taken zero in slide 14.
2. What is the epsilon in plane wave equation in slide 15?
3. What is it meant by orthogonality of plane waves and how it changed the result of field energy in slide 16?

From Surya –

1. Its is well known that operators of Quantum Mechanics are hermitian in nature, while creation and annihilation are non-hermitian, why do we consider these non-hermitian operators? Isn't this against the rules of QM?
2. Slide 6 and 7 present the matrix element of X, P and a, a* for different phonon numbers, is there any physical significance of doing so is just a way to represent states systematically?
3. Can we explain uncertainty principle in terms of zero-point oscillation of harmonic oscillator?

From Vincent --

1. Why is it called “phonon number”? I’ve also heard it being called a "fermion number”.

Question:

1. It is well known that operators of Quantum Mechanics are hermitian in nature, while creation and annihilation are non-hermitian, why do we consider these non-hermitian operators? Isn't this against the rules of QM?
2. Slide 6 and 7 present the matrix element of X, P and a, a* for different phonon numbers, is there any physical significance of doing so is just a way to represent states systematically?

One-dimensional harmonic oscillator

$$H\psi(x) = \left(\frac{P^2}{2m} + \frac{m\omega^2}{2}X^2\right)\psi(x) = E\psi(x)$$

Define:

$$a = \left(\frac{m\omega}{2\hbar}\right)^{1/2} X + i\left(\frac{1}{2m\omega\hbar}\right)^{1/2} P$$

$$a^\dagger = \left(\frac{m\omega}{2\hbar}\right)^{1/2} X - i\left(\frac{1}{2m\omega\hbar}\right)^{1/2} P$$

Note that:

$$[a, a^\dagger] = 1$$

It follows that:

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) \qquad \psi \Rightarrow |n\rangle \qquad E \Rightarrow E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

This material is covered in Chapter V of your textbook. Presumably you have previously derived these equations. Are they still true? At this point the operators a and a† seem to be “cute” curiosities?

Representation of the position and momentum operators in terms of the energy eigenstates of the harmonic oscillator:

$n=$ 0 1 2 3 .....

$$X \leftrightarrow \left(\frac{\hbar}{2m\omega}\right)^{1/2} \begin{bmatrix} 0 & 1^{1/2} & 0 & 0 & \cdots \\ 1^{1/2} & 0 & 2^{1/2} & 0 & \\ 0 & 2^{1/2} & 0 & 3^{1/2} & \\ 0 & 0 & 3^{1/2} & 0 & \\ \vdots & & & & \end{bmatrix}$$

$$P \leftrightarrow i\left(\frac{m\omega\hbar}{2}\right)^{1/2} \begin{bmatrix} 0 & -1^{1/2} & 0 & 0 & \cdots \\ 1^{1/2} & 0 & -2^{1/2} & 0 & \\ 0 & 2^{1/2} & 0 & -3^{1/2} & \\ 0 & 0 & 3^{1/2} & 0 & \\ \vdots & & & & \end{bmatrix}$$

It is convenient to evaluate the position and momentum operators in the basis of energy eigenstates of the harmonic oscillator denoted by the integer *n*.

Representation of the raising and lowering operators in terms of the energy eigenstates of the harmonic oscillator:

$$a^\dagger \leftrightarrow \begin{array}{c} \\ n=0 \\ n=1 \\ n=2 \\ \cdot \\ \cdot \end{array} \begin{array}{c} \begin{array}{cccc} n=0 & n=1 & n=2 & \dots \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & \dots \\ 1^{1/2} & 0 & 0 & \\ 0 & 2^{1/2} & 0 & \\ 0 & 0 & 3^{1/2} & \\ \vdots & & & \end{bmatrix} \end{array}$$

$$a \leftrightarrow \begin{bmatrix} 0 & 1^{1/2} & 0 & 0 & \cdots \\ 0 & 0 & 2^{1/2} & 0 & \\ 0 & 0 & 0 & 3^{1/2} & \\ \vdots & & & & \end{bmatrix}$$

The a and a† operators can also be evaluated in this basis.

Contributing to the discussion –

The creation and annihilation operators within the harmonic oscillator formalism seem to have been introduced by mathematical logic and found to have very interesting properties. In fact, as shown in Chapter V, starting from the creation and annihilation operators, one can deduce the Harmonic Oscillator spectrum. These operators do not by themselves represent physical quantities and therefore do not "have" to be Hermitian. The matrix form of X and P in the basis of |n> is just one of many ways to represent these operators.

Related question –

Why is it called "phonon number"? I've also heard it being called a "fermion number".

Answer:

The harmonic oscillator states clearly have an associated quantum number *n.* I am calling it phonon number for the moment. This formalism has a built in symmetry in that $a_1a_2= a_2a_1$ consistent with "Bose" particles. Fermi particles are different.

Question:

1. I do not see why Potential is taken zero in slide 14.

Equations within the Lorenz gauge --

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \qquad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = 0$$

It is further convenient to seek solutions with $\Phi \equiv 0 \quad \Rightarrow \nabla \cdot \mathbf{A} = 0$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Answer: It is one of many possible choices and it turns out to be convenient.

Question:

What is the epsilon in plane wave equation in slide 15?

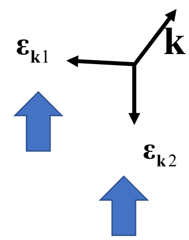
Plane wave solutions to electromagnetic waves in terms of vector potentials

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = 0 \qquad \nabla \cdot \mathbf{A} = 0$$

A pure plane wave takes the form

$$\mathbf{A}_{\mathbf{k}\sigma}(\mathbf{r},t) = A_{\mathbf{k}\sigma}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} \qquad \omega_{\mathbf{k}} = |\mathbf{k}|c$$

$$\mathbf{k}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\sigma} = 0 \qquad \boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\sigma'} = \delta_{\sigma\sigma'}$$



3 mutually perpendicular vectors

These are unit polarization vectors.

For the pure plane wave, the following relations hold:

$$\frac{\partial \mathbf{A}_{\mathbf{k}\sigma}(\mathbf{r},t)}{\partial t} = -i\omega_{\mathbf{k}}A_{\mathbf{k}\sigma}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}$$

$$\nabla \times \mathbf{A}_{\mathbf{k}\sigma}(\mathbf{r},t) = i\mathbf{k}\times A_{\mathbf{k}\sigma}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}$$

From the equations for the vector potential, we find that there are two plane wave solutions with two different polarizations as indicated by the index σιγμα.