PHY 742 Quantum Mechanics II
1-1:50 AM MWF via video link:
https://wakeforest-university.zoom.us/my/natalie.holzwarth

Extra notes for Lecture 23

Quantization of the Electromagnetic fields

Complete the reading of XVII. Quantizing Electromagnetic Fields.

1. **Quantum Hamiltonian for the electromagnetic fields**
2. **Eigenstates of the electromagnetic Hamiltonian**
3. **Quantum expressions for the electromagnetic fields**
4. **Variance of measurable properties of the electromagnetic fields**
5. **Properties of a single mode coherent state**

Please finish reading Chapter 17 of Professor Carlson's textbook.

21	Mon: 03/23/2020	Chap. 17	Quantization of the Electromagnetic Field	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 17	Quantization of the Electromagnetic Field	#18	03/27/2020
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27	Mon: 04/06/2020				
28	Wed: 04/08/2020				
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020				
30	Wed: 04/15/2020				
31	Fri: 04/17/2020				
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The assigned homework for today's lecture involves verifying some of the equations discussed in this lecture.

Your questions???

Apologies for posting the lecture late last night and then not completing the link correctly (It was corrected before 10 am this morning, but no excuse for me.) I am trying to do better, but perhaps you could also post questions from the reading materials?? Other suggestions?

Summary of previous results for the electromagnetic Hamiltonian

In terms of the operators $a_{\mathbf{k}\sigma}$ and $a^{\dagger}_{\mathbf{k}\sigma}$ operators for wavevector $\mathbf{k}$ and polarization σ.

With commutation relations: $\left[a_{\mathbf{k}\sigma}, a^{\dagger}_{\mathbf{k}'\sigma'}\right] = \delta_{\mathbf{k}\mathbf{k}'}\delta_{\sigma\sigma'}$ $\left[a_{\mathbf{k}\sigma}, a_{\mathbf{k}'\sigma'}\right] = 0$ $\left[a^{\dagger}_{\mathbf{k}\sigma}, a^{\dagger}_{\mathbf{k}'\sigma'}\right] = 0$

The eigenstates of the EM Field Hamiltonian (omitting diverging term) are integers $n_{\mathbf{k}\sigma}$:

$$H^{\text{fixed}}_{\text{field}} \left| n_{\mathbf{k}\sigma} \right\rangle = \sum_{\mathbf{k}'\sigma'} \left(\hbar\omega_{\mathbf{k}'} a^{\dagger}_{\mathbf{k}'\sigma'} a_{\mathbf{k}'\sigma'} \right) \left| n_{\mathbf{k}\sigma} \right\rangle = \hbar\omega_{\mathbf{k}} n_{\mathbf{k}\sigma} \left| n_{\mathbf{k}\sigma} \right\rangle$$

It is convenient to define the photon number operator

$\mathbf{N}_{\mathbf{k}'\sigma'} \equiv a^{\dagger}_{\mathbf{k}'\sigma'} a_{\mathbf{k}'\sigma'}$ such that $\mathbf{N}_{\mathbf{k}\sigma} \left| n_{\mathbf{k}\sigma} \right\rangle = n_{\mathbf{k}\sigma} \left| n_{\mathbf{k}\sigma} \right\rangle$

The is a review of equations discussed in Lecture 22.

Properties of the creation and annihilation operators:

$$a_{\mathbf{k}\sigma}\left|n_{\mathbf{k}\sigma}\right\rangle = \sqrt{n_{\mathbf{k}\sigma}}\left|n_{\mathbf{k}\sigma}-1\right\rangle$$

$$a^{\dagger}_{\mathbf{k}\sigma}\left|n_{\mathbf{k}\sigma}\right\rangle = \sqrt{n_{\mathbf{k}\sigma}+1}\left|n_{\mathbf{k}\sigma}+1\right\rangle$$

Quantum mechanical form of vector potential --

$$\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} + a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

Note: We are assuming that the polarization vector is real.

Continuing review of previous results.

Quantum mechanical form of vector potential and corresponding fields --

$$\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}\sigma} \sqrt{\frac{\hbar}{2V\epsilon_0\omega_\mathbf{k}}} \boldsymbol{\varepsilon}_{\mathbf{k}\sigma} \left(a_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_\mathbf{k}t} + a^\dagger_{\mathbf{k}\sigma} e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_\mathbf{k}t)} \right)$$

Electric field:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad \Rightarrow \mathbf{E}(\mathbf{r},t) = i\sum_{\mathbf{k}\sigma} \sqrt{\frac{\hbar\omega_\mathbf{k}}{2V\epsilon_0}} \boldsymbol{\varepsilon}_{\mathbf{k}\sigma} \left(a_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_\mathbf{k}t} - a^\dagger_{\mathbf{k}\sigma} e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_\mathbf{k}t)} \right)$$

Magnetic field:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \Rightarrow \mathbf{B}(\mathbf{r},t) = i\sum_{\mathbf{k}\sigma} \sqrt{\frac{\hbar}{2V\epsilon_0\omega_\mathbf{k}}} \mathbf{k} \times \boldsymbol{\varepsilon}_{\mathbf{k}\sigma} \left(a_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_\mathbf{k}t} - a^\dagger_{\mathbf{k}\sigma} e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_\mathbf{k}t)} \right)$$

From the quantum expression of the vector potential, we can also write expressions for the electric and magnetic fields.

Embarassing/puzzling expectation values --

$$\langle n_{\mathbf{k}'\sigma'} | \mathbf{A}(\mathbf{r},t) | n_{\mathbf{k}'\sigma'} \rangle = \sum_{\mathbf{k}\sigma} \sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}} \boldsymbol{\varepsilon}_{\mathbf{k}\sigma} \langle n_{\mathbf{k}'\sigma'} | \left(a_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} + a^{\dagger}_{\mathbf{k}\sigma} e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)} \right) | n_{\mathbf{k}'\sigma'} \rangle = 0$$

Electric field:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad \Rightarrow \langle n_{\mathbf{k}'\sigma'} | \mathbf{E}(\mathbf{r},t) | n_{\mathbf{k}'\sigma'} \rangle = i \sum_{\mathbf{k}\sigma} \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}} \boldsymbol{\varepsilon}_{\mathbf{k}\sigma} \langle n_{\mathbf{k}'\sigma'} | \left(a_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - a^{\dagger}_{\mathbf{k}\sigma} e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)} \right) | n_{\mathbf{k}'\sigma'} \rangle = 0$$

Magnetic field:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \Rightarrow \langle n_{\mathbf{k}'\sigma'} | \mathbf{B}(\mathbf{r},t) | n_{\mathbf{k}'\sigma'} \rangle = i \sum_{\mathbf{k}\sigma} \sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}} \mathbf{k} \times \boldsymbol{\varepsilon}_{\mathbf{k}\sigma} \langle n_{\mathbf{k}'\sigma'} | \left(a_{\mathbf{k}\sigma} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - a^{\dagger}_{\mathbf{k}\sigma} e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)} \right) | n_{\mathbf{k}'\sigma'} \rangle = 0$$

Consider evaluating the expectation values of these fields for a pure photon eigenstate. Embarrassingly, they are 0.

In order to compare the classical treatment to the quantum approach we need to calculate expectation values of the observables. In addition to mean value of an observable, its statistical properties are also of interest, particularly the variance and the standard deviation (its square root) which is defined in terms of the average of the squared value of the observable and the average value itself:

$$\text{Standard deviation:} \quad \Delta V \equiv \sqrt{\left\langle V^2 \right\rangle - \left|\left\langle V \right\rangle\right|^2}$$

The next few slides review the relationship of this variance to observables in quantum mechanics which have non trivial commutation relationships and thus have built in variance values.

Digression -- Commutator formalism in quantum mechanics

Definition:

Given two Hermitian operators A and B, their commutator is

$[A,B] \equiv AB - BA$

Theorem:

Given Hermitian operators A, B, C such that

$[A,B] = iC,$

it follows that $\Delta A \Delta B \geq \frac{1}{2}\left|\langle C \rangle\right|$

In order to understand how the previous results can be true, we need to review the notion of variance in quantum mechanics. In particular, the variance often is controlled by non-trivial commutation relations. In this slide and the following, the relationship between variance and commutators is reviewed.

Proof --

Note that:

$$[A,B]^\dagger = (iC)^\dagger$$

$$(AB - BA)^\dagger = B^\dagger A^\dagger - A^\dagger B^\dagger = -iC^\dagger$$

$$= BA - AB = -iC$$

Calculation of the variance:

$$(\Delta A)^2 \equiv \langle \psi | (A - \langle A \rangle)^2 | \psi \rangle$$
$$= \langle (A - \langle A \rangle)\psi | (A - \langle A \rangle)\psi \rangle$$

Similarly,

$$(\Delta B)^2 \equiv \langle \psi | (B - \langle B \rangle)^2 | \psi \rangle$$
$$= \langle (B - \langle B \rangle)\psi | (B - \langle B \rangle)\psi \rangle$$

Define $|\psi_A\rangle \equiv |(A - \langle A \rangle)\psi\rangle$

$$|\psi_B\rangle \equiv |(B - \langle B \rangle)\psi\rangle$$

Schwarz inequality:

$$\langle \psi_A | \psi_A \rangle \langle \psi_B | \psi_B \rangle \geq |\langle \psi_A | \psi_B \rangle|^2$$

Define $\left|\psi_A\right\rangle \equiv \left|\left(A-\langle A\rangle\right)\psi\right\rangle$ and $\left|\psi_B\right\rangle \equiv \left|\left(B-\langle B\rangle\right)\psi\right\rangle$

Schwarz inequality:

$$\langle\psi_A|\psi_A\rangle\langle\psi_B|\psi_B\rangle \geq \left|\langle\psi_A|\psi_B\rangle\right|^2$$

$$\langle\psi_A|\psi_B\rangle = \langle\psi|\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right)|\psi\rangle$$

$$\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right) = \frac{1}{2}\left(\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right)+\left(B-\langle B\rangle\right)\left(A-\langle A\rangle\right)\right)$$

$$+\frac{1}{2}\left(\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right)-\left(B-\langle B\rangle\right)\left(A-\langle A\rangle\right)\right)$$

$$= F+\frac{i}{2}C$$

$$\langle \psi_A | \psi_B \rangle = \langle \psi | \left(A - \langle A \rangle \right) \left(B - \langle B \rangle \right) | \psi \rangle \qquad = \langle \psi | F | \psi \rangle + \frac{i}{2} \langle \psi | C | \psi \rangle$$

$$\left| \langle \psi_A | \psi_B \rangle \right|^2 = \left| \langle \psi | F | \psi \rangle \right|^2 + \frac{1}{4} \left| \langle \psi | C | \psi \rangle \right|^2 \geq \frac{1}{4} \left| \langle \psi | C | \psi \rangle \right|^2$$

Putting it all together:

$$\langle \psi_A | \psi_A \rangle \langle \psi_B | \psi_B \rangle \geq \left| \langle \psi_A | \psi_B \rangle \right|^2 \geq \frac{1}{4} \left| \langle \psi | C | \psi \rangle \right|^2$$

$$\Rightarrow \left(\Delta A \right)^2 \left(\Delta B \right)^2 \geq \frac{1}{4} \left| \langle C \rangle \right|^2$$

Therefore: $[A, B] = iC$ implies $\Delta A \Delta B \geq \frac{1}{2} \left| \langle C \rangle \right|$

Example: $A = X, \quad B = P$

$[X, P] = i\hbar$ implies $\Delta X \Delta P \geq \frac{\hbar}{2}$

Wrapping up the commutator discussion with the example of the uncertainty principle applied to position and momentum.

What does this have to do with quantum EM fields?

In fact, your textbook shows that although

$$\langle n_{\mathbf{k}'\sigma'}|\mathbf{E}(\mathbf{r},t)|n_{\mathbf{k}'\sigma'}\rangle = 0 \quad \text{and} \quad \langle n_{\mathbf{k}'\sigma'}|\mathbf{B}(\mathbf{r},t)|n_{\mathbf{k}'\sigma'}\rangle = 0,$$

the variances of the fields are both infinite for a pure eigenstate --

$$\langle 0|\mathbf{E}^2(\mathbf{r})|0\rangle = |\mathbf{E}(\mathbf{r})|0\rangle|^2 = \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k}\sigma}\sum_{\mathbf{k}'\sigma'}\sqrt{\omega_k\omega_{k'}}\left(\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\cdot\boldsymbol{\varepsilon}^*_{\mathbf{k}'\sigma'}\right)e^{i\mathbf{k}\cdot\mathbf{r}-i\mathbf{k}'\cdot\mathbf{r}}\langle 1,\mathbf{k},\sigma|1,\mathbf{k}',\sigma'\rangle$$

$$= \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k}\sigma}\omega_k = \frac{\hbar c}{\varepsilon_0 V}\sum_{\mathbf{k}} k = \frac{\hbar c}{\varepsilon_0}\int\frac{d^3\mathbf{k}}{(2\pi)^3}k, \qquad (17.19a)$$

infinite

$$\langle 0|\mathbf{B}^2(\mathbf{r})|0\rangle = |\mathbf{B}(\mathbf{r})|0\rangle|^2 = \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k},\sigma}\sum_{\mathbf{k}',\sigma'}\frac{e^{i\mathbf{k}\cdot\mathbf{r}-i\mathbf{k}'\cdot\mathbf{r}}}{\sqrt{\omega_k\omega_{k'}}}\left(\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\right)\cdot\left(\mathbf{k}'\times\boldsymbol{\varepsilon}^*_{\mathbf{k}'\sigma'}\right)\langle 1,\mathbf{k},\sigma|1,\mathbf{k}',\sigma'\rangle$$

$$= \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k},\sigma}\frac{|\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}|^2}{\omega_k} = \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k},\sigma}\frac{k^2}{\omega_k} = \frac{\hbar}{\varepsilon_0 V c}\sum_{\mathbf{k}} k = \frac{\hbar}{\varepsilon_0 c}\int\frac{d^3\mathbf{k}}{(2\pi)^3}k, \qquad (17.19b)$$

infinite

For the E and B fields themselves, the variance is not a result of non trivial commutation relations. Here we calculate the variances for pure photon states.

It is also possible to show that components of the E and B field have nontrivial commutation relations, indicating that in general it is not possible to simultaneously determine E and B at the same point in space to arbitrary accuracy.

Effects of the phase of each mode.

In deriving these equations, we neglected the phase of each mode. A more careful treatment of photon number and phase show that these also have nontrivial commutation relations.

How is this quantum treatment of the electromagnetic fields consistent with the classical picture?

1. **There is no need for consistency.?**
2. **There should be consistency in certain ranges of the parameters.?**

Summary of what we have learned so far. What do you think about how the quantum equations could be related to the classical picture?

Gauber's coherent state: $\left|c_{\alpha}\right\rangle \equiv \sum_{n=0}^{\infty} \frac{\alpha^{n} e^{-|\alpha|^{2}/2}}{\sqrt{n!}}|n\rangle$ based on a single mode $n \rightarrow n_{\mathbf{k}\sigma}$

Electric field: $\left\langle c_{\alpha}\right|\mathbf{E}(\mathbf{r},t)\left|c_{\alpha}\right\rangle = i\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_{0}}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(\alpha_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - \alpha_{\mathbf{k}\sigma}^{*}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$

Magnetic field: $\left\langle c_{\alpha}\right|\mathbf{B}(\mathbf{r},t)\left|c_{\alpha}\right\rangle = i\sqrt{\frac{\hbar}{2V\epsilon_{0}\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(\alpha_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - \alpha_{\mathbf{k}\sigma}^{*}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$

Note that α is a complex number which can be written in terms of a real amplitude and phase: E_0 and ψ :

$$\left\langle c_{\alpha}\right|\mathbf{E}(\mathbf{r},t)\left|c_{\alpha}\right\rangle = -2\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_{0}}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}E_{0}\sin\left(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t+\psi\right)$$

$$\left\langle c_{\alpha}\right|\mathbf{B}(\mathbf{r},t)\left|c_{\alpha}\right\rangle = -2\sqrt{\frac{\hbar}{2V\epsilon_{0}\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}E_{0}\sin\left(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t+\psi\right)$$

Here we are assuming that

$\alpha = E_0 e^{i\psi}$

Here we introduce the single mode coherent state as a particular linear combination of eigenstates of the electromagnetic Hamiltonian.

Single mode coherent state continued

It can also be shown that

$$\left\langle c_\alpha \left| \left| \mathbf{E}(\mathbf{r},t) \right|^2 \right| c_\alpha \right\rangle = \frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}\left(4E_0^2 \sin^2(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t+\psi)+1\right)$$

Therefore

$$\left\langle c_\alpha \left| \left| \mathbf{E}(\mathbf{r},t) \right|^2 \right| c_\alpha \right\rangle - \left| \left\langle c_\alpha \left| \mathbf{E}(\mathbf{r},t) \right| c_\alpha \right\rangle \right|^2 = \frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}$$

This means that variance of the E field for the coherent state is independent of the amplitude E_0. Therefore, for large E_0 the variance is small in comparison.

For these coherent states, we can evaluate the variance of the quantum mechanical electric field. You should verify these equations for your homework.

Single mode coherent state continued

Now consider the expectation values of the number operator and its square:

$$\mathbf{N}_{\mathbf{k}\sigma} \equiv a^{\dagger}_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}$$

$$\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle = |\alpha|^2 \qquad \langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma}\mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle = |\alpha|^4 + |\alpha|^2$$

Square of the variance: $\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma}\mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle - \left| \langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle \right|^2 = |\alpha|^2$

Fractional uncertainty in the number of photons for the coherent state:

$$\frac{\sqrt{\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma}\mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle - \left| \langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle \right|^2}}{\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle} = \frac{1}{|\alpha|}$$

Again using the coherent states, we can evaluate the variance of the photon number. What do you think is the significance of these results?

Interpretation of a single mode coherent state

$$\left|c_\alpha\right\rangle \equiv \sum_{n=0}^{\infty} \frac{\alpha^n e^{-|\alpha|^2/2}}{\sqrt{n!}}\left|n\right\rangle \quad \text{based on a single mode } n \to n_{\mathbf{k}\sigma}$$

The probability of finding n photons in this state is given by:

$$\left|\left\langle n \middle| c_\alpha \right\rangle\right|^2 = \frac{|\alpha|^{2n} e^{-|\alpha|^2}}{n!}$$

This is the form of a Poisson distribution for a mean value of $|\alpha|^2$.

Here we see that the coherent state is related to a Poisson distribution, important in statistical analysis.

More reading -- **page 231**

REVIEWS OF

MODERN PHYSICS

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Coherence Properties of Optical Fields*

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This article presents a review of coherence properties of electromagnetic fields and their measurements, with special emphasis on the optical region of the spectrum. Analyses based on both the classical and quantum theories are described. After a brief historical introduction, the elementary concepts which are frequently employed in the discussion of interference phenomena are summarized. The measure of second-order coherence is then introduced in connection with the analysis of a simple interference experiment and some of the more important second-order coherence effects are studied. Their uses in stellar interferometry and interference spectroscopy are described. Analysis of partial polarization from the standpoint of correlation theory is also outlined. The general statistical description of the field is discussed in some detail. The recently discovered universal "diagonal" representation of the density operator for free fields is also considered and it is shown how, with the help of the associated generalized phase-space distribution function, the quantum-mechanical correlation functions may be expressed in the same form as the classical ones. The sections which follow deal with the statistical properties of thermal and nonthermal light, and with the temporal and spatial coherence of black-body radiation. Later sections, dealing with fourth- and higher-order coherence effects include a discussion of the photoelectric detection process. Among the fourth-order effects described in detail are bunching phenomena, the Hanbury Brown–Twiss effect and its application to astronomy. The article concludes with a discussion of various transient superposition effects, such as light beats and interference fringes produced by independent light beams.

There are many more interesting aspects of the statistical properties of quantum electromagnetic fields. Here is an example of an interesting review article.