

$$\begin{aligned}
&> \text{assume}(Z > 0); \text{assume}(a > 0); \\
&> f1s := r \rightarrow \frac{1}{\sqrt{\pi}} \cdot \left(\frac{Z}{a} \right)^{\frac{3}{2}} \exp\left(-\frac{Z \cdot r}{a}\right); \\
&\quad f1s := r \mapsto \frac{\left(\frac{Z}{a}\right)^{\frac{3}{2}} e^{-\frac{Zr}{a}}}{\sqrt{\pi}}
\end{aligned} \tag{1}$$

$$\begin{aligned}
&> f2s := r \rightarrow \frac{1}{\sqrt{32\pi}} \cdot \left(\frac{Z}{a} \right)^{\frac{3}{2}} \cdot \left(2 - \frac{Z \cdot r}{a} \right) \exp\left(-\frac{Z \cdot r}{2 \cdot a}\right); \\
&\quad f2s := r \mapsto \frac{\left(\frac{Z}{a}\right)^{\frac{3}{2}} \left(2 - \frac{Zr}{a}\right) e^{-\frac{Zr}{2a}}}{\sqrt{32\pi}}
\end{aligned} \tag{2}$$

$$\begin{aligned}
&> v1 := (4 \cdot \pi)^2 \cdot (int(x \cdot (f1s(x))^2 \cdot int(y^2 \cdot (f2s(y))^2, y=0..x), x=0..infinity) + int(x^2 \cdot (f1s(x))^2 \\
&\quad \cdot int(y \cdot (f2s(y))^2, y=x..infinity), x=0..infinity));
&\quad v1 := \frac{17 Z \sim}{81 a \sim}
\end{aligned} \tag{3}$$

$$\begin{aligned}
&> v2 := (4 \cdot \pi)^2 \cdot (int(x \cdot (f1s(x) \cdot f2s(x)) \cdot int(y^2 \cdot (f1s(y) \cdot f2s(y)), y=0..x), x=0..infinity) \\
&\quad + int(x^2 \cdot (f1s(x) \cdot f2s(x)) \cdot int(y \cdot (f1s(y) \cdot f2s(y)), y=x..infinity), x=0..infinity));
&\quad v2 := \frac{16 Z \sim}{729 a \sim}
\end{aligned} \tag{4}$$

Checking --

$$\begin{aligned}
&> (4 \cdot \pi)^2 \cdot (int(x \cdot (f1s(x))^2 \cdot int(y^2 \cdot (f1s(y))^2, y=0..x), x=0..infinity) + int(x^2 \cdot (f1s(x))^2 \cdot int(y \\
&\quad \cdot (f1s(y))^2, y=x..infinity), x=0..infinity));
&\quad \frac{5 Z \sim}{8 a \sim}
\end{aligned} \tag{5}$$

$$\begin{aligned}
&> (4 \cdot \pi) \cdot int(x^2 \cdot (f1s(x))^2, x=0..infinity);
&\quad 1
\end{aligned} \tag{6}$$

$$\begin{aligned}
&> (4 \cdot \pi) \cdot int(x^2 \cdot (f2s(x))^2, x=0..infinity);
&\quad 1
\end{aligned} \tag{7}$$

$$\begin{aligned}
&> TE := -4 - 1 + \frac{68}{81} - \frac{64}{729}; TS := -4 - 1 + \frac{68}{81} + \frac{64}{729} \\
&\quad TE := -\frac{3097}{729} \\
&\quad TS := -\frac{2969}{729}
\end{aligned} \tag{8}$$

$$\begin{aligned}
&> evalf(%);
&\quad -4.072702332
\end{aligned} \tag{9}$$

$$\begin{aligned}
&> TE \cdot 13.605693122994 \\
&\quad -57.80086638
\end{aligned} \tag{10}$$

$$\begin{aligned} > GS := -4 - 4 + \frac{20}{8}; \\ & GS := -\frac{11}{2} \end{aligned} \tag{11}$$

$$\begin{aligned} > GS \cdot 13.605693122994; \\ & -74.83131216 \end{aligned} \tag{12}$$

$$\begin{aligned} > (TS - GS) \cdot 13.605693122994 \\ & 19.41937406 \end{aligned} \tag{13}$$

$$\begin{aligned} > (TE - GS) \cdot 13.605693122994 \\ & 17.03044578 \end{aligned} \tag{14}$$

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