```
> assume(Z > 0); assume(a > 0);
```

```
> f1s := r -> 1/sqrt(Pi) * (Z/a)^(3/2) * exp(-Z*r/a);
```

$$f1s := r \mapsto \frac{\left(\frac{Z}{a}\right)^{3/2} e^{-\frac{Z r}{a}}}{\sqrt{\pi}} \tag{1}$$

```
> f2s := r -> 1/sqrt(32*Pi) * (Z/a)^(3/2) * (2 - Z*r/a) * exp(-Z*r/(2*a));
```

$$f2s := r \mapsto \frac{\left(\frac{Z}{a}\right)^{3/2} \left(2 - \frac{Z r}{a}\right) e^{-\frac{Z r}{2 a}}}{\sqrt{32\,\pi}} \tag{2}$$

```
> v1 := (4*Pi)^2 * (int(x*(f1s(x))^2 * int(y^2*(f2s(y))^2, y=0..x), x=0..infinity)
    + int(x^2*(f1s(x))^2 * int(y*(f2s(y))^2, y=x..infinity), x=0..infinity));
```

$$v1 := \frac{17\,Z\sim}{81\,a\sim} \tag{3}$$

```
> v2 := (4*Pi)^2 * (int(x*(f1s(x)*f2s(x)) * int(y^2*(f1s(y)*f2s(y)), y=0..x), x=0..infinity)
    + int(x^2*(f1s(x)*f2s(x)) * int(y*(f1s(y)*f2s(y)), y=x..infinity), x=0..infinity));
```

$$v2 := \frac{16\,Z\sim}{729\,a\sim} \tag{4}$$

Checking --

```
> (4*Pi)^2 * (int(x*(f1s(x))^2 * int(y^2*(f1s(y))^2, y=0..x), x=0..infinity) + int(x^2*(f1s(x))^2 * int(y
    *(f1s(y))^2, y=x..infinity), x=0..infinity));
```

$$\frac{5\,Z\sim}{8\,a\sim} \tag{5}$$

```
> (4*Pi) * int(x^2*(f1s(x))^2, x=0..infinity);
```

$$1 \tag{6}$$

```
> (4*Pi) * int(x^2*(f2s(x))^2, x=0..infinity);
```

$$1 \tag{7}$$

```
> TE := -4 - 1 + 68/81 - 64/729; TS := -4 - 1 + 68/81 + 64/729
```

$$TE := -\frac{3097}{729}$$

$$TS := -\frac{2969}{729} \tag{8}$$

```
> evalf(%);
```

$$-4.072702332 \tag{9}$$

```
> TE * 13.605693122994
```

$$-57.80086638 \tag{10}$$

> $GS := -4 - 4 + \frac{20}{8};$

$$GS := -\frac{11}{2} \quad \textbf{(11)}$$

> $GS \cdot 13.605693122994;$

$$-74.83131216 \quad \textbf{(12)}$$

> $(TS - GS) \cdot 13.605693122994$

$$19.41937406 \quad \textbf{(13)}$$

> $(TE - GS) \cdot 13.605693122994$

$$17.03044578 \quad \textbf{(14)}$$

>