PHY 341/641 Thermodynamics and Statistical Mechanics
MWF: Online at 12 PM & FTF at 2 PM

Record!!!

Discussion for Lecture 12:

Variations in the number of particles
Begin discussion of heat engines

Reading: Chapters 4.1-4.2

1. **Details of Carnot cycle (using consistent PV and ST diagrams)**
2. **Efficiency of engines**
3. **Efficiency of refrigerators and heat pumps**

Course schedule for Spring 2021

ary schedule -- subject to frequent adjustment.) Reading assignments are for the **An Introduction to** **Physics** by Daniel V. Schroeder. The HW assignment numbers refer to problems in that text.

	Lecture date	Reading	Topic	HW	Due date
1	Wed: 01/27/2021	Chap. 1.1-1.3	Introduction and ideal gas equations	1.21	01/29/2021
2	Fri: 01/29/2021	Chap. 1.2-1.4	First law of thermodynamics	1.17	02/03/2021
3	Mon: 02/01/2021	Chap. 1.5-1.6	Work and heat for an ideal gas		
4	Wed: 02/03/2021	Chap. 1.1-1.6	Review of energy, heat, and work	1.45	02/05/2021
5	Fri: 02/05/2021	Chap. 2.1-2.2	Aspects of entropy		
6	Mon: 02/08/2021	Chap. 2.3-2.4	Multiplicity distributions	2.24	02/10/2021
7	Wed: 02/10/2021	Chap. 2.5-2.6	Entropy and macrostate multiplicity	2.26	02/12/2021
8	Fri: 02/12/2021	Chap. 2.1-2.6	Review of entropy and macrostates	2.32	02/15/2021
9	Mon: 02/15/2021	Chap. 3.1-3.2	Temperature, entropy, heat	3.10a-b	02/17/2021
10	Wed: 02/17/2021	Chap. 3.3-3.4	Temperature, entropy, heat	3.23	02/19/2021
11	Fri: 02/19/2021	Chap. 3.5-3.6	Temperature, entropy, heat	3.28	02/22/2021
12	Mon: 02/22/2021	Chap. 4.1-4.3	Ideal engines and refrigerators	4.1	02/24/2021
13	Wed: 02/24/2021	Chap. 4.3-4.4	Real engines and refrigerators	4.20	02/26/2021
14	Fri: 02/26/2021	Chap. 5.1	Free energy		
15	Mon: 03/01/2021				

Your questions –

From Parker -- My question for this reading is.what is what does the compression ratio gamma mean, it comes into play for an adiabatic exponent? What does it physically correspond to? Comment – gamma=C_p/C_v. Compression ratio is something else.

From Michael -- Why do more cars not have diesel engines if they are more efficient than combustible engines?

From Kristen -- 1. Could you explain why exactly the Carnot cycle is the most efficient? Because with the formula 1-Tc/Th for efficiency, would it mean that Tc was almost zero? 2. Why is it that refrigerators can be much more "efficient" than engines? 3. How come in the Internal Combustion Engine you can simply get rid of the cold reservoir?

Your questions – continued

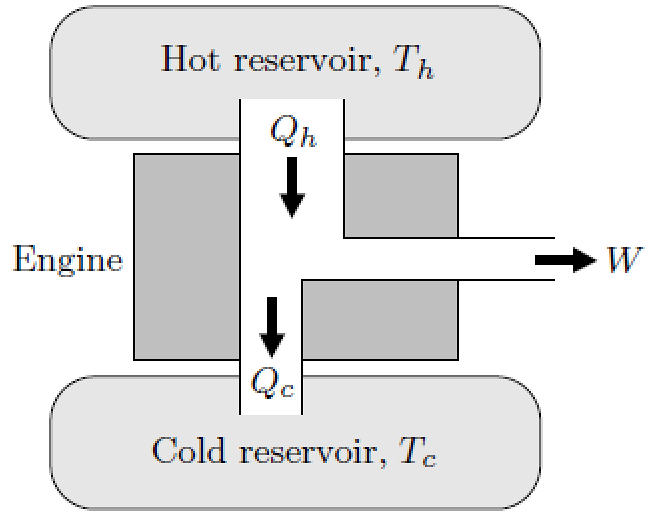
From Annelise -- why are the engines we have so inefficient? Is it because no one has figured out how to make the engines run more efficiently, or are they the way they are because of the laws of thermodynamics?

From Rich -- Is the Carnot cycle efficiency ever truly attainable since the temperature differences must be so marginally small and unchanging, or is this efficiency only true at a limit?

From Chao -- For the Otto circle, why does it happen that in the ignition process, the pressure increase dramatically, while the Volume stays constant?

Introduction to the thermo(statics) of heat engines

Figure 4.1. Energy-flow diagram for a heat engine. Energy enters as heat from the hot reservoir, and leaves both as work and as waste heat expelled to the cold reservoir. Copyright ©2000, Addison-Wesley.



In the engine, the desirable output is the net work while the necessary input is the heat input

Engine efficiency

$$\epsilon \equiv \frac{W_{net}}{Q_{in}}$$

Note that the net work of interest is the work done **by the system**.

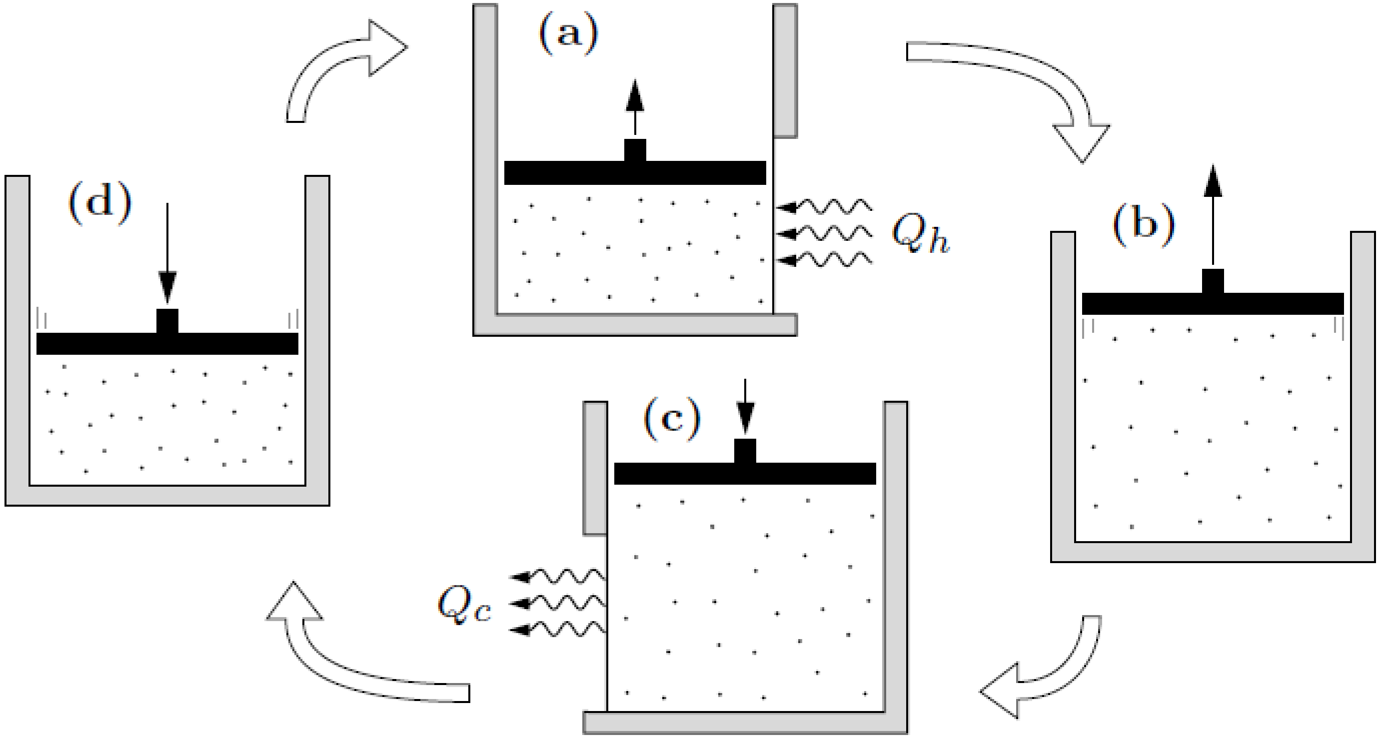
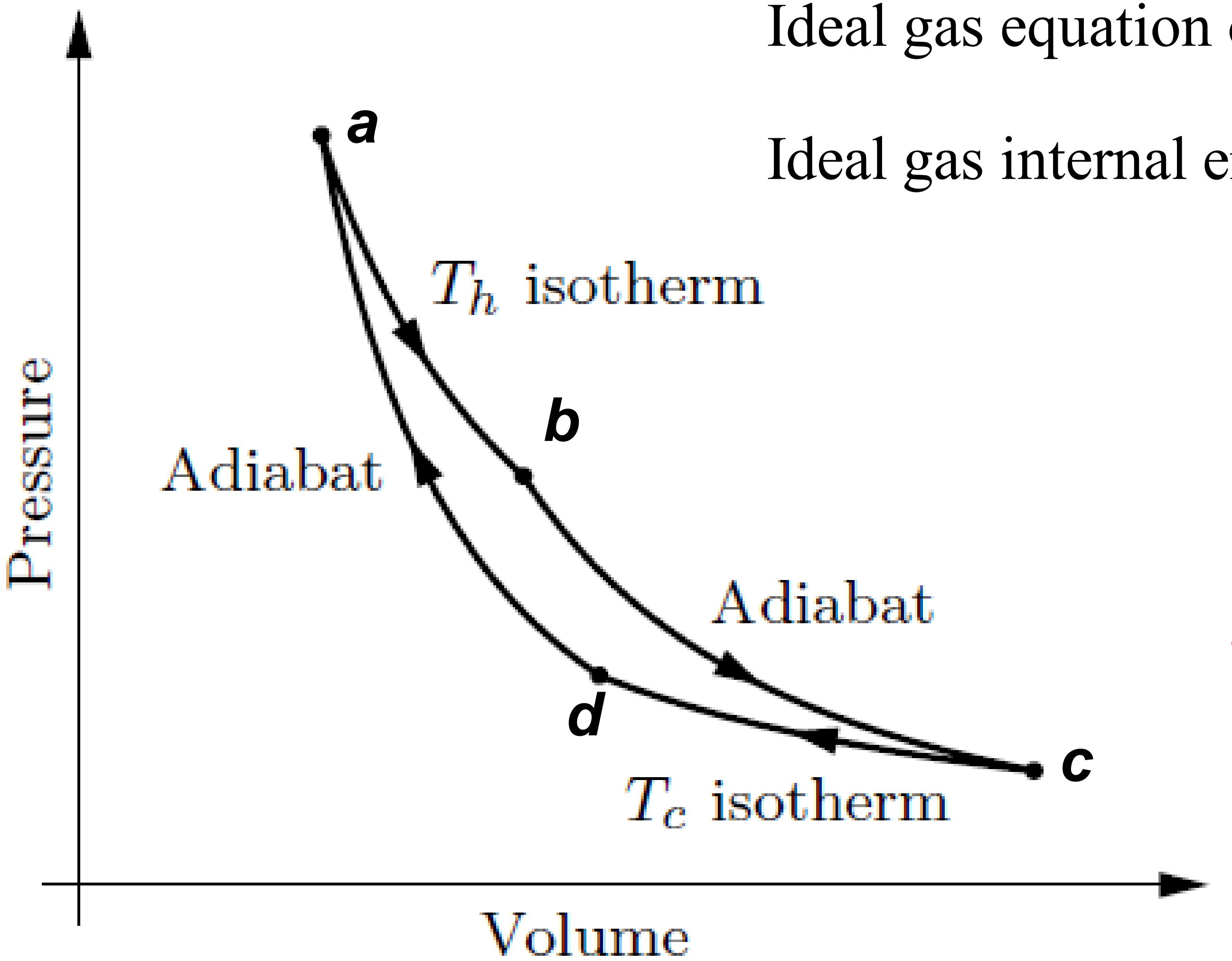


Figure 4.2. The four steps of a Carnot cycle: (a) isothermal expansion at T_h while absorbing heat; (b) adiabatic expansion to T_c; (c) isothermal compression at T_c while expelling heat; and (d) adiabatic compression back to T_h. The system must be put in thermal contact with the hot reservoir during step (a) and with the cold reservoir during step (c). Copyright ©2000, Addison-Wesley.

Note that in previous lectures, the ST diagrams used different numberings and notations. In this lecture we will label all steps consistently.

PV diagram from your textbook



Ideal gas relations:

Ideal gas equation of state $PV = Nk_B T$

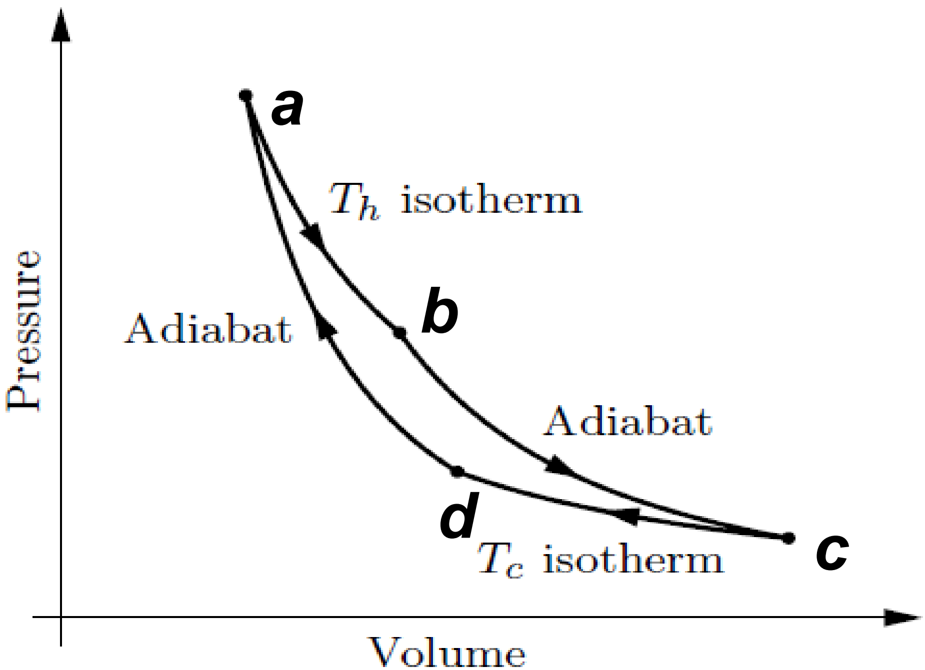
Ideal gas internal energy $U = \frac{Nk_B T}{\gamma - 1}$

Recall that

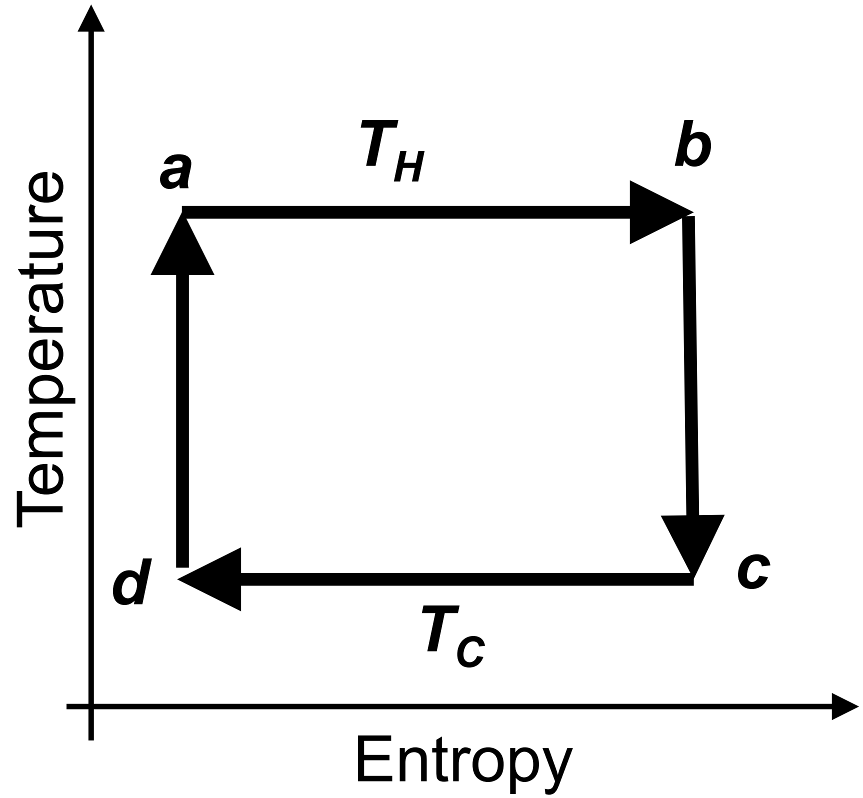
$$\gamma \equiv \frac{C_P}{C_V}$$

γ is an empirically measured quantity related to degrees of freedom of ideal gas

P versus V diagram

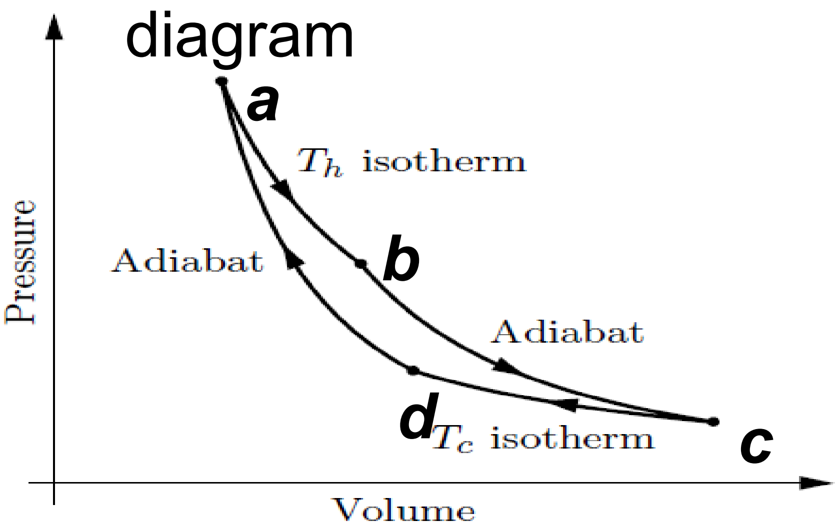


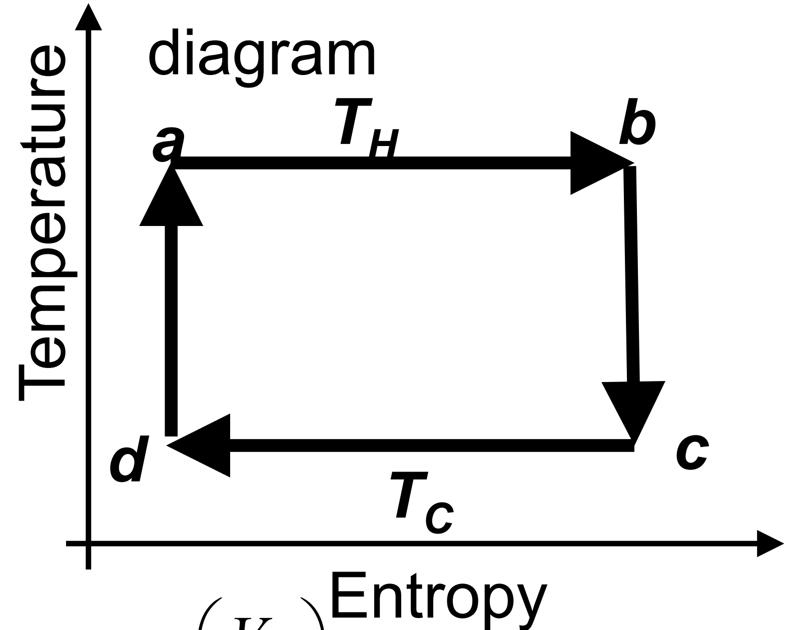
T versus S diagram



For step a → b: $Q_{ab} = -W_{ab} = Nk_B T_H \ln\left(\frac{V_b}{V_a}\right)$ For step b → c: $Q_{bc} = 0 \quad V_b T_H^{1/(\gamma-1)} = V_c T_C^{1/(\gamma-1)}$

For step c → d: $Q_{cd} = -W_{cd} = Nk_B T_C \ln\left(\frac{V_d}{V_c}\right)$ For step d → a: $Q_{da} = 0 \quad V_d T_C^{1/(\gamma-1)} = V_a T_H^{1/(\gamma-1)}$





$$Q_{ab} = Nk_B T_H \ln\left(\frac{V_b}{V_a}\right) \qquad Q_{cd} = Nk_B T_C \ln\left(\frac{V_d}{V_c}\right)$$

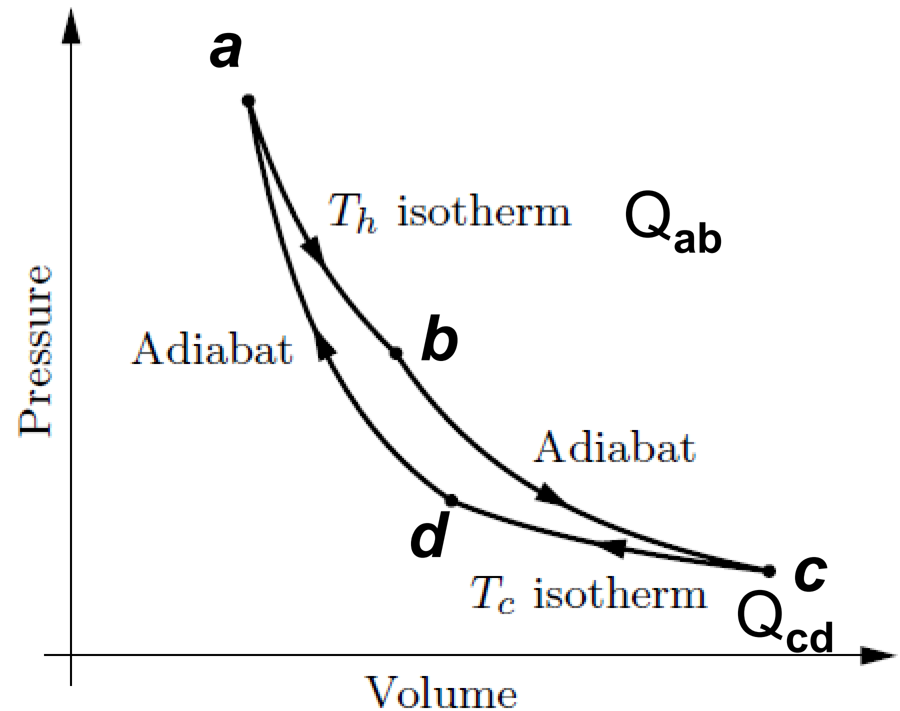
$$V_b T_H^{1/(\gamma-1)} = V_c T_C^{1/(\gamma-1)} \qquad V_d T_C^{1/(\gamma-1)} = V_a T_H^{1/(\gamma-1)} \quad \Rightarrow \left(\frac{T_H}{T_C}\right)^{1/(\gamma-1)} = \frac{V_c}{V_b} = \frac{V_d}{V_a}$$

$$\Rightarrow \frac{V_b}{V_a} = \frac{V_c}{V_d} \qquad \Rightarrow Q_{cd} = -Nk_B T_C \ln\left(\frac{V_b}{V_a}\right) = -Q_{ab}\frac{T_C}{T_H}$$

Engine efficiency

$$\epsilon \equiv \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}}$$

$$= \frac{Q_{ab} + Q_{cd}}{Q_{ab}}$$



For the Carnot cycle -- $Q_{cd} = -Q_{ab}\frac{T_C}{T_H}$

$$\epsilon = 1 - \frac{T_C}{T_H}$$

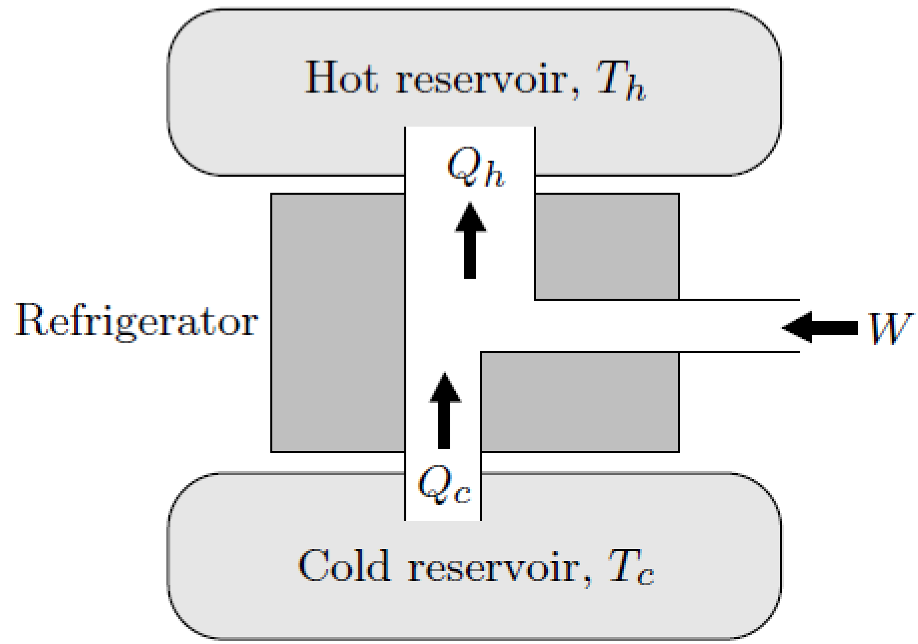
Example: Suppose $T_C = 300K$ and $T_H = 600K$

$$\epsilon = 1 - \frac{300}{600} = 0.5$$

Comment: Here we have assumed that there are no energy losses and that all of the processes are reversible.

Using the Carnot cycle for heating and cooling

Figure 4.4. Energy-flow diagram for a refrigerator or air conditioner. For a kitchen refrigerator, the space inside it is the cold reservoir and the space outside it is the hot reservoir. An electrically powered compressor supplies the work. Copyright ©2000, Addison-Wesley.



Coefficient of performance $$COP = \frac{|Q_C|}{|W_{net}|}$$

For Carnot cycle $$COP = \frac{|Q_{cd}|}{Q_{ab} + Q_{cd}} = \frac{T_C}{T_H - T_C}$$

Example COP for refrigerator having T_H=300K and T_C=280K -- COP=14

Question: For this refrigerator, suppose that Q_{cd} is 300 Watt s. What power is needed for an ideal compressor to achieve heat removal at a rate of 300 Watts

Coefficient of performance $$COP = \frac{|Q_C|}{|W_{net}|}$$

$$|W_{net}| = \frac{|Q_C|}{COP} \qquad \frac{d|W_{net}|}{dt} = \frac{1}{COP}\frac{d|Q_C|}{dt}$$

In this case: $$\frac{d|W_{net}|}{dt} = \frac{1}{14}\ 300\ \text{Watts} = 21\ \text{Watts}$$

Example based on problem 4.14

A heat pump is an electrical device that heats a building by pumping heat to from the cold outside. In other words, it's the same as a refrigerator, but its purpose is to warm the hot reservoir rather than to cool the cold reservoir (although it does both). Define the following symbols:

- T_H – temperature inside building
- T_C – temperature outside building
- Q_H – heat pumped into the building per day
- Q_C – heat pumped out of the building per day
- W – energy needed for mechanical pump per day

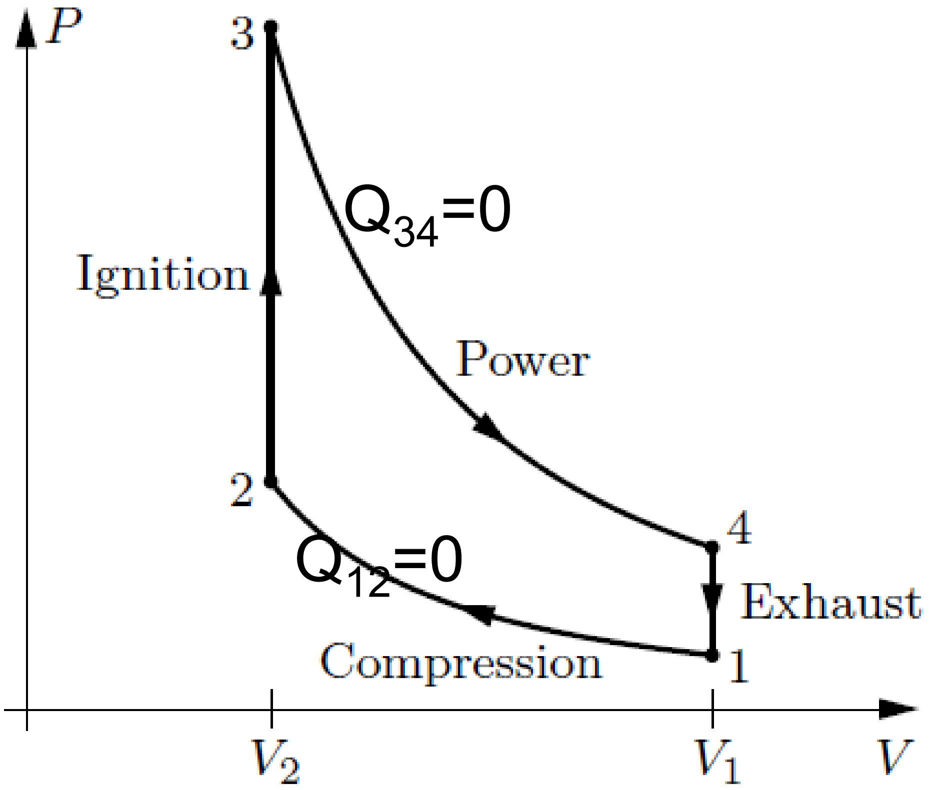
$$COP_{\text{Heat Pump}} = \frac{Q_H}{|W|} = \frac{T_H}{T_H - T_C} \quad \text{for a Carnot cycle}$$

$$\text{For} \quad T_H = 300K \text{ and } T_C = 273K \quad COP_{\text{Heat Pump}} = 11$$

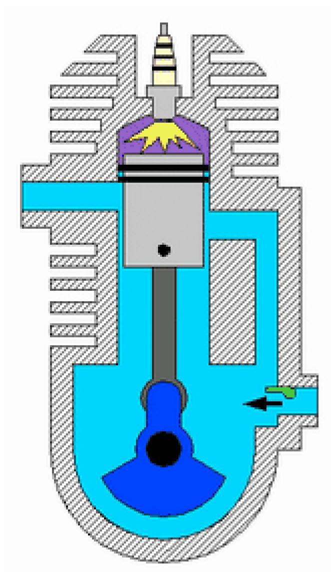
In practice, the performance is less than this….

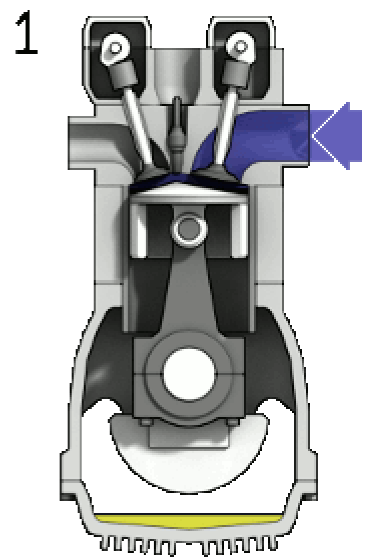
Another thermodynamic cycle example

Figure 4.5. The idealized Otto cycle, an approximation of what happens in a gasoline engine. In real engines the compression ratio V_1/V_2 is larger than shown here, typically 8 or 10. Copyright ©2000, Addison-Wesley.

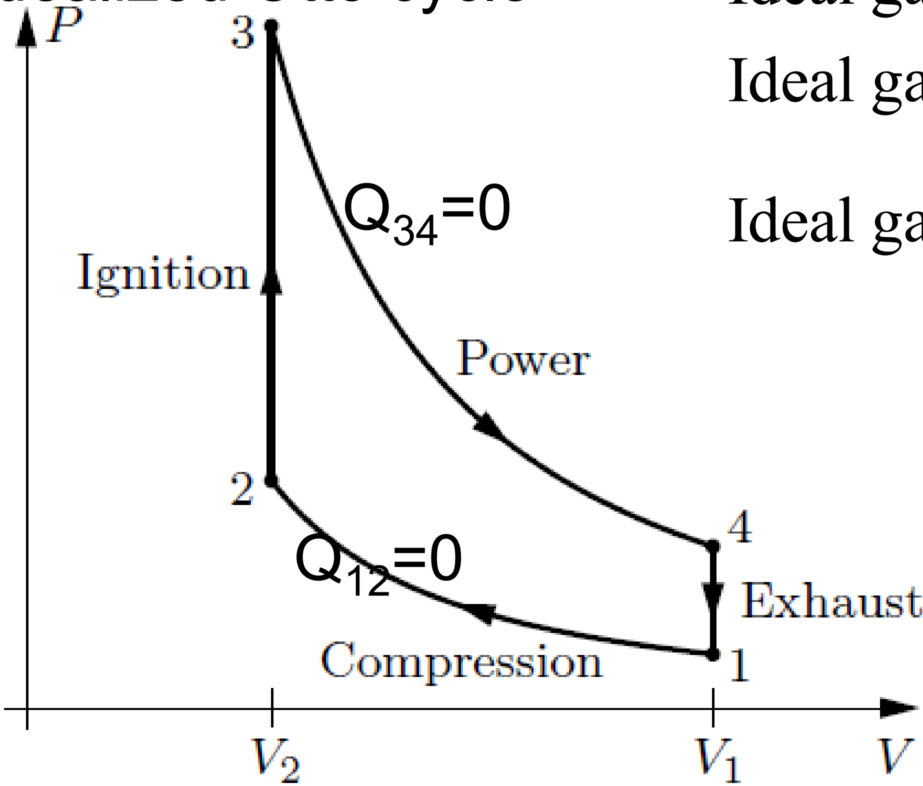


Animations of the Otto cycle from web page
https://energyeducation.ca/encyclopedia/Otto_cycle





Idealized Otto cycle



Ideal gas relations:

Ideal gas equation of state $PV = Nk_BT$

Ideal gas internal energy $U = \dfrac{Nk_BT}{\gamma - 1}$

$$Q_{23} = U_{23} = \frac{Nk_B}{\gamma - 1}(T_3 - T_2)$$

$$Q_{41} = U_{41} = \frac{Nk_B}{\gamma - 1}(T_1 - T_4)$$

$$Q_{12} = 0 \quad V_1 T_1^{1/(\gamma-1)} = V_2 T_2^{1/(\gamma-1)}$$

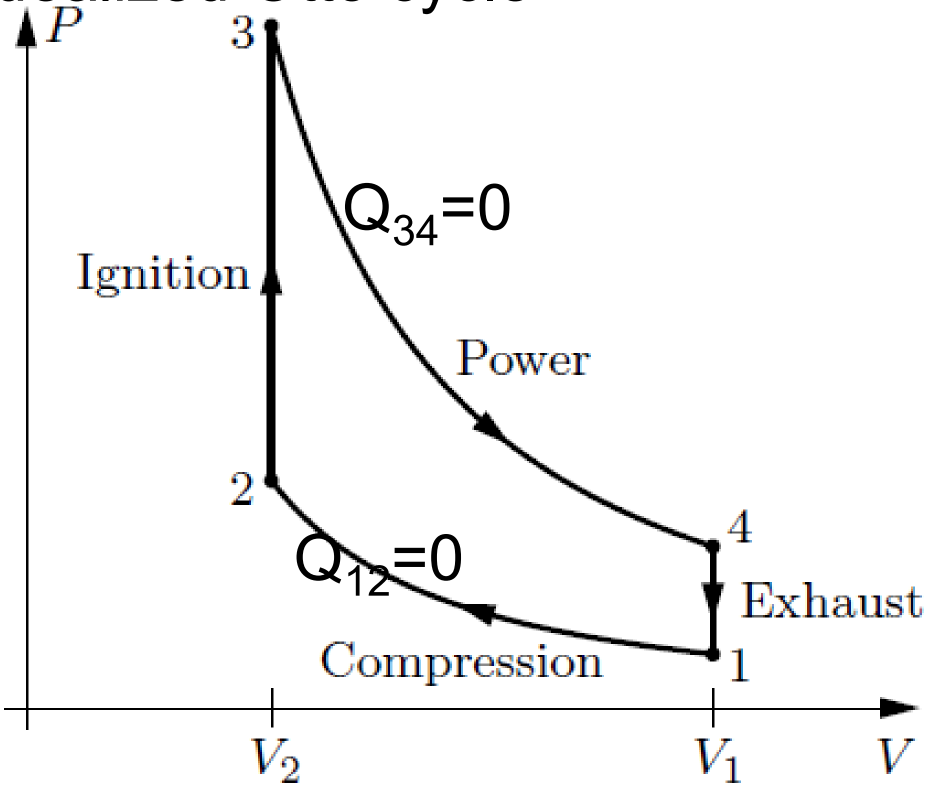
$$Q_{34} = 0 \quad V_2 T_3^{1/(\gamma-1)} = V_1 T_4^{1/(\gamma-1)}$$

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} = \left(\frac{T_3}{T_4}\right)^{1/(\gamma-1)}$$

$$\epsilon = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}}$$

$$= 1 - \frac{T_4}{T_3} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

Idealized Otto cycle



$$\epsilon = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}}$$

$$= 1 - \frac{T_4}{T_3} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

For $\gamma = 1.4$ and $\frac{V_2}{V_1} = \frac{1}{8}$ $\quad \epsilon = 0.56$