PHY 341/641 Thermodynamics and Statistical Mechanics
MWF: Online at 12 PM & FTF at 2 PM

Record!!!

Discussion for Lecture 13:

Practical thermodynamic cycles
Engines, refrigerators, etc.

Reading: Chapters 4.3-4.4

1. **Otto and Diesel cycles**
2. **Rankine cycle**
3. **Other processes**

Thursday's Physics Colloquium --

Online Colloquium: "Pulsars – Their Discovery and Impact" — February 25, 2021 at 4 PM

Dr. Jocelyn Bell Burnell

Visiting Professor of Astrophysics, University of Oxford

Professorial Fellow in Physics, Mansfield College

Chancellor, University of Dundee

Thursday, February 25, 2021, 4 PM EST

Via Video Conference (contact wfuphys@wfu.edu for link information)

All interested persons are cordially invited to join the Zoom call.

Course schedule for Spring 2021

schedule -- subject to frequent adjustment.) Reading assignments are for the **An Introduction to Therm**
by Daniel V. Schroeder. The HW assignment numbers refer to problems in that text.

	Lecture date	Reading	Topic	HW	Due date
1	Wed: 01/27/2021	Chap. 1.1-1.3	Introduction and ideal gas equations	1.21	01/29/2021
2	Fri: 01/29/2021	Chap. 1.2-1.4	First law of thermodynamics	1.17	02/03/2021
3	Mon: 02/01/2021	Chap. 1.5-1.6	Work and heat for an ideal gas		
4	Wed: 02/03/2021	Chap. 1.1-1.6	Review of energy, heat, and work	1.45	02/05/2021
5	Fri: 02/05/2021	Chap. 2.1-2.2	Aspects of entropy		
6	Mon: 02/08/2021	Chap. 2.3-2.4	Multiplicity distributions	2.24	02/10/2021
7	Wed: 02/10/2021	Chap. 2.5-2.6	Entropy and macrostate multiplicity	2.26	02/12/2021
8	Fri: 02/12/2021	Chap. 2.1-2.6	Review of entropy and macrostates	2.32	02/15/2021
9	Mon: 02/15/2021	Chap. 3.1-3.2	Temperature, entropy, heat	3.10a-b	02/17/2021
10	Wed: 02/17/2021	Chap. 3.3-3.4	Temperature, entropy, heat	3.23	02/19/2021
11	Fri: 02/19/2021	Chap. 3.5-3.6	Temperature, entropy, heat	3.28	02/22/2021
12	Mon: 02/22/2021	Chap. 4.1-4.3	Ideal engines and refrigerators	4.1	02/24/2021
13	Wed: 02/24/2021	Chap. 4.3-4.4	Real engines and refrigerators	4.20	02/26/2021
14	Fri: 02/26/2021	Chap. 5.1	Free energy	5.5	03/01/2021
15	Mon: 03/01/2021				
16	Wed: 03/03/2021				
17	Fri: 03/05/2021				
18	Mon: 03/08/2021				
19	Wed: 03/10/2021				
20	Fri: 03/12/2021	Chap. 1-5	Review		

Your questions –

From Kristen -- 1. For the refrigerator discussed in the beginning of 4.4, (Figure 4.9), how can something be compressed without changing its volume?   2. With laser cooling, how does hitting the atoms cause them to slow down?

From Parker -- Why is the force between any two molecules weakly attractive at long distances and strongly repulsive at short distances, and usually the attraction dominates making U potential energy negative?

From Michael -- Can you further explain how a throttling valve operates in order to make the incoming gas cold enough to liquefy? I can't conceptually understand this concept currently.

From Rich -- In a heat exchanger, does the throttled gas heat the liquid in the evaporator? Where is the excess heat sent? In laser cooling, why wouldn't the effects/forces of all lasers cancel out?

Questions continued –

From Chao -- Why when temperature reaches absolute zero, the entropy will become Zero? If we look at the formula of microstate, Omega, it seems like there is a factor of 5/2 out there.

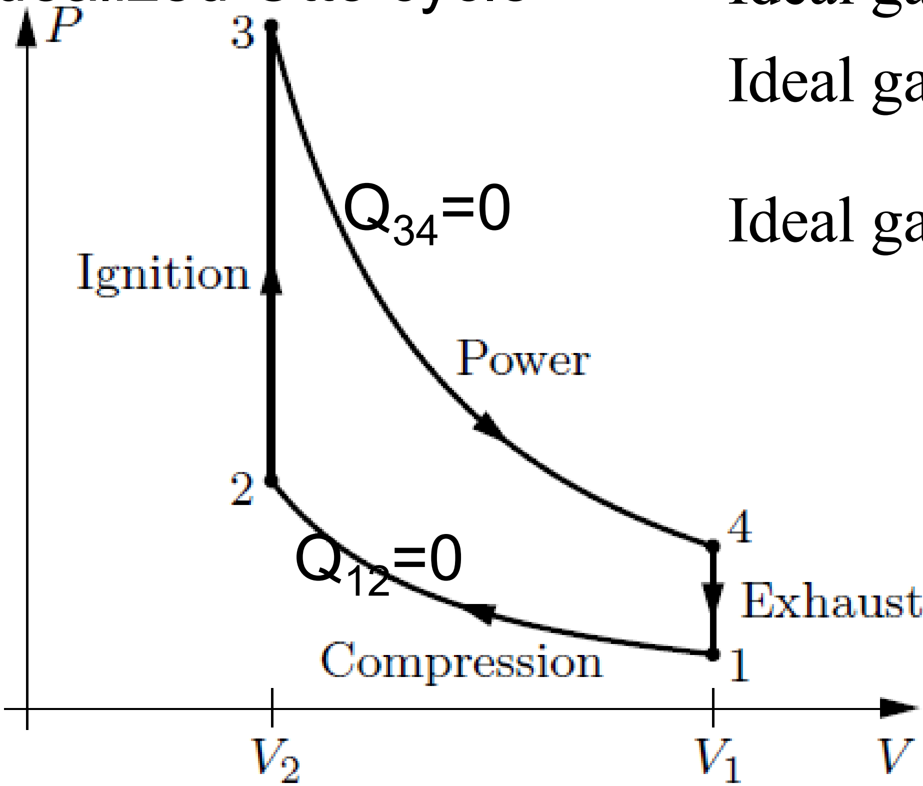
From Zezhong -- Is there any difference on heat cycle between the real refrigerator and helium dilution refrigerator although their structures are quite different.

Comment – from Sackur-Tetrode equation

$$S(N,V,U) \approx Nk_B\left(\ln\left(\frac{V}{N}\left(\frac{4\pi MU}{3Nh^2}\right)^{3/2}\right)+\frac{5}{2}\right)$$

Probably need to consider details of derivation in the limit that T →0. Is the ideal gas law still true?

Idealized Otto cycle



Ideal gas relations:

Ideal gas equation of state $PV = Nk_BT$

Ideal gas internal energy $U = \dfrac{Nk_BT}{\gamma - 1}$

$$Q_{23} = U_{23} = \frac{Nk_B}{\gamma - 1}(T_3 - T_2)$$

$$Q_{41} = U_{41} = \frac{Nk_B}{\gamma - 1}(T_1 - T_4)$$

$$Q_{12} = 0 \quad V_1T_1^{1/(\gamma-1)} = V_2T_2^{1/(\gamma-1)}$$

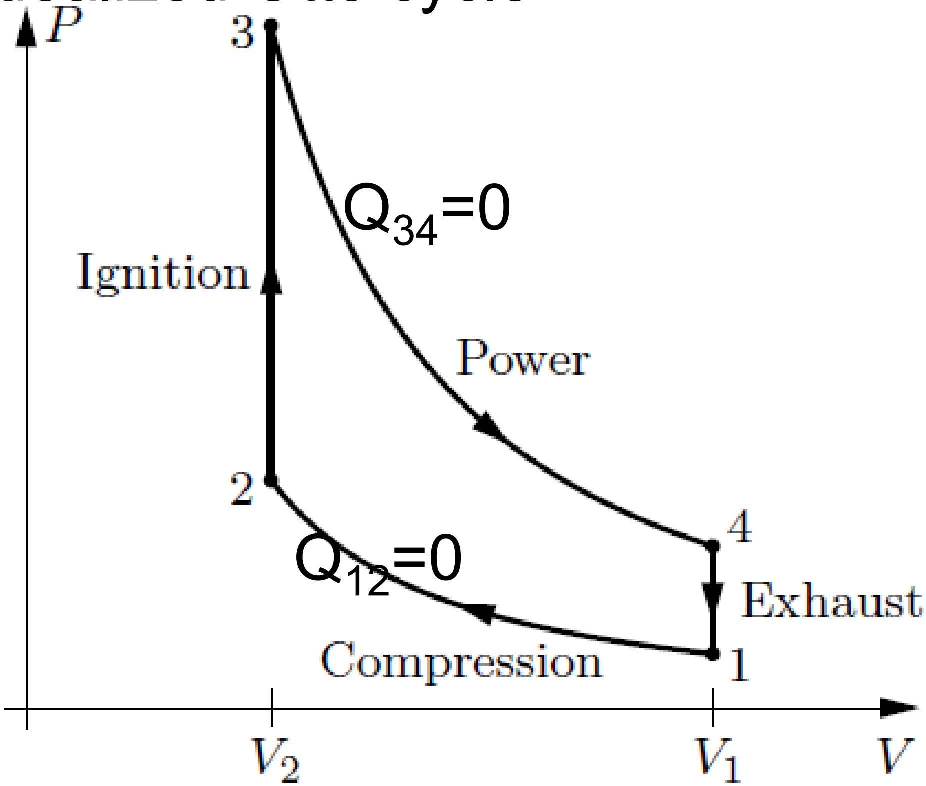
$$Q_{34} = 0 \quad V_2T_3^{1/(\gamma-1)} = V_1T_4^{1/(\gamma-1)}$$

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} = \left(\frac{T_3}{T_4}\right)^{1/(\gamma-1)}$$

$$\epsilon = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}}$$

$$= 1 - \frac{T_4}{T_3} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

Idealized Otto cycle



$$\epsilon = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}}$$

$$= 1 - \frac{T_4}{T_3} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

$$\text{For } \gamma = 1.4 \text{ and } \frac{V_2}{V_1} = \frac{1}{8} \qquad \epsilon = 0.56$$

Nikolaus Otto
1832-1891

https://www.britannica.com/biography/Nikolaus-Otto

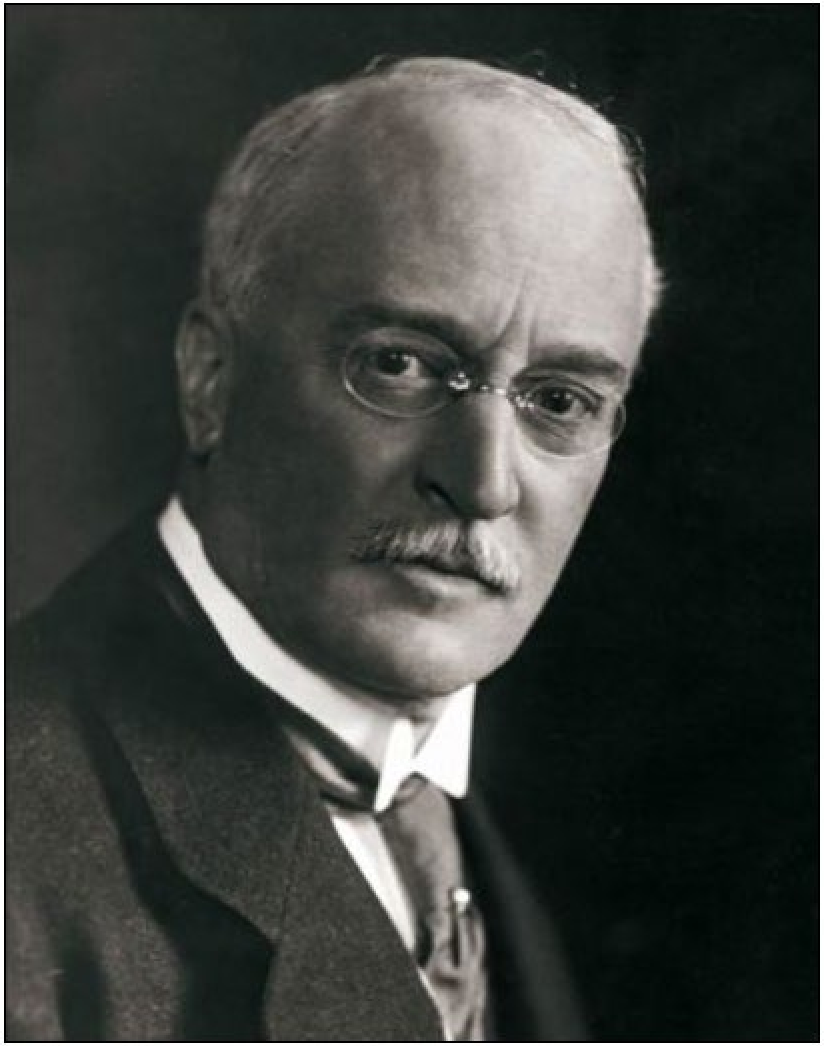
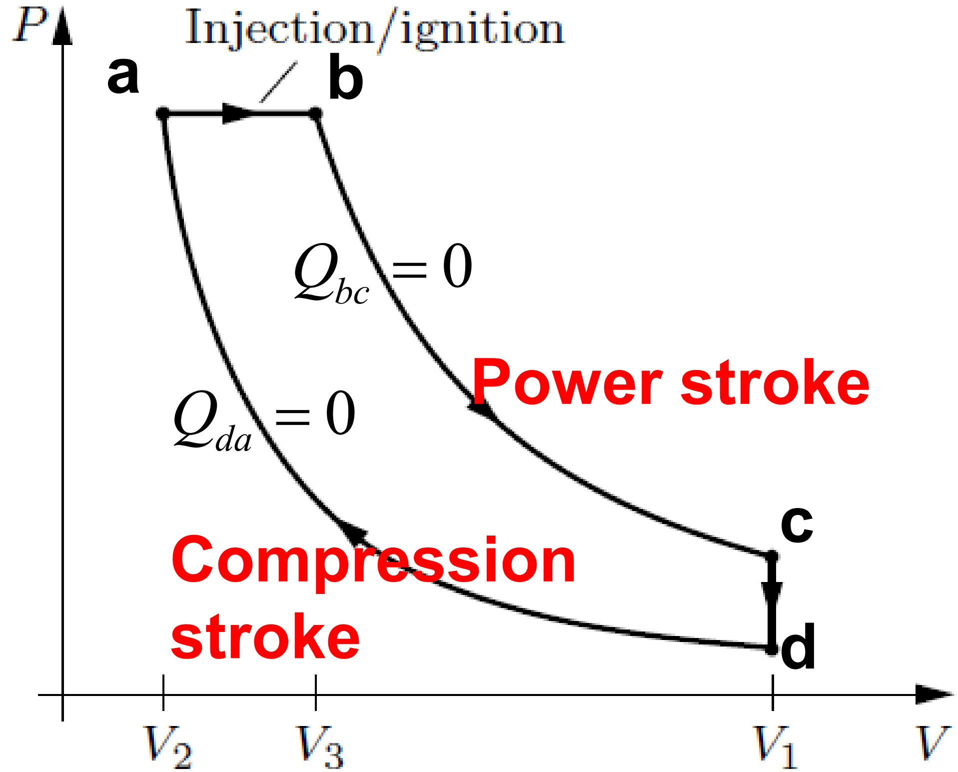


Figure 1. Rudolf Diesel (1858-1913)

https://dieselnet.com/tech/diesel_history.php#diesel

Idealized Diesel cycle

Figure 4.6. *PV* diagram for the Diesel cycle. Copyright ©2000, Addison-Wesley.



Ideal gas relations:

Ideal gas equation of state $PV = Nk_BT$

Ideal gas internal energy $U = \dfrac{Nk_BT}{\gamma - 1}$

Typically, $\gamma = 1.4$ for air

Compression ratio: $\dfrac{V_1}{V_2}$

Cutoff ratio: $\dfrac{V_3}{V_2}$

Example of a cycle NOT using an ideal gas – the Rankine cycle

Developed the steam turbine cycle

William John Macquorn Rankine (1820-1872)

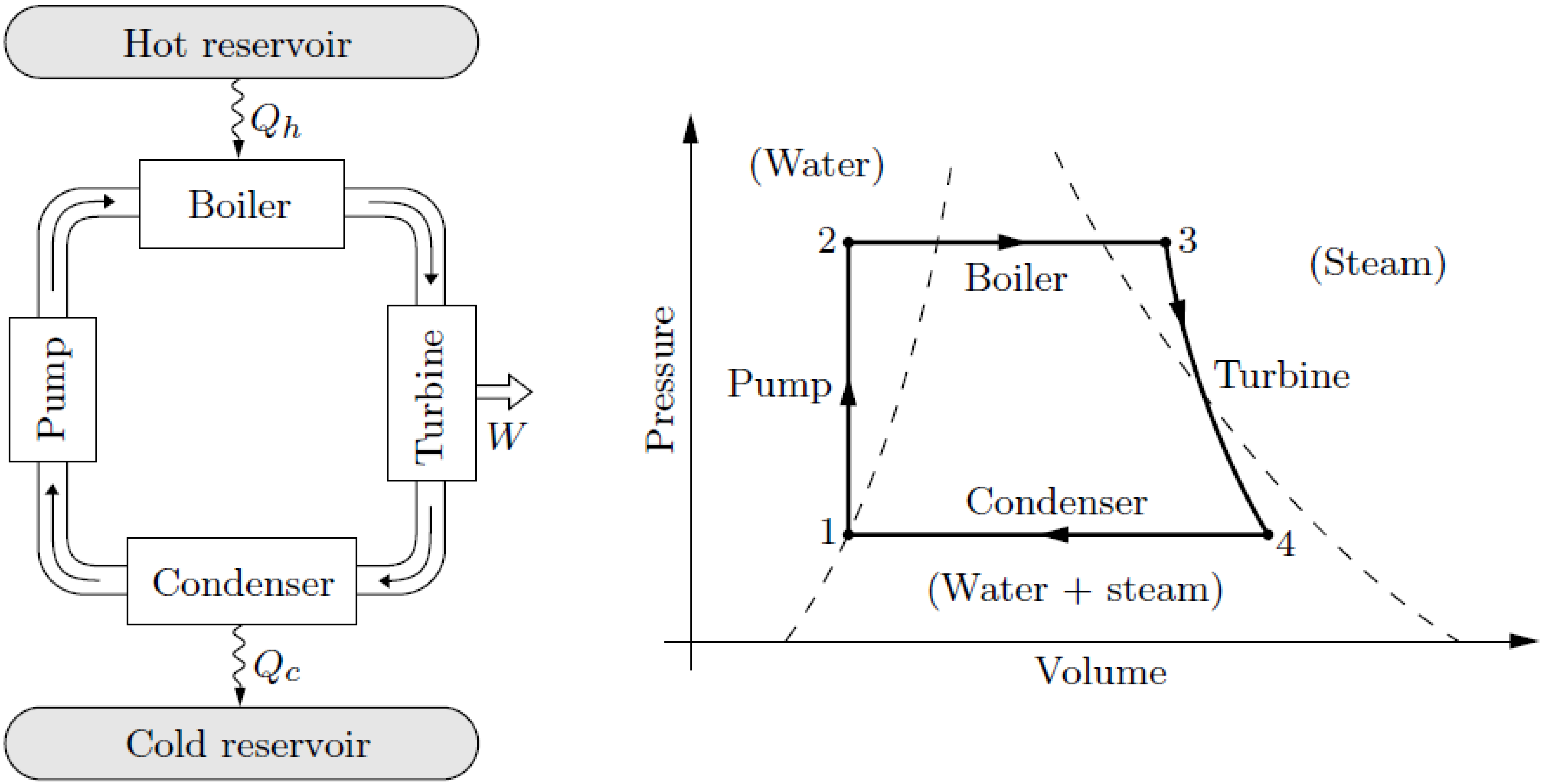
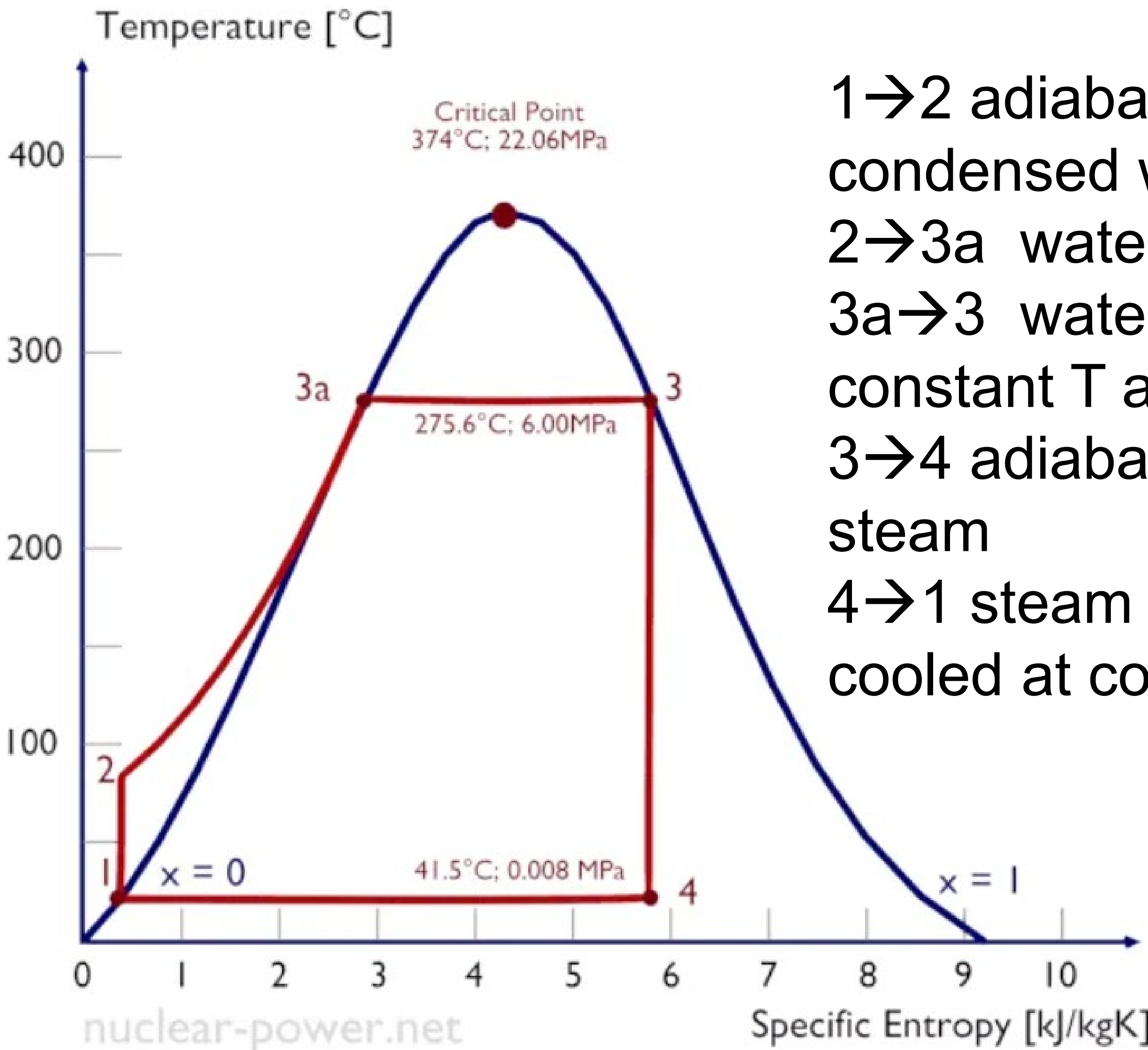


Figure 4.8. Schematic diagram of a steam engine and the associated PV cycle (not to scale), called the **Rankine cycle**. The dashed lines show where the fluid is liquid water, where it is steam, and where it is part water and part steam. Copyright ©2000, Addison-Wesley.

Other representations of the Rankine cycle –

https://www.nuclear-power.net/nuclear-engineering/thermodynamics/thermodynamic-cycles/rankine-cycle-steam-turbine-cycle/



1→2 adiabatic compression of condensed water to high P

2→3a water is heated to boiling

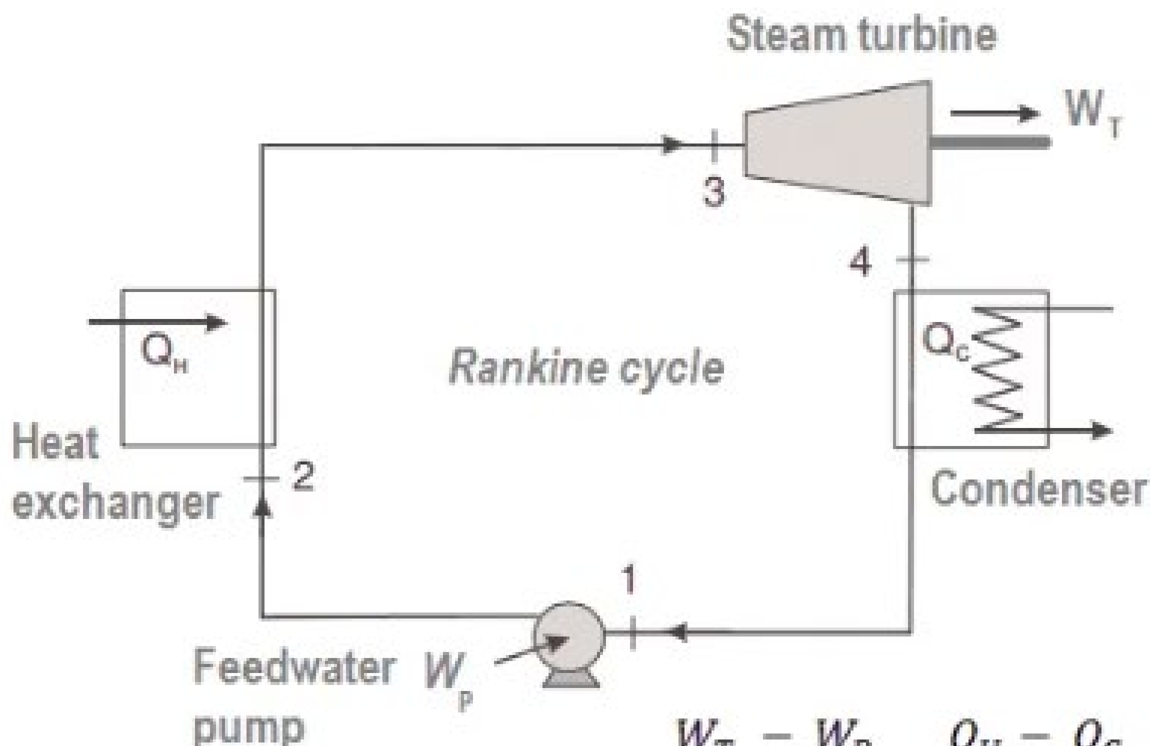
3a→3 water is vaporized at constant T and high P

3→4 adiabatic expansion of steam

4→1 steam is condensed and cooled at constant T

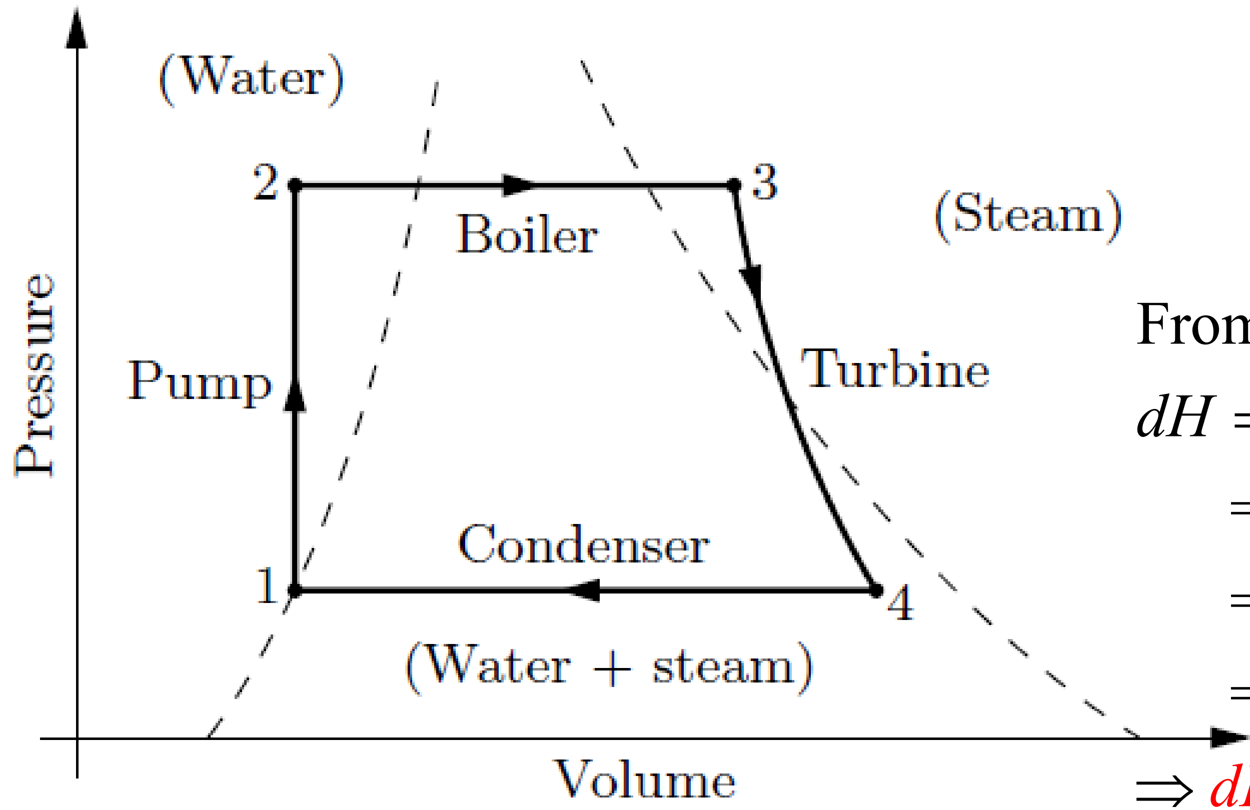
Rankine Cycle – Ts Diagram

1→2 adiabatic compression of condensed water to high P
2→3a water is heated to boiling
3a→3 water is vaporized at constant T and high P
3→4 adiabatic expansion of steam
4→1 steam is condensed and cooled at constant T



$$\eta = \frac{W_T - W_P}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

Some details from your textbook --



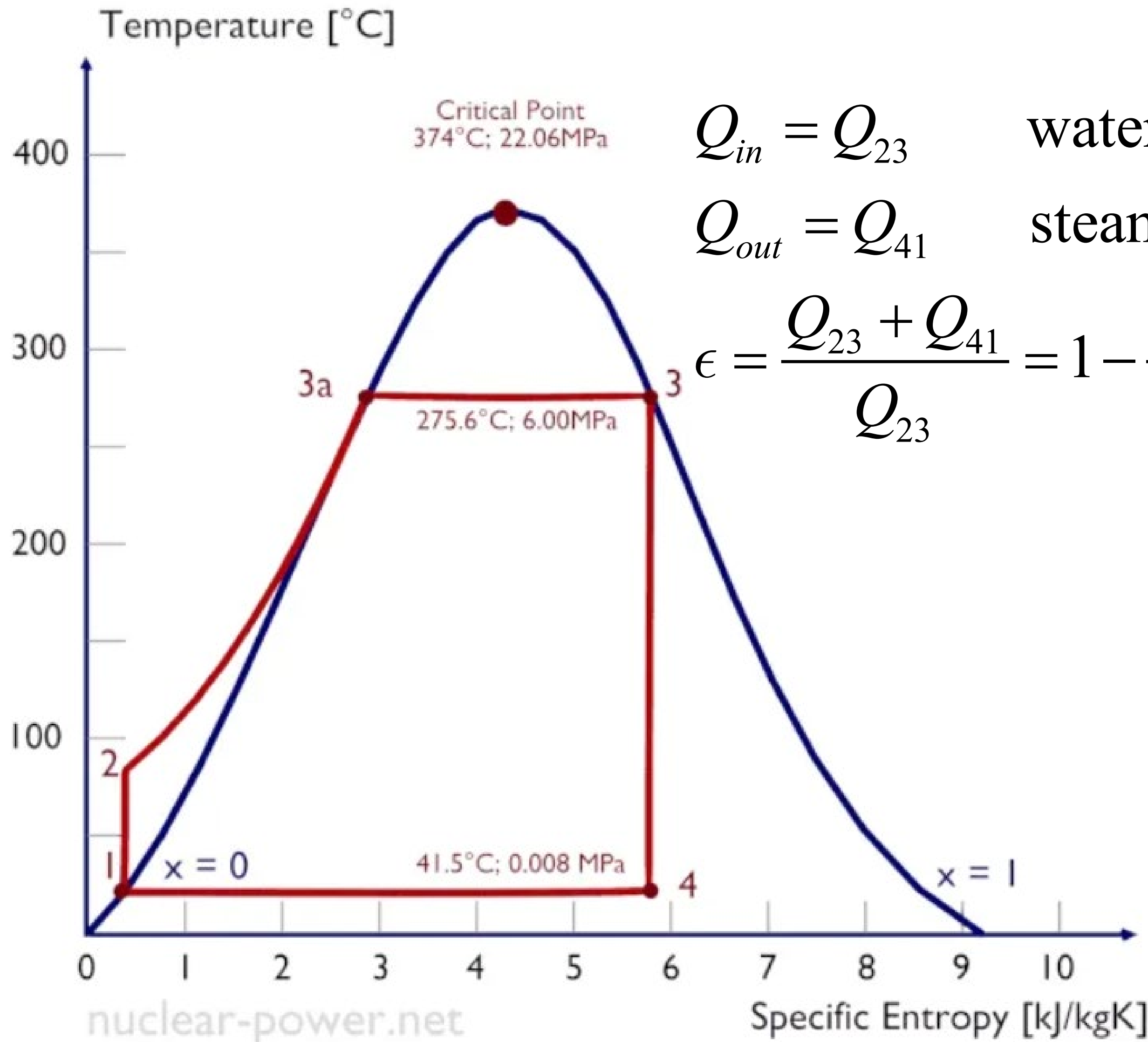
From first law of thermo --

$$dH = dU + d(PV)$$
$$= dQ + dW + d(PV)$$
$$= dQ - PdV + d(PV)$$
$$= dQ + VdP$$
$$\Rightarrow dH = dQ \quad \text{for constant } P$$

Digression -- note that constant pressure processes are conveniently analyzed in terms of enthalpy $H = U + PV$

Why?

Efficiency of Rankine cycle



$$Q_{in} = Q_{23} \qquad \text{water} \rightarrow \text{steam at high } P$$

$$Q_{out} = Q_{41} \qquad \text{steam at low } P \rightarrow \text{water}$$

$$\epsilon = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 - \frac{H_4 - H_1}{H_3 - H_2} \approx 1 - \frac{H_4 - H_1}{H_3 - H_1}$$

Rankine Cycle – Ts Diagram

Conditions in textbook:

1. T= 20°C, P=0.023 bar
3. T=600° C P=300 bar
4. S=6.233 kJ/K

Some numerical values from your textbook –

T (°C)	P (bar)	H_{water} (kJ)	H_{steam} (kJ)	S_{water} (kJ/K)	S_{steam} (kJ/K)
0	0.006	0	2501	0	9.156
10	0.012	42	2520	0.151	8.901
20	0.023	84	2538	0.297	8.667
30	0.042	126	2556	0.437	8.453
50	0.123	209	2592	0.704	8.076
100	1.013	419	2676	1.307	7.355

H_1

Table 4.1. Properties of saturated water/steam. Pressures are given in bars, where 1 bar = 10^5 Pa ≈ 1 atm. All values are for 1 kg of fluid, and are measured relative to liquid water at the triple point (0.01°C and 0.006 bar). Excerpted from Keenan et al. (1978). Copyright ©2000, Addison-Wesley.

P (bar)		Temperature (°C) 200	300	400	500	600
1.0	H (kJ)	2875	3074	3278	3488	3705
	S (kJ/K)	7.834	8.216	8.544	8.834	9.098
3.0	H (kJ)	2866	3069	3275	3486	3703
	S (kJ/K)	7.312	7.702	8.033	8.325	8.589
10	H (kJ)	2828	3051	3264	3479	3698
	S (kJ/K)	6.694	7.123	7.465	7.762	8.029
30	H (kJ)		2994	3231	3457	3682
	S (kJ/K)		6.539	6.921	7.234	7.509
100	H (kJ)			3097	3374	3625
	S (kJ/K)			6.212	6.597	6.903
300	H (kJ)			2151	3081	3444 H_3
	S (kJ/K)			4.473	5.791	6.233 S_3

Table 4.2. Properties of superheated steam. All values are for 1 kg of fluid, and are measured relative to liquid water at the triple point. Excerpted from Keenan et al. (1978). Copyright ©2000, Addison-Wesley.

At step 4 we have a mixture of water and steam at

T=20^o C and P=0.023 atm and S=6.233 kJ/K

$$xS_{water} + (1-x)S_{steam} = 6.233 kJ/K \quad \Rightarrow x = 0.29$$

$$H_4 \approx xH_{water} + (1-x)H_{steam} = 1824 kJ$$ (using values from the tables)

$Q_{in} = Q_{23}$ water $\rightarrow$ steam at high P

$Q_{out} = Q_{41}$ steam at low $P \rightarrow$ water

$$\epsilon = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 - \frac{H_4 - H_1}{H_3 - H_2} \approx 1 - \frac{H_4 - H_1}{H_3 - H_1}$$

$$= 1 - \frac{1824 - 84}{3444 - 84} = 0.48$$

Carnot efficiency for lowest and highest temperatures

$$\epsilon_{Carnot} = 1 - \frac{293}{873} = 0.66$$

Thermodynamics for cooling

Note that it is possible for an ideal gas expansion process to lower the temperature. For example for an adiabatic ideal gas with $\gamma = 5/3$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{1-1/\gamma} \qquad \text{for } T_1 = 300K, P_2 = P_1/32$$

$$T_2 = 300K \left(\frac{1}{32}\right)^{0.4} = 75 \text{ K}$$

Note that this example corresponds to $V_2 = 8V_1$

However, in order to cool even further, phase change processes are needed. While water works well in a steam engine, the temperatures involved are not appropriate for refrigeration.

Refrigeration process based on Freon

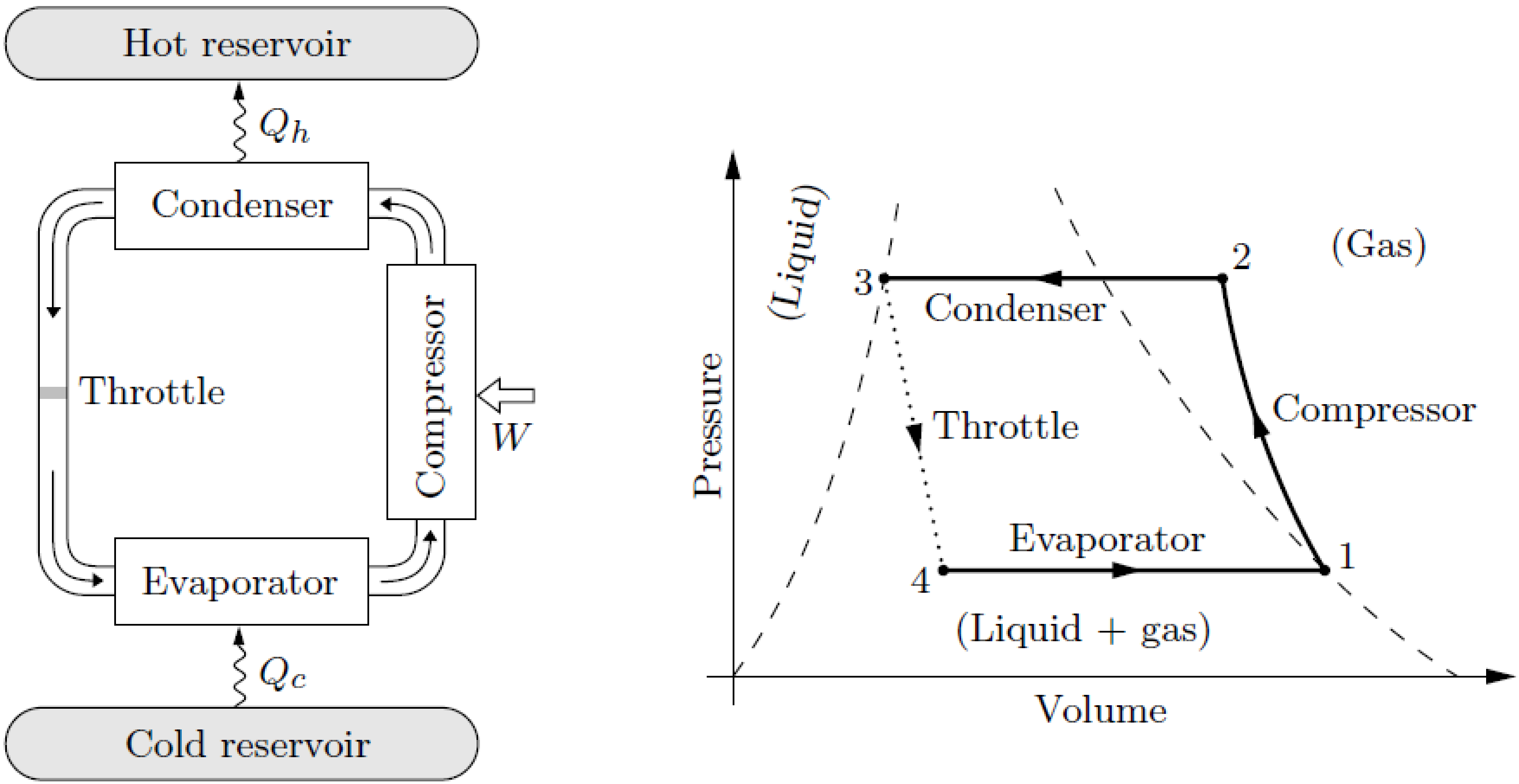
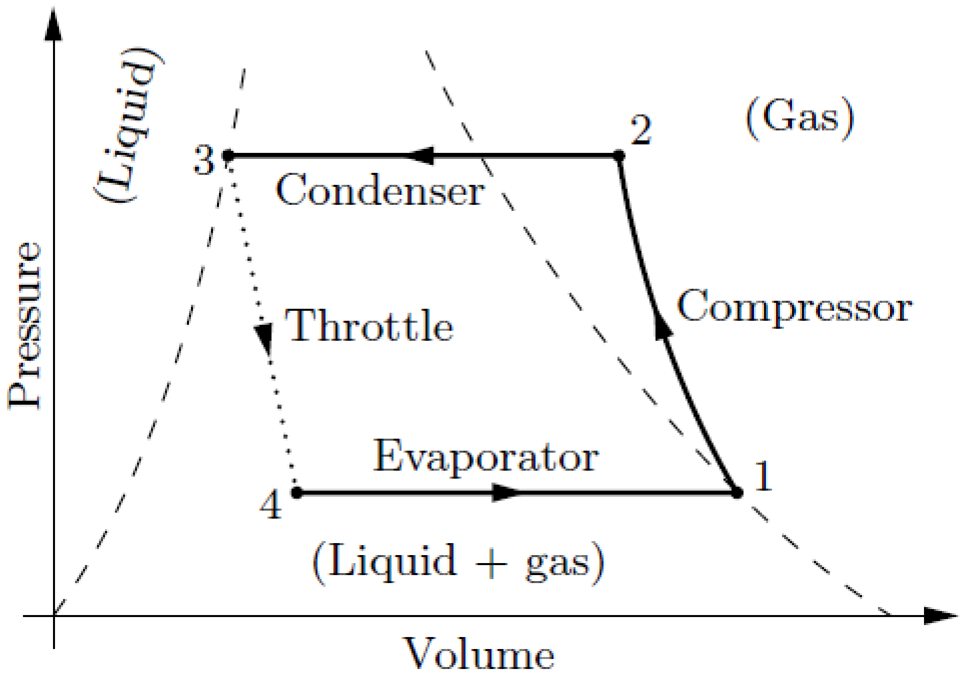


Figure 4.9. A schematic drawing and PV diagram (not to scale) of the standard refrigeration cycle. The dashed lines indicate where the refrigerant is liquid, gas, and a combination of the two. Copyright ©2000, Addison-Wesley.



1 – gas compressed adiabatically to higher T and P, smaller V
2 – gas liquifies and heat removed
3 – pressure reduced on liquid forming liquid and gas -- “throttle”
4 – gasification at constant pressure

Details of throttle process for ideal gas

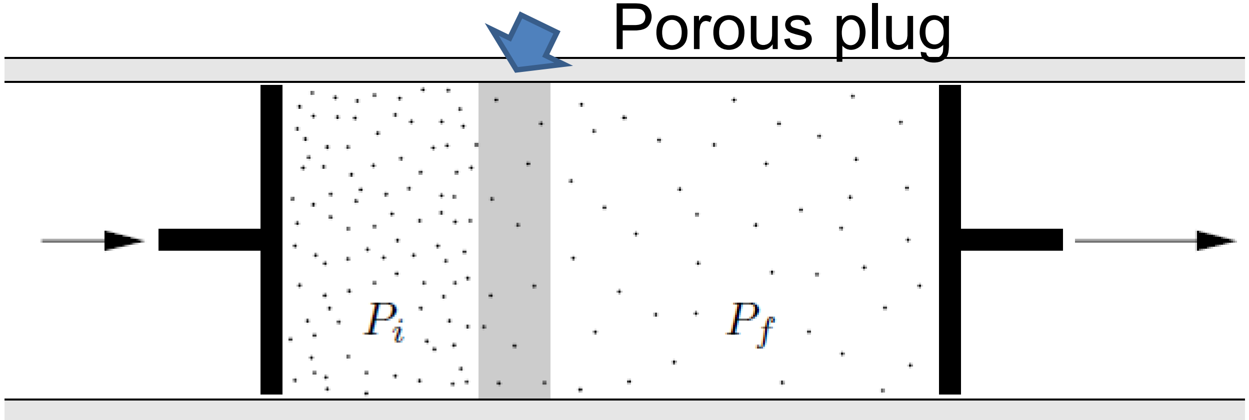


Figure 4.10. The throttling process, in which a fluid is pushed through a porous plug and then expands into a region of lower pressure. Copyright ©2000, Addison-Wesley.

$$U_f - U_i = Q + W = P_i V_i - P_f V_f \quad \text{(for adiabatic process)}$$

$$\Rightarrow U_f + P_f V_f = U_i + P_i V_i \qquad \Rightarrow H_f = H_i = \frac{\gamma}{\gamma - 1} N k_B T$$

Note that for an ideal gas, $H = \frac{\gamma}{\gamma - 1} N k_B T$ so that

the process would be isothermal. However more generally non ideal gas behavior may involve phase change due to interactions between particles.

Coefficient of performance

$$\text{COP}=\frac{H_1-H_4}{H_2-H_3-H_1+H_4}=\frac{H_1-H_3}{H_2-H_1}$$

Because of its convenient melting point (-158^{o}C) and boiling point (-30^{o}C) Freon has been used throughout the 20th century as the refrigerant of choice. However, since 2010 has been gradually replaced because of it's contribution to ozone deletion in the atmosphere.

Extreme refrigeration due to quantum mechanics

1. Helium dilution refrigeration <0.001K the two isotopes of He – ^{3}He and ^{4}He have very different behaviors near T ~ 0 having Fermi and Bose statics, respectively. Dissolving small amounts of ^{3}He into ^{4}He absorbs heat. The cycle involves steps separating and then mixing ^{3}He and ^{4}He.
   https://nationalmaglab.org/about/around-the-lab/what-the/dilfridge

2. Lasers can be used to generate strong electromagnetic fields that can control the motions of atoms a low temperature resulting in laser cooling. The process involves quantum mechanical absorption and emission of light.
   http://info.phys.unm.edu/~ideutsch/Classes/Phys500S09/Downloads/newoptics.pdf