PHY 341/641 Thermodynamics and Statistical Mechanics
MWF: Online at 12 PM & FTF at 2 PM

Record!!!

Discussion for Lecture 25:

Introduction to statistical mechanics – Canonical ensembles and their thermodynamic properties

Reading: Appendix A and begin Chapter 7

1. **Elements of quantum mechanics**
2. **Fermi quantum particles and Bose quantum particles compared with distinguishable particle combinations**

PHYSICS COLLOQUIUM

4 PM

THURSDAY

APRIL 1, 2021

ZOOM link

"From Fukushima to the Future: Lessons Learned and New Developments"

The day before a huge tsunami hit the coast of Japan on March 11, 2011, nuclear power appeared to be poised for a "renaissance" in much of the world. However, the tsunami resulted in a major accident at the Fukushima nuclear power plant, causing the world to hit the pause button on nuclear power development. In the 9 years since that accident, the industry has focused on understanding the underlying causes of the accident and modifying current nuclear plants and operations based on the lessons learned. Now, new nuclear power plants are being built in several countries, and more are being planned. This talk will address the accident and its aftermath, including major changes that have been made at existing plants, as well as the status of nuclear power today in different countries, and how advanced nuclear reactor concepts might affect the future of nuclear power.

Dr. Gail Marcus

Independent Consultant
Nuclear Power Technology and Policy
Washington, DC

4:00 pm

21	Mon: 03/22/2021	Chap. 6.1 & 6.5	Microcanonical and canonical ensembles		
22	Wed: 03/24/2021	Chap. 6.1-6.2	Canonical distributions	#18	03/26/2021
23	Fri: 03/26/2021	Chap. 6.1-6.7	Canonical distributions	6.49	03/29/2021
24	Mon: 03/29/2021	Chap. 6.1-6.7	Canonical distributions	#20	03/31/2021
25	Wed: 03/31/2021	App. A & Chap. 7.1	Quantum mechanical effects	#21	04/02/2021
26	Fri: 04/02/2021	Chap. 7.1-7.2	Quantum mechanical effects		

PHY 341/641 -- Assignment #21

March 31, 2021

Read Appendix A and start reading Chapter 7 in **Schroeder** .

Consider three different states A,B,C each with a different energy and three particles 1,2,3. Enumerate the possible distinct distributions of the particles among the states and their corresponding energies for the following cases:

1. The three particles are distinguishable
2. The three particles are indistinguishable and obey Bose statistics
3. The three particles are indistinguishable and obey Fermi statistics

Your questions – To be answered on Friday --

From Kristen -- 1. Could you explain why in this case we keep the mu dN term (equation 7.3)? 2. Why must Z be an integer in the example given in the beginning of section 7.2? 3. Could you explain what the quantum volume represents?

From Rich -- How do you calculate Zint for equation 7.10?

From Chao –

The grand partition function for this single-site system has just two terms:

$$\mathcal{Z} = 1 + e^{-(\epsilon-\mu)/kT}. \qquad (7.9)$$

From Michael -- How do we distinguish the Gibbs sum/grand partition function from other partition functions we have learned about thus far, and when would we use it exactly?

What new physics does Quantum Mechanics bring?

Classical mechanics

Newton's Laws of motion –

Particle motion described in term of position and momentum **r, p**. In many cases, energy is conserved and equal to the Hamiltonian function:

$$H(\mathbf{r},\mathbf{p}) = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) = E$$

Particles can be distinguishable

Quantum mechanics

Schoedinger Equation --

Particle motion is described in terms of a probability amplitude ψ; momentum becomes an operator; particles have intrinsic spin; energy is an eigenvalue of the Schroedinger equation:

$$\mathcal{H}\Psi = E\Psi$$

Particles are generally indistinguishable and have characteristic properties depending on their intrinsic spin

Examples of solutions to the Schroedinger resulting in discrete states

$$\mathcal{H}\Psi = E\Psi$$

Intrinsic S spin of a particle having a gyromagnetic ratio γ

The quantum operators associated with spin are:

$$\mathbf{S}^2\Psi_{s,m_s} = s(s+1)\Psi_{s,m_s}$$

$$S_z\Psi_{s,m_s} = m_s\Psi_{s,m_s} \quad \text{for } m_s = -s, -s+1, \ldots s$$

Energy eigenstates are realized in a magnetic field along the z-axis

$$\mathcal{H} = -\mu B_z \quad \text{for } \mu = \hbar\gamma S_z$$

Examples:

Electron: $s = 1/2$ $\gamma = \mathbf{1.760\,859\,630\,23 \text{ x } 10^{11} s^{-1} T^{-1}}$

Proton: $s = 1/2$ $\gamma = \mathbf{2.675\,221\,8744 \text{ x } 10^{8} s^{-1} T^{-1}}$

Neutron: $s = 1/2$ $\gamma = \mathbf{1.832\,471\,71 \text{ x } 10^{8} s^{-1}}\mathbf{T}^{-1}$

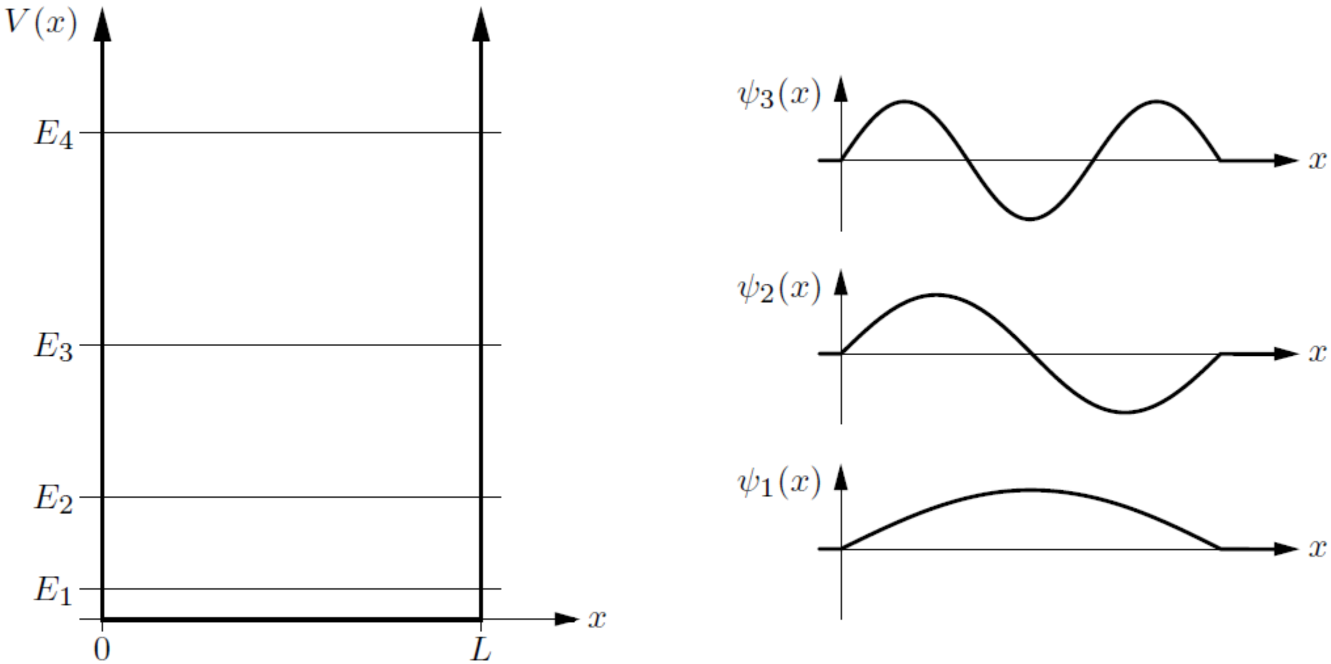
Deuterium (^{2}H): $s = 1$ $\gamma = 4.11 \text{ x } 10^{7}\mathbf{s^{-1}T}^{-1}$

Helium (^{4}He): $s = 0$

Examples of solutions to the Schroedinger resulting in discrete states

$$\mathcal{H}\Psi = E\Psi \qquad -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi = E\Psi \qquad \Psi(0) = \Psi(L) = 0$$

Particle confined within a one-dimensional box of length L



$$\Psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2} \qquad n = 1, 2, \ldots$$

Figure A.9. A few of the lowest energy levels and corresponding definite-energy wavefunctions for a particle in a one-dimensional box. Copyright ©2000, Addison-Wesley.

Examples of solutions to the Schroedinger resulting in discrete states

$$\mathcal{H}\Psi = E\Psi \qquad \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega^2x^2\right)\Psi = E\Psi$$

Harmonic oscillator in one-dimension

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right)$$

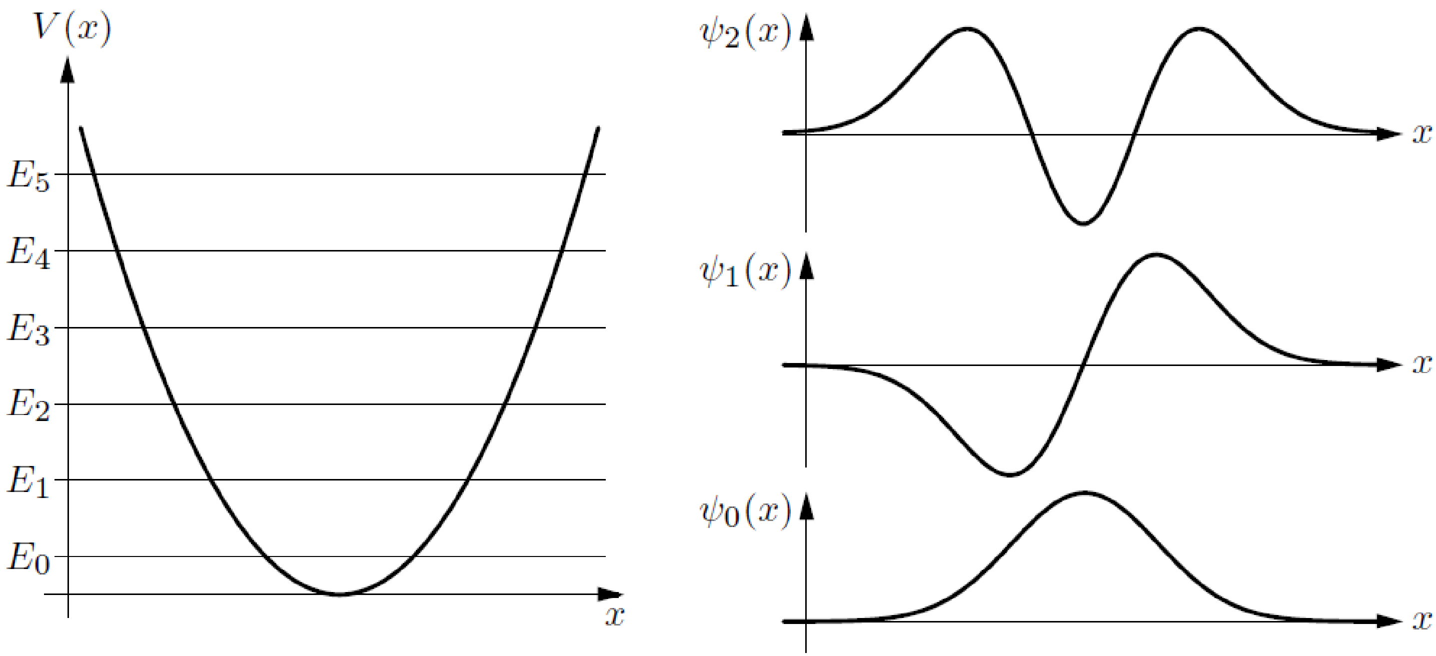


Figure A.10. A few of the lowest energy levels and corresponding wavefunctions for a one-dimensional quantum harmonic oscillator. Copyright ©2000, Addison-Wesley.

Examples of solutions to the Schroedinger resulting in discrete states

$$\mathcal{H}\Psi = E\Psi \qquad \left(-\frac{\hbar^2}{2I_{rot}R^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)\right)\Psi = E\Psi$$

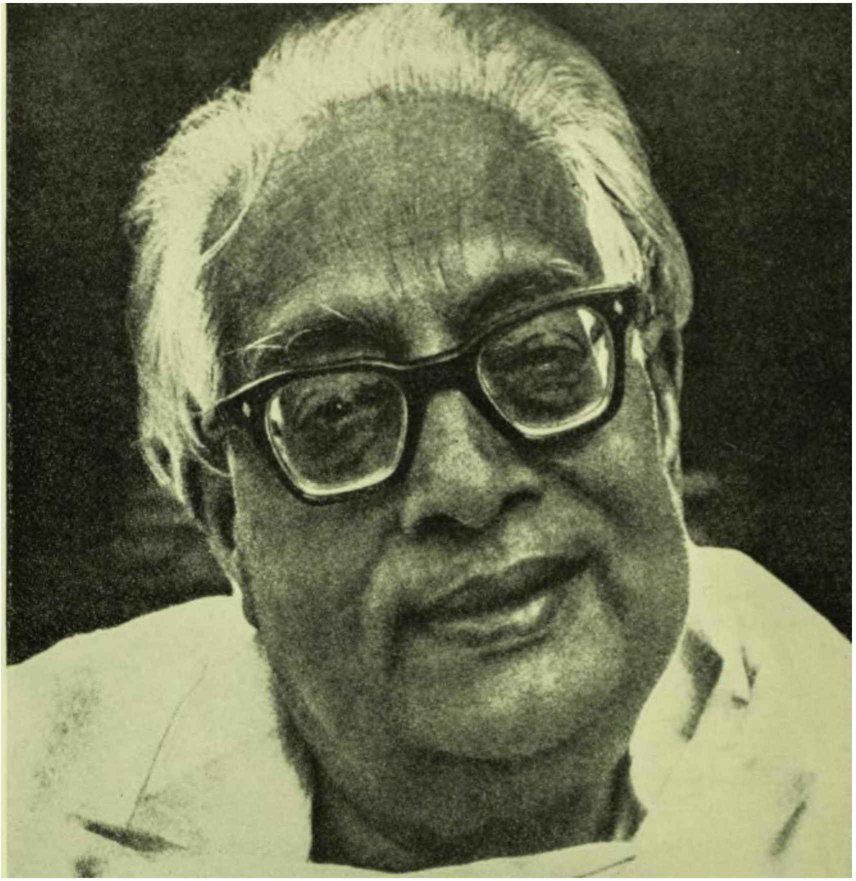
Rigid rotor with moment of inertia I_{rot}

$$E = \frac{\hbar^2}{2I_{rot}R^2}l(l+1) \qquad l = 0,1,2,....$$

$$\Psi(\theta,\varphi) = Y_{lm}(\theta,\varphi) \qquad m = -l, -l+1 ... l \quad (2l+1)$$

$$Y_{00}(\theta,\varphi) = \sqrt{\frac{1}{4\pi}}$$

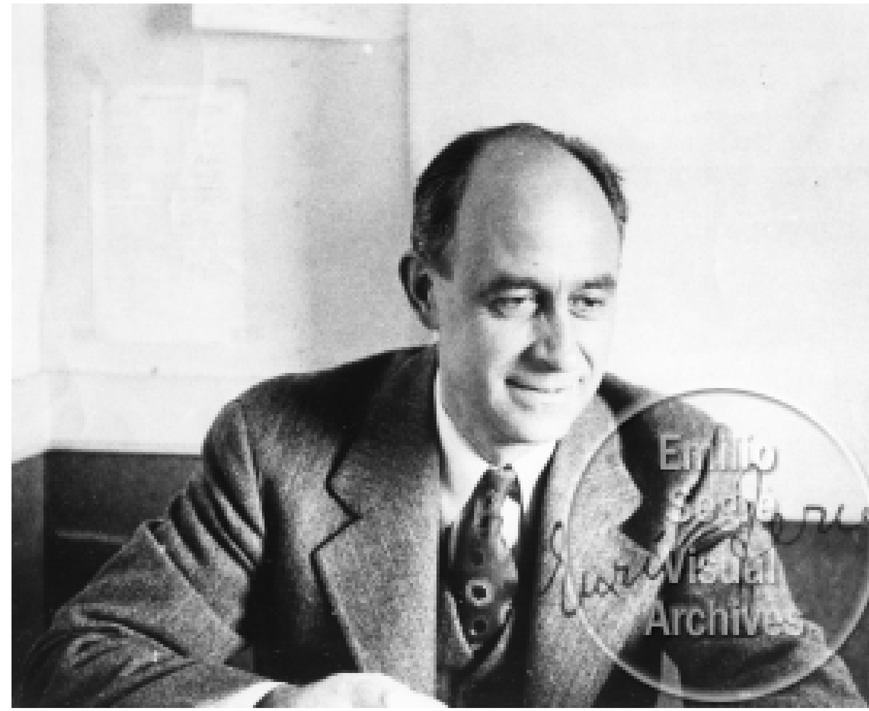
$$Y_{1-1}(\theta,\varphi) = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\varphi} \quad Y_{10}(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos\theta \quad Y_{11}(\theta,\varphi) = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi}$$



SATYENDRA NATH BOSE

1 January 1894—4 February 1974

Bose particles have integer *s*



Enrico Fermi
1901-1954

Fermi particles have half integer *s*

Experimentally observed properties of Bose and Fermi particles --

Bose particles have integer *s.* It is observed that non interacting identical Bose particles are symmetrical under particle swapping – $\Psi(1,2)=\Psi(2,1)$. In terms of single particle states

$$\Psi_{Bose}(1,2)=\sqrt{\frac{1}{2}}\left(\Psi_A(1)\Psi_B(2)+\Psi_A(2)\Psi_B(1)\right)$$

Fermi particles have half integer *s.* It is observed that non interacting identical Fermi particles are anti-symmetrical under particle swapping – $\Psi(1,2)=-\Psi(2,1)$. In terms of single particle states

$$\Psi_{Fermi}(1,2)=\sqrt{\frac{1}{2}}\left(\Psi_A(1)\Psi_B(2)-\Psi_A(2)\Psi_B(1)\right)$$

$$\Psi_{Bose}(1,2)=\sqrt{\frac{1}{2}}\left(\Psi_A(1)\Psi_B(2)+\Psi_A(2)\Psi_B(1)\right)$$

$$\Psi_{Fermi}(1,2)=\sqrt{\frac{1}{2}}\left(\Psi_A(1)\Psi_B(2)-\Psi_A(2)\Psi_B(1)\right)$$

What happens if the wavefunctions for A and B are the same?

a. This affects both Bose and Fermi particles the same way
b. This affects Bose particles significantly.
c. This affects Fermi particles significantly.

$$\Psi_{Bose}(1,2)=\sqrt{\frac{1}{2}}\left(\Psi_A(1)\Psi_B(2)+\Psi_A(2)\Psi_B(1)\right)$$

$$\Psi_{Fermi}(1,2)=\sqrt{\frac{1}{2}}\left(\Psi_A(1)\Psi_B(2)-\Psi_A(2)\Psi_B(1)\right)$$

Conclusions for properties of non interacting identical Bose particles ➔ Many Bose particles may occupy the same quantum state

- Explains behavior of quanta of electromagnetic radiation (photons)
- Explains "Bose condensation"

Conclusions for properties of non interacting identical Fermi particles ➔ A quantum state may accommodate 1 or 0 Fermi"

- Explains behavior of electrons in an ideal metal

Suppose that you have three states A, B, C with different wavefunctions and energies and two non interacting particles. What valid combinations exist for your system

System of distinguishable classical particles 1 & 2:

A	B	C	Config. energy
12			2A
	12		2B
		12	2C
1	2		A+B
2	1		A+B
	1	2	B+C
	2	1	B+C
1		2	A+C
2		1	A+C

System of indistinguishable Bose particles 1 & 2:

A	B	C	Config. energy
12			2A
	12		2B
		12	2C
1	2		A+B
	1	2	B+C
1		2	A+C

System of indistinguishable Fermi particles 1 & 2:

A	B	C	Config. energy
1	2		A+B
	1	2	B+C
1		2	A+C