

PHY 712 Electrodynamics

10-10:50 AM MWF Online

Plan for Lecture 13:

Continue reading Chapter 5

A. Examples of magnetostatic fields

B. Magnetic dipoles

C. Hyperfine interaction

Course schedule for Spring 2021

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Wed: 01/27/2021	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/29/2021
2	Fri: 01/29/2021	Chap. 1	Electrostatic energy calculations	#2	02/01/2021
3	Mon: 02/01/2021	Chap. 1 & 2	Electrostatic potentials and fields	#3	02/03/2021
4	Wed: 02/03/2021	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions	#4	02/05/2021
5	Fri: 02/05/2021	Chap. 1 - 3	Brief introduction to numerical methods	#5	02/08/2021
6	Mon: 02/08/2021	Chap. 2 & 3	Image charge constructions	#6	02/10/2021
7	Wed: 02/10/2021	Chap. 2 & 3	Cylindrical and spherical geometries		
8	Fri: 02/12/2021	Chap. 3 & 4	Spherical geometry and multipole moments	#7	02/15/2021
9	Mon: 02/15/2021	Chap. 4	Dipoles and Dielectrics	#8	02/19/2021
10	Wed: 02/17/2021	Chap. 4	Dipoles and Dielectrics		
11	Fri: 02/19/2021	Chap. 4	Polarization and Dielectrics	#9	02/24/2021
12	Mon: 02/22/2021	Chap. 5	Magnetostatics	#10	02/26/2021
13	Wed: 02/24/2021	Chap. 5	Magnetic dipoles and hyperfine interaction	#11	03/01/2021
14	Fri: 02/26/2021	Chap. 5	Magnetic dipoles and dipolar fields		

PHY 712 -- Assignment #11

February 24, 2021

Finish reading Chapter 5 in **Jackson** .

1. Work problem 5.13 in **Jackson**.

Various forms of Ampere's law :

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

Vector potential: $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$

For Coulomb gauge: $\nabla \cdot \mathbf{A}(\mathbf{r}) = 0$

$$\Rightarrow \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r})$$

For confined current density :

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3 r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Other examples of current density sources:

Quantum mechanical expression for current density

for a particle of mass M and charge e and of probability amplitude $\Psi(\mathbf{r})$:

$$\mathbf{J}(\mathbf{r}) = -\frac{e\hbar}{2Mi} (\Psi^*(\mathbf{r}) \nabla \Psi(\mathbf{r}) - \Psi(\mathbf{r}) \nabla \Psi^*(\mathbf{r}))$$

For an electron in a spherical potential (such as in an atom):

$$\Psi(\mathbf{r}) \equiv \Psi_{nlm_l}(\mathbf{r}) = R_{nl}(r) Y_{lm_l}(\hat{\mathbf{r}})$$

$$\begin{aligned} \mathbf{J}(\mathbf{r}) &= \frac{e\hbar}{2Mi} |R_{nl}(r)|^2 \frac{1}{r \sin \theta} \left(Y_{lm_l}^*(\hat{\mathbf{r}}) \frac{\partial Y_{lm_l}(\hat{\mathbf{r}})}{\partial \varphi} - Y_{lm_l}(\hat{\mathbf{r}}) \frac{\partial Y_{lm_l}^*(\hat{\mathbf{r}})}{\partial \varphi} \right) \hat{\boldsymbol{\phi}} \\ &= \frac{e\hbar}{M} \frac{m_l}{r \sin \theta} |\Psi_{nlm_l}(\mathbf{r})|^2 \hat{\boldsymbol{\phi}} \end{aligned}$$

Note that: $\hat{\boldsymbol{\phi}} = -\sin \varphi \hat{\mathbf{x}} + \cos \varphi \hat{\mathbf{y}} = \frac{\hat{\mathbf{z}} \times \mathbf{r}}{r \sin \theta}$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{M} \frac{m_l}{r^2 \sin^2 \theta} |\Psi_{nlm_l}(\mathbf{r})|^2 (\hat{\mathbf{z}} \times \mathbf{r})$$

Details of the electron orbital magnetic dipole moment

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{m_e} \frac{m_l}{r \sin \theta} \left| \Psi_{nlm_l}(\mathbf{r}) \right|^2 \hat{\boldsymbol{\phi}}$$

Note that: $\hat{\boldsymbol{\phi}} = -\sin \varphi \hat{\mathbf{x}} + \cos \varphi \hat{\mathbf{y}}$

Magnetic dipole moment:

$$\begin{aligned} \mathbf{m} &= \frac{1}{2} \int d^3 r' \mathbf{r}' \times \mathbf{J}(\mathbf{r}') = -\frac{e\hbar m_l}{2m_e} \int d^3 r' \frac{\mathbf{r}' \times \hat{\boldsymbol{\phi}}'}{r' \sin \theta'} \left| \Psi_{nlm_l}(\mathbf{r}') \right|^2 \\ &= -\frac{e\hbar m_l}{2m_e} \int d^3 r' \frac{-r' \hat{\boldsymbol{\theta}}'}{r' \sin \theta'} \left| \Psi_{nlm_l}(\mathbf{r}') \right|^2 \end{aligned}$$

Note that: $\hat{\boldsymbol{\theta}} = \cos \theta \cos \varphi \hat{\mathbf{x}} + \cos \theta \sin \varphi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}}$

$$\begin{aligned} \mathbf{m} &= -\frac{e\hbar m_l \hat{\mathbf{z}}}{2m_e} \int d^3 r' \left| \Psi_{nlm_l}(\mathbf{r}') \right|^2 \\ &= -\frac{e\hbar m_l}{2m_e} \hat{\mathbf{z}} \end{aligned}$$

Summary of magnetic field generated by point magnetic dipole moment:

$$\mathbf{B}_{\mu_e}(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\frac{3\hat{\mathbf{r}}(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}) - \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3} \boldsymbol{\mu}_e \delta(\mathbf{r}) \right)$$

Magnetic field near nucleus due to orbiting electron:

$$\mathbf{B}_O(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{e}{m_e} L_z \hat{\mathbf{z}} \left\langle \frac{1}{r^3} \right\rangle$$

"Hyperfine" interaction energy:

$$\begin{aligned} \mathcal{H}_{HF} &= -\boldsymbol{\mu}_N \cdot (\mathbf{B}_{\mu_e}(\mathbf{r}) + \mathbf{B}_O(\mathbf{r})) \\ &= \frac{\mu_0}{4\pi} \left(\frac{3(\boldsymbol{\mu}_N \cdot \hat{\mathbf{r}})(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}) - \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3} \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e \delta(\mathbf{r}) + \frac{e}{m_e} \left\langle \frac{\mathbf{L} \cdot \boldsymbol{\mu}_N}{r^3} \right\rangle \right) \end{aligned}$$

$$\mathcal{H}_{HF} = \frac{\mu_0}{4\pi} \left(\frac{3(\boldsymbol{\mu}_N \cdot \hat{\mathbf{r}})(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}) - \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3} \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e \delta(\mathbf{r}) + \frac{e}{m_e} \left\langle \frac{\mathbf{L} \cdot \boldsymbol{\mu}_N}{r^3} \right\rangle \right)$$

