PHY 712 Electrodynamics
10-10:50 AM MWF Online

Class discussion for Lecture 13:

Continue reading Chapter 5

A. Examples of magnetostatic fields

B. Magnetic dipoles

C. Hyperfine interaction

Thursday's Physics Colloquium --

Online Colloquium: "Pulsars – Their Discovery and Impact" — February 25, 2021 at 4 PM

Dr. Jocelyn Bell Burnell

Visiting Professor of Astrophysics, University of Oxford

Professorial Fellow in Physics, Mansfield College

Chancellor, University of Dundee

Thursday, February 25, 2021, 4 PM EST

Via Video Conference (contact wfuphys@wfu.edu for link information)

All interested persons are cordially invited to join the Zoom call.

Course schedule for Spring 2021

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Wed: 01/27/2021	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/29/2021
2	Fri: 01/29/2021	Chap. 1	Electrostatic energy calculations	#2	02/01/2021
3	Mon: 02/01/2021	Chap. 1 & 2	Electrostatic potentials and fields	#3	02/03/2021
4	Wed: 02/03/2021	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions	#4	02/05/2021
5	Fri: 02/05/2021	Chap. 1 - 3	Brief introduction to numerical methods	#5	02/08/2021
6	Mon: 02/08/2021	Chap. 2 & 3	Image charge constructions	#6	02/10/2021
7	Wed: 02/10/2021	Chap. 2 & 3	Cylindrical and spherical geometries		
8	Fri: 02/12/2021	Chap. 3 & 4	Spherical geometry and multipole moments	#7	02/15/2021
9	Mon: 02/15/2021	Chap. 4	Dipoles and Dielectrics	#8	02/19/2021
10	Wed: 02/17/2021	Chap. 4	Dipoles and Dielectrics		
11	Fri: 02/19/2021	Chap. 4	Polarization and Dielectrics	#9	02/24/2021
12	Mon: 02/22/2021	Chap. 5	Magnetostatics	#10	02/26/2021
13	Wed: 02/24/2021	Chap. 5	Magnetic dipoles and hyperfine interaction	#11	03/01/2021
14	Fri: 02/26/2021	Chap. 5	Magnetic dipoles and dipolar fields		

PHY 712 -- Assignment #11

February 24, 2021

Finish reading Chapter 5 in **Jackson** .

1. Work problem 5.13 in **Jackson**.

Your questions –

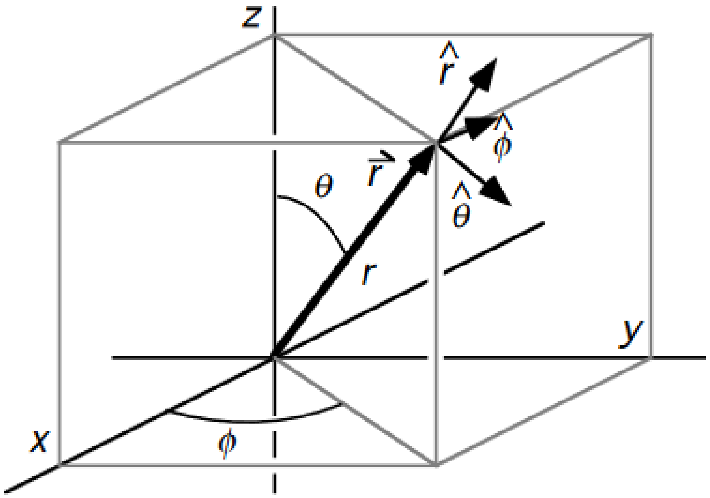
From Nick -- Can you discuss briefly how to transition unit vectors among coordinate systems. You gave us phi-hat last time, but this time we have theta-hat and I can't quite see it. Also, what is the meaning of the bracket notation we're using?

From Gao -- What is fermi contact?

Comment about spherical polar coordinates

Ref: https://www.cpp.edu/~ajm/materials/delsph.pdf

Spherical Coordinates



Transforms

The forward and reverse coordinate transformations are

$$r = \sqrt{x^2 + y^2 + z^2} \qquad x = r\sin\theta\cos\phi$$

$$\theta = \arctan\left(\sqrt{x^2 + y^2}, z\right) \qquad y = r\sin\theta\sin\phi$$

$$\phi = \arctan(y, x) \qquad z = r\cos\theta$$

where we *formally* take advantage of the *two argument* arctan function to eliminate quadrant confusion.

Unit Vectors

The unit vectors in the spherical coordinate system are functions of position. It is convenient to express them in terms of the *spherical* coordinates and the unit vectors of the *rectangular* coordinate system which are *not* themselves functions of position.

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\hat{\phi} = \frac{\hat{z}\times\hat{r}}{\sin\theta} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$$\hat{\theta} = \hat{\phi}\times\hat{r} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$$

Various forms of Ampere's law :

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

$$\text{Vector potential:} \quad \mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

$$\text{For Coulomb gauge:} \quad \nabla \cdot \mathbf{A}(\mathbf{r}) = 0$$

$$\Rightarrow \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r})$$

For confined current density :

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Other examples of current density sources:

Quantum mechanical expression for current density

for a particle of mass M and charge e and of probability amplitude $\Psi(\mathbf{r})$:

$$\mathbf{J}(\mathbf{r}) = -\frac{e\hbar}{2Mi}\left(\Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r}) - \Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r})\right)$$

For an electron in a spherical potential (such as in an atom):

$$\Psi(\mathbf{r}) \equiv \Psi_{nlm_l}(\mathbf{r}) = R_{nl}(r)Y_{lm_l}(\hat{\mathbf{r}})$$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{2Mi}\left|R_{nl}(r)\right|^2 \frac{1}{r\sin\theta}\left(Y_{lm_l}{}^*(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}(\hat{\mathbf{r}})}{\partial\varphi} - Y_{lm_l}(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}{}^*(\hat{\mathbf{r}})}{\partial\varphi}\right)\hat{\boldsymbol{\varphi}}$$

$$= \frac{e\hbar}{M}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})^2\right|\hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}} = \frac{\hat{\mathbf{z}}\times\mathbf{r}}{r\sin\theta}$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{M}\frac{m_l}{r^2\sin^2\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2(\hat{\mathbf{z}}\times\mathbf{r})$$

Details of the electron orbital magnetic dipole moment

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{m_e}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2 \hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}}$

Magnetic dipole moment:

$$\mathbf{m} = \frac{1}{2}\int d^3r'\mathbf{r}' \times \mathbf{J}(\mathbf{r}') = -\frac{e\hbar m_l}{2m_e}\int d^3r' \frac{\mathbf{r}' \times \hat{\boldsymbol{\varphi}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2$$

$$= -\frac{e\hbar m_l}{2m_e}\int d^3r' \frac{-r'\hat{\boldsymbol{\theta}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2$$

Note that: $\hat{\boldsymbol{\theta}} = \cos\theta\cos\varphi\,\hat{\mathbf{x}} + \cos\theta\sin\varphi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$

$$\mathbf{m} = -\frac{e\hbar m_l\hat{\mathbf{z}}}{2m_e}\int d^3r'\left|\Psi_{nlm_l}(\mathbf{r})\right|^2$$

$$= -\frac{e\hbar m_l}{2m_e}\hat{\mathbf{z}}$$

Summary of magnetic field generated by point magnetic dipole moment:

$$\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\hat{\mathbf{r}}\left(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}\right) - \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_e \delta(\mathbf{r}) \right)$$

Magnetic field near nucleus due to orbiting electron:

$$\mathbf{B}_O(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{e}{m_e} L_z \hat{\mathbf{z}} \left\langle \frac{1}{r^3} \right\rangle$$

"Hyperfine" interaction energy:

$$\mathcal{H}_{HF} = -\boldsymbol{\mu}_N \cdot \left(\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r}) + \mathbf{B}_O(\mathbf{r}) \right)$$

$$= \frac{\mu_0}{4\pi}\left(\frac{3\left(\boldsymbol{\mu}_N \cdot \hat{\mathbf{r}}\right)\left(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}\right) - \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e \delta(\mathbf{r}) + \frac{e}{m_e}\left\langle \frac{\mathbf{L} \cdot \boldsymbol{\mu}_N}{r^3} \right\rangle \right)$$

$$\mathcal{H}_{HF} = \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{\mu}_N \cdot \hat{\mathbf{r}})(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}) - \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e \delta(\mathbf{r}) + \frac{e}{m_e}\left\langle \frac{\mathbf{L}\cdot\boldsymbol{\mu}_N}{r^3} \right\rangle \right)$$