

PHY 712 Electrodynamics

10-10:50 AM Online

Plan for Lecture 17:

Read Chapter 7

- 1. Plane polarized electromagnetic waves**
- 2. Reflectance and transmittance of electromagnetic waves – extension to anisotropy and complexity**

Course schedule for Spring 2021

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Wed: 01/27/2021	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/29/2021
2	Fri: 01/29/2021	Chap. 1	Electrostatic energy calculations	#2	02/01/2021
3	Mon: 02/01/2021	Chap. 1 & 2	Electrostatic potentials and fields	#3	02/03/2021
4	Wed: 02/03/2021	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions	#4	02/05/2021
5	Fri: 02/05/2021	Chap. 1 - 3	Brief introduction to numerical methods	#5	02/08/2021
6	Mon: 02/08/2021	Chap. 2 & 3	Image charge constructions	#6	02/10/2021
7	Wed: 02/10/2021	Chap. 2 & 3	Cylindrical and spherical geometries		
8	Fri: 02/12/2021	Chap. 3 & 4	Spherical geometry and multipole moments	#7	02/15/2021
9	Mon: 02/15/2021	Chap. 4	Dipoles and Dielectrics	#8	02/19/2021
10	Wed: 02/17/2021	Chap. 4	Dipoles and Dielectrics		
11	Fri: 02/19/2021	Chap. 4	Polarization and Dielectrics	#9	02/24/2021
12	Mon: 02/22/2021	Chap. 5	Magnetostatics	#10	02/26/2021
13	Wed: 02/24/2021	Chap. 5	Magnetic dipoles and hyperfine interaction	#11	03/01/2021
14	Fri: 02/26/2021	Chap. 5	Magnetic dipoles and dipolar fields		
15	Mon: 03/01/2021	Chap. 6	Maxwell's Equations	#12	03/08/2021
16	Wed: 03/03/2021	Chap. 6	Electromagnetic energy and forces		
17	Fri: 03/05/2021	Chap. 7	Electromagnetic plane waves		
18	Mon: 03/08/2021	Chap. 7	Electromagnetic plane waves		
19	Wed: 03/10/2021	Chap. 7	Optical effects of refractive indices		
20	Fri: 03/12/2021	Chap. 1-7	Review		
	Mon: 03/15/2021 03/05/2021	No class	APS March Meeting PHY 712 Spring 2021 -- Lecture 17	Take Home Exam	2

Maxwell's equations

For linear isotropic media and no sources: $\mathbf{D} = \epsilon \mathbf{E}$; $\mathbf{B} = \mu \mathbf{H}$

Coulomb's law: $\nabla \cdot \mathbf{E} = 0$

Ampere-Maxwell's law: $\nabla \times \mathbf{B} - \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} = 0$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

Analysis of Maxwell's equations without sources -- continued:

Coulomb's law : $\nabla \cdot \mathbf{E} = 0$

Ampere - Maxwell's law : $\nabla \times \mathbf{B} - \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} = 0$

Faraday's law : $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles : $\nabla \cdot \mathbf{B} = 0$

$$\begin{aligned}\nabla \times \left(\nabla \times \mathbf{B} - \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} \right) &= -\nabla^2 \mathbf{B} - \mu\epsilon \frac{\partial (\nabla \times \mathbf{E})}{\partial t} \\ &= -\nabla^2 \mathbf{B} + \mu\epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0\end{aligned}$$

$$\begin{aligned}\nabla \times \left(\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} \right) &= -\nabla^2 \mathbf{E} + \frac{\partial (\nabla \times \mathbf{B})}{\partial t} \\ &= -\nabla^2 \mathbf{E} + \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0\end{aligned}$$

Analysis of Maxwell's equations without sources -- continued:
Both $\mathbf{E}$ and $\mathbf{B}$ fields are solutions to a wave equation:

$$\nabla^2 \mathbf{B} - \frac{1}{v^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$\nabla^2 \mathbf{E} - \frac{1}{v^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\text{where } v^2 \equiv c^2 \frac{\mu_0 \epsilon_0}{\mu \epsilon} \equiv \frac{c^2}{n^2}$$

Plane wave solutions to wave equation :

$$\mathbf{B}(\mathbf{r}, t) = \Re(\mathbf{B}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}) \quad \mathbf{E}(\mathbf{r}, t) = \Re(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t})$$

Analysis of Maxwell's equations without sources -- continued:

Plane wave solutions to wave equation :

$$\mathbf{B}(\mathbf{r}, t) = \Re(\mathbf{B}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}) \quad \mathbf{E}(\mathbf{r}, t) = \Re(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t})$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

Note: ϵ, μ, n, k can all be complex; for the moment we will assume that they are all real (no dissipation).

Note that $\mathbf{E}_0$ and $\mathbf{B}_0$ are not independent;

$$\mathbf{k} = n \frac{\omega}{c} \hat{\mathbf{k}}$$

from Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

$$\Rightarrow \mathbf{B}_0 = \frac{\mathbf{k} \times \mathbf{E}_0}{\omega} = \frac{n \hat{\mathbf{k}} \times \mathbf{E}_0}{c}$$

also note: $\hat{\mathbf{k}} \cdot \mathbf{E}_0 = 0$ and $\hat{\mathbf{k}} \cdot \mathbf{B}_0 = 0$

Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

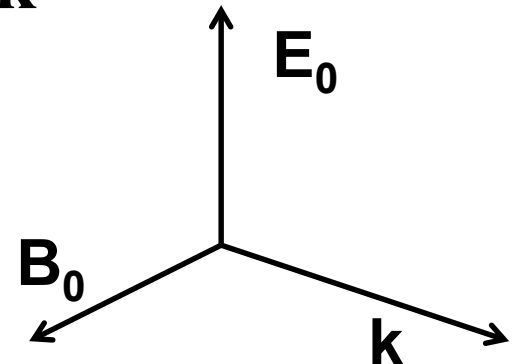
$$\mathbf{B}(\mathbf{r}, t) = \Re\left(\frac{n\hat{\mathbf{k}} \times \mathbf{E}_0}{c} e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}\right) \quad \mathbf{E}(\mathbf{r}, t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} \quad \text{and } \hat{\mathbf{k}} \cdot \mathbf{E}_0 = 0$$

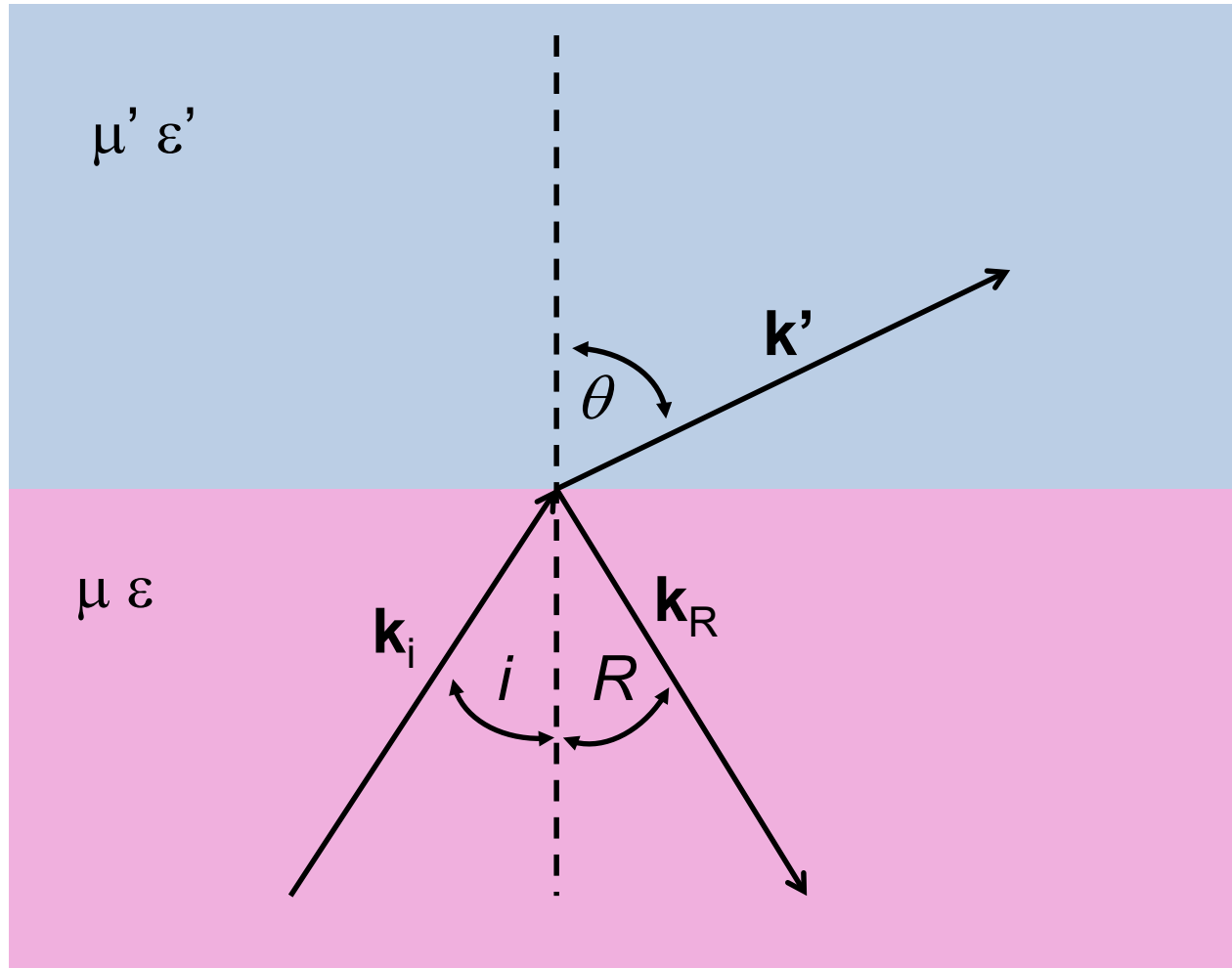
Poynting vector and energy density:

$$\langle \mathbf{S} \rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c} \hat{\mathbf{k}} = \frac{1}{2} \sqrt{\frac{\epsilon}{\mu}} |\mathbf{E}_0|^2 \hat{\mathbf{k}}$$

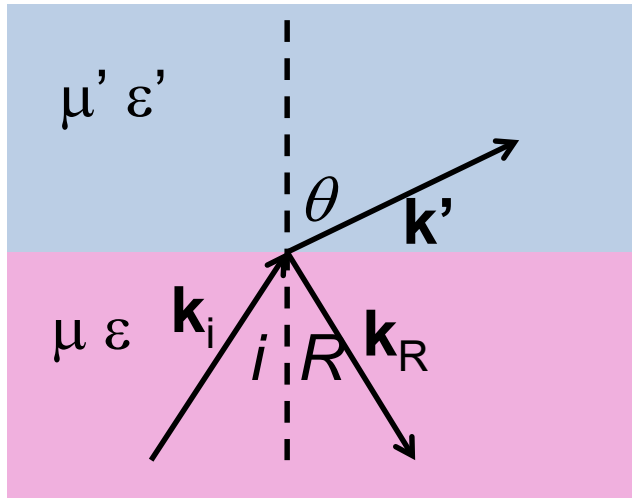
$$\langle u \rangle_{avg} = \frac{1}{2} \epsilon |\mathbf{E}_0|^2$$



Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



Reflection and refraction -- continued



In medium $\mu' \epsilon'$:

$$\mathbf{E}'(\mathbf{r}, t) = \Re \left(\mathbf{E}'_0 e^{i \frac{\omega}{c} (n' \hat{\mathbf{k}}' \cdot \mathbf{r} - ct)} \right)$$

$$\mathbf{B}'(\mathbf{r}, t) = \frac{n'}{c} \hat{\mathbf{k}}' \times \mathbf{E}'(\mathbf{r}, t) = \sqrt{\mu' \epsilon'} \hat{\mathbf{k}}' \times \mathbf{E}(\mathbf{r}, t)$$

In medium $\mu \epsilon$:

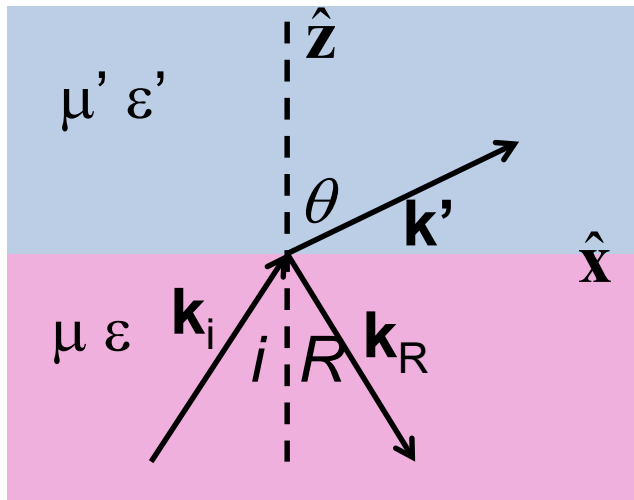
$$\mathbf{E}_i(\mathbf{r}, t) = \Re \left(\mathbf{E}_{0i} e^{i \frac{\omega}{c} (n \hat{\mathbf{k}}_i \cdot \mathbf{r} - ct)} \right)$$

$$\mathbf{B}_i(\mathbf{r}, t) = \frac{n}{c} \hat{\mathbf{k}}_i \times \mathbf{E}_i(\mathbf{r}, t) = \sqrt{\mu \epsilon} \hat{\mathbf{k}}_i \times \mathbf{E}_i(\mathbf{r}, t)$$

$$\mathbf{E}_R(\mathbf{r}, t) = \Re \left(\mathbf{E}_{0R} e^{i \frac{\omega}{c} (n \hat{\mathbf{k}}_R \cdot \mathbf{r} - ct)} \right)$$

$$\mathbf{B}_R(\mathbf{r}, t) = \frac{n}{c} \hat{\mathbf{k}}_R \times \mathbf{E}_R(\mathbf{r}, t) = \sqrt{\mu \epsilon} \hat{\mathbf{k}}_R \times \mathbf{E}_R(\mathbf{r}, t)$$

Reflection and refraction -- continued



Snell's law – matching phase factors at boundary plane $z=0$.

$$e^{i\frac{\omega}{c}(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct)} \Big|_{z=0} = e^{i\frac{\omega}{c}(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct)} \Big|_{z=0}$$

$$= e^{i\frac{\omega}{c}(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct)} \Big|_{z=0}$$

matching plane: $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$

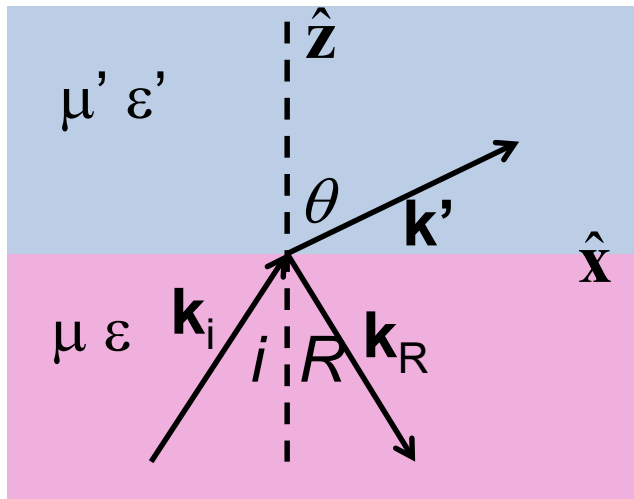
$$\hat{\mathbf{k}}'\cdot\mathbf{r} = x \sin \theta$$

$$\hat{\mathbf{k}}_i\cdot\mathbf{r} = x \sin i = \hat{\mathbf{k}}_R\cdot\mathbf{r} = x \sin R \quad \Rightarrow i = R$$

$$n'\hat{\mathbf{k}}'\cdot\mathbf{r} = n\hat{\mathbf{k}}_i\cdot\mathbf{r} \quad \Rightarrow n'x \sin \theta = nx \sin i$$

Snell's law : $n' \sin \theta = n \sin i$

Reflection and refraction -- continued



Continuity equations at boundary with no sources :

$$\nabla \cdot \mathbf{D} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0$$

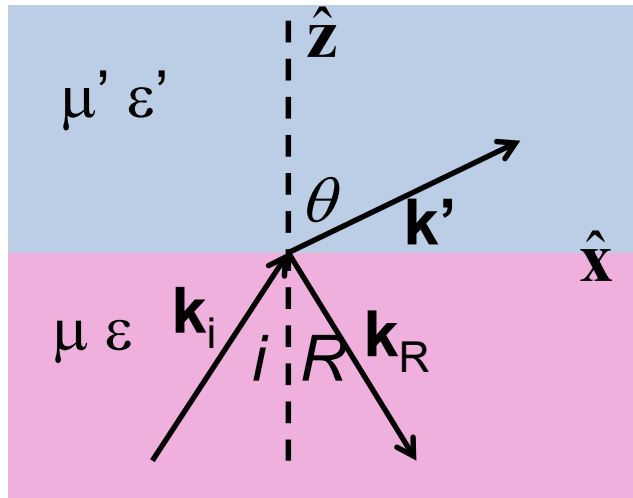
$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

Matching field amplitudes at boundary plane :

$$\mathbf{D} \cdot \hat{\mathbf{z}}, \mathbf{B} \cdot \hat{\mathbf{z}} \quad \text{continuous}$$

$$\mathbf{H} \times \hat{\mathbf{z}}, \mathbf{E} \times \hat{\mathbf{z}} \quad \text{continuous}$$

Reflection and refraction -- continued



$\mathbf{E} \times \hat{\mathbf{z}}$ continuous:

$$(\mathbf{E}_{0i} + \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \mathbf{E}'_0 \times \hat{\mathbf{z}}$$

$\mathbf{H} \times \hat{\mathbf{z}}$ continuous:

$$\begin{aligned} \frac{n}{\mu} (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \times \hat{\mathbf{z}} \\ = \frac{n'}{\mu'} \hat{\mathbf{k}}' \times \mathbf{E}'_0 \times \hat{\mathbf{z}} \end{aligned}$$

Matching field amplitudes at boundary plane:

$\mathbf{D} \cdot \hat{\mathbf{z}}$ continuous:

$$\epsilon (\mathbf{E}_{0i} + \mathbf{E}_{0R}) \cdot \hat{\mathbf{z}} = \epsilon' \mathbf{E}'_0 \cdot \hat{\mathbf{z}}$$

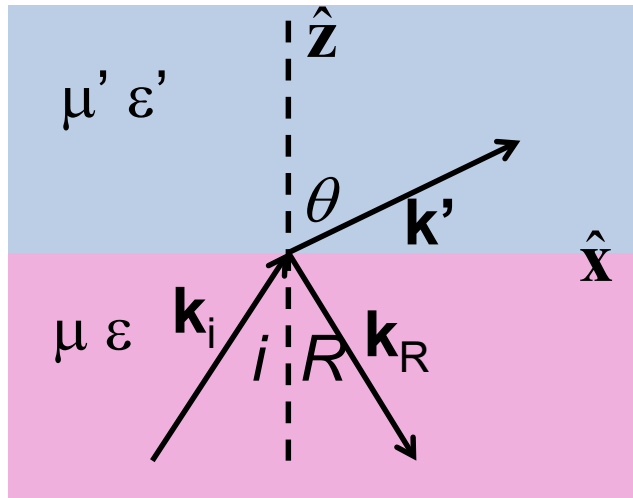
$\mathbf{B} \cdot \hat{\mathbf{z}}$ continuous:

$$\begin{aligned} n (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \cdot \hat{\mathbf{z}} = \\ n' \hat{\mathbf{k}}' \times \mathbf{E}'_0 \cdot \hat{\mathbf{z}} \end{aligned}$$

Known: $\mathbf{E}_{0i}, \hat{\mathbf{k}}_i$

Unknown: $\mathbf{E}'_0, \mathbf{E}_{0R}, \hat{\mathbf{k}}'$

Reflection and refraction -- continued



s-polarization – $\mathbf{E}$ field “polarized”
perpendicular to plane of incidence

$\mathbf{E} \times \hat{\mathbf{z}}$ continuous:

$$(\mathbf{E}_{0i} + \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \mathbf{E}'_0 \times \hat{\mathbf{z}}$$

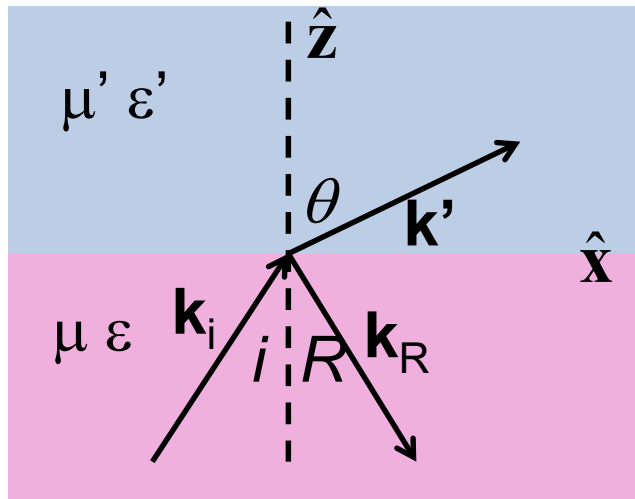
$\mathbf{H} \times \hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu} (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \frac{n'}{\mu'} \hat{\mathbf{k}}' \times \mathbf{E}'_0 \times \hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}} = \frac{n \cos i - \frac{\mu}{\mu'} n' \cos \theta}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta} \quad \frac{E'_0}{E_{0i}} = \frac{2n \cos i}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta}$$

Note that: $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

Reflection and refraction -- continued



p-polarization – $\mathbf{E}$ field “polarized” parallel to plane of incidence

$\mathbf{D} \cdot \hat{\mathbf{z}}$ continuous:

$$\varepsilon (\mathbf{E}_{0i} + \mathbf{E}_{0R}) \cdot \hat{\mathbf{z}} = \varepsilon' \mathbf{E}'_0 \cdot \hat{\mathbf{z}}$$

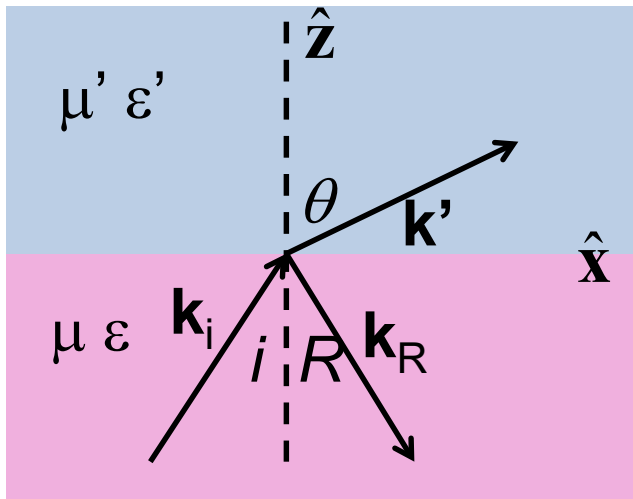
$\mathbf{H} \times \hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu} (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \frac{n'}{\mu'} \hat{\mathbf{k}}' \times \mathbf{E}'_0 \times \hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos \theta}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta} \quad \frac{E'_0}{E_{0i}} = \frac{2n \cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta}$$

Note that: $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

Reflection and refraction -- continued



Intensity in terms of Poynting vector:

$$\langle \mathbf{S} \rangle_{avg} = \frac{n |\mathbf{E}_0|^2}{2\mu c} \hat{\mathbf{k}} = \frac{1}{2} \sqrt{\frac{\epsilon}{\mu}} |\mathbf{E}_0|^2 \hat{\mathbf{k}}$$

Reflectance, transmittance:

$$R = \left| \frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} \right| = \left| \frac{E_{0R}}{E_{0i}} \right|^2$$

$$T = \frac{\mathbf{S}' \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n'}{n} \frac{\mu}{\mu'} \frac{\cos \theta}{\cos i}$$

Note that $R + T = 1$

For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n \cos i - \frac{\mu}{\mu'} n' \cos \theta}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta} \quad \frac{E'_0}{E_{0i}} = \frac{2n \cos i}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta}$$

Note that : $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

$$R = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{n \cos i - \frac{\mu}{\mu'} n' \cos \theta}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta} \right|^2$$

$$T = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n'}{n} \frac{\mu}{\mu'} \frac{\cos \theta}{\cos i} = \frac{4nn' \cos i \cos \theta}{\left| n \cos i + \frac{\mu}{\mu'} n' \cos \theta \right|^2} \frac{\mu}{\mu'}$$

For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos \theta}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta} \quad \frac{E'_0}{E_{0i}} = \frac{2n \cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta}$$

Note that: $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

$$R = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{\frac{\mu}{\mu'} n' \cos i - n \cos \theta}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta} \right|^2$$

$$T = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n'}{n} \frac{\mu}{\mu'} \frac{\cos \theta}{\cos i} = \frac{4nn' \cos i \cos \theta}{\left| \frac{\mu}{\mu'} n' \cos i + n \cos \theta \right|^2} \frac{\mu}{\mu'}$$

Special case: normal incidence ($i=0$, $\theta=0$)

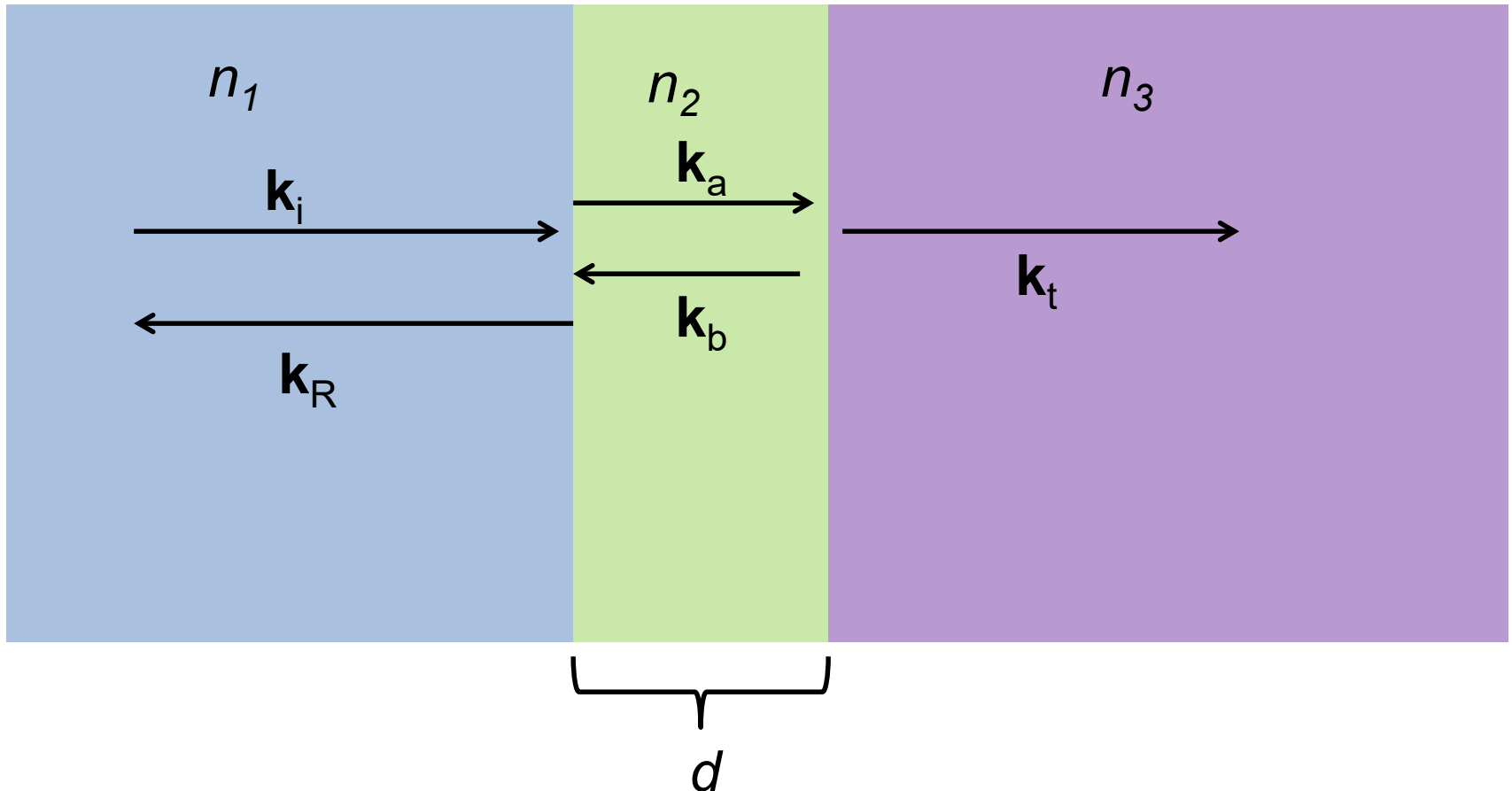
$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' - n}{\frac{\mu}{\mu'} n' + n} \quad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'} n' + n}$$

Reflectance, transmittance:

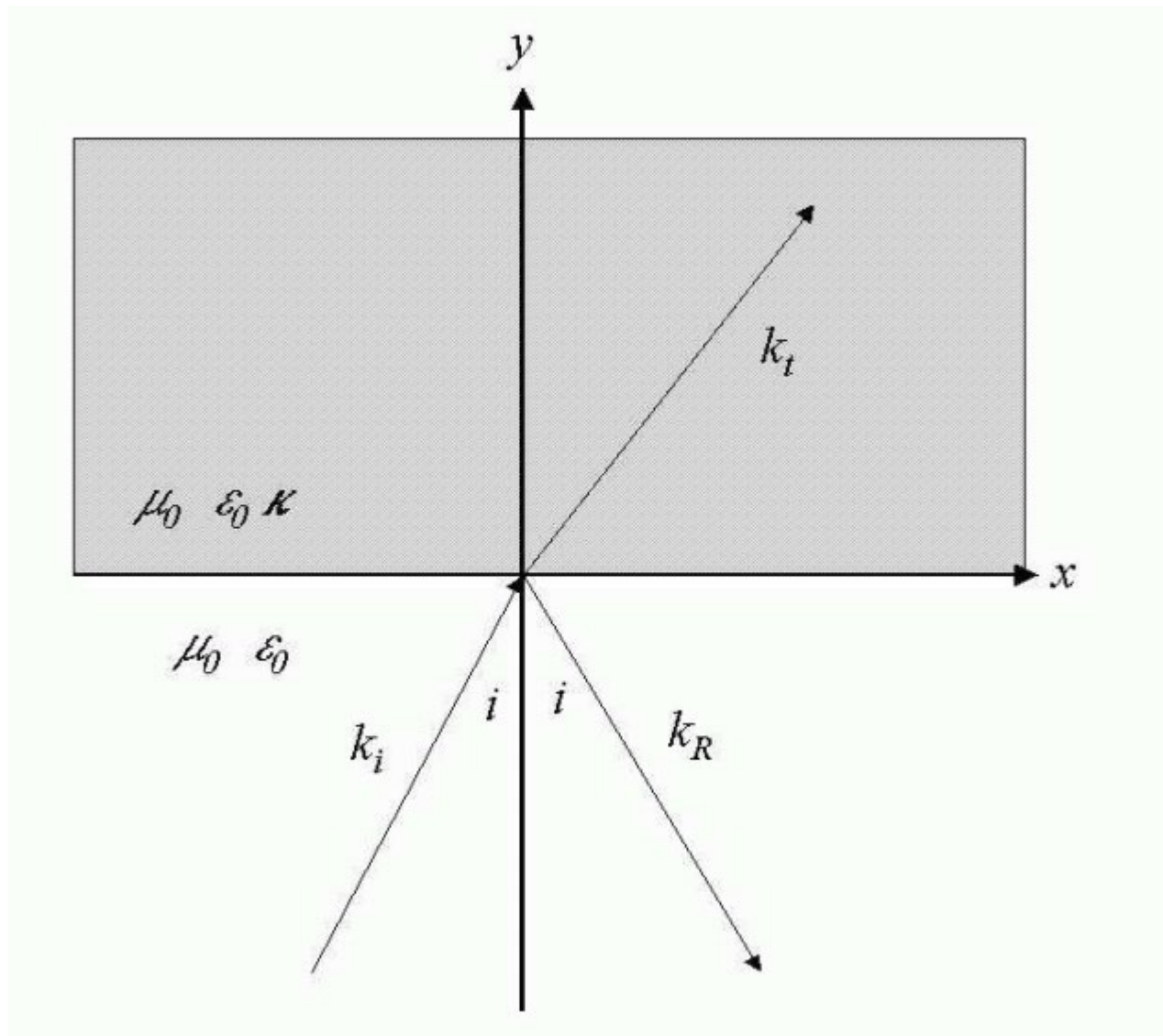
$$R = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{\frac{\mu}{\mu'} n' - n}{\frac{\mu}{\mu'} n' + n} \right|^2$$

$$T = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n' \mu}{n \mu'} = \left| \frac{2n}{\frac{\mu}{\mu'} n' + n} \right|^2 \frac{n' \mu}{n \mu'}$$

Multilayer dielectrics (Problem #7.2)



Extension of analysis to anisotropic media --



Consider the problem of determining the reflectance from an anisotropic medium with isotropic permeability μ_0 and anisotropic permittivity $\varepsilon_0 \mathbf{\kappa}$ where:

$$\mathbf{\kappa} \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}$$

By assumption, the wave vector in the medium is confined to the x - y plane and will be denoted by

$$\mathbf{k}_t \equiv \frac{\omega}{c} (n_x \hat{\mathbf{x}} + n_y \hat{\mathbf{y}}), \text{ where } n_x \text{ and } n_y \text{ are to be determined.}$$

The electric field inside the medium is given by:

$$\mathbf{E} = (E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{z}}) e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}.$$

Inside the anisotropic medium, Maxwell's equations are:

$$\nabla \cdot \mathbf{H} = 0 \qquad \nabla \cdot \boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} - i\omega\mu_0\mathbf{H} = 0 \qquad \nabla \times \mathbf{H} + i\omega\epsilon_0\boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

After some algebra, the equation for $\mathbf{E}$ is:

$$\begin{pmatrix} \kappa_{xx} - n_y^2 & n_x n_y & 0 \\ n_x n_y & \kappa_{yy} - n_x^2 & 0 \\ 0 & 0 & \kappa_{zz} - (n_x^2 + n_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0.$$

From $\mathbf{E}$, $\mathbf{H}$ can be determined from

$$\mathbf{H} = \frac{1}{\mu_0 c} \left\{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) + (E_y n_x - E_x n_y) \hat{\mathbf{z}} \right\} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

The fields for the incident and reflected waves are the same as for the isotropic case.

$$\mathbf{k}_i = \frac{\omega}{c} (\sin i \hat{\mathbf{x}} + \cos i \hat{\mathbf{y}}),$$

$$\mathbf{k}_R = \frac{\omega}{c} (\sin i \hat{\mathbf{x}} - \cos i \hat{\mathbf{y}}).$$

Note that, consistent with Snell's law: $n_x = \sin i$

Continuity conditions at the $y=0$ plane must be applied for the following fields:

$\mathbf{H}(x, 0, z, t)$, $E_x(x, 0, z, t)$, $E_z(x, 0, z, t)$, and $D_y(x, 0, z, t)$.

There will be two different solutions, depending of the polarization of the incident field.

Solution for s-polarization

$$E_x = E_y = 0 \quad \Rightarrow \quad n_y^2 = \kappa_{zz} - n_x^2$$

$$\mathbf{E} = E_z \hat{\mathbf{z}} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t} \quad \mathbf{H} = \frac{1}{\mu_0 c} \left\{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) \right\} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}$$

E_z must be determined from the continuity conditions:

$$E_0 + E_0'' = E_z \quad (E_0 - E_0'') \cos i = E_z n_y \quad (E_0 + E_0'') \sin i = E_z n_x$$

$$\frac{E_0''}{E_0} = \frac{\cos i - n_y}{\cos i + n_y}.$$

Solution for p-polarization

$$E_z = 0 \quad \Rightarrow \quad n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}} (\kappa_{yy} - n_x^2).$$

$$\mathbf{E} = E_x \left(\hat{\mathbf{x}} - \frac{\kappa_{xx} n_x}{\kappa_{yy} n_y} \hat{\mathbf{y}} \right) e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}.$$

$$\mathbf{H} = -\frac{E_x}{\mu_0 c} \frac{\kappa_{xx}}{n_y} \hat{\mathbf{z}} e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}.$$

E_x must be determined from the continuity conditions:

$$(E_0 - E_0'') \cos i = E_x \quad (E_0 + E_0'') = \frac{\kappa_{xx}}{n_y} E_x \quad (E_0 + E_0'') \sin i = \frac{\kappa_{xx} n_x}{n_y} E_x.$$

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx} \cos i - n_y}{\kappa_{xx} \cos i + n_y}.$$

Extension of analysis to complex dielectric functions

For simplicity assume that $\mu = \mu_0$

Suppose the dielectric function is complex :

$$\epsilon = \epsilon_R + i\epsilon_I \qquad \frac{\epsilon}{\epsilon_0} = (n_R + in_I)^2 \equiv \alpha + i\beta$$

$$n_R = \left(\frac{\sqrt{\alpha^2 + \beta^2} + \alpha}{2} \right)^{1/2} \qquad n_I = \left(\frac{\sqrt{\alpha^2 + \beta^2} - \alpha}{2} \right)^{1/2}$$

$$\mathbf{E}(\mathbf{r}, t) = \Re \left(\mathbf{E}_0 e^{i\frac{\omega}{c}(\mathbf{n}\hat{\mathbf{k}}\cdot\mathbf{r} - ct)} \right) = \Re \left(\mathbf{E}_0 e^{i\frac{\omega}{c}(n_R\hat{\mathbf{k}}\cdot\mathbf{r} - ct)} \right) e^{-\frac{\omega}{c}n_I\hat{\mathbf{k}}\cdot\mathbf{r}}$$