PHY 712 Electrodynamics
10-10:50 AM MWF Online

Discussion notes for Lecture 1:

Reading: Appendix 1 and Chapters I&1

1. Course structure and expectations
2. Units – SI vs Gaussian
3. Electrostatics – Poisson equation

Physics Colloquium series – Thursdays 4-5 PM online

SEMINARS-2021-SPRING

WFU Physics Colloquium Schedule — Spring 2021

Previous and future colloquia

All colloquia will be held at 4 PM online (unless noted otherwise).

Thurs. Jan. 28, 2021 — Professor George C. Schatz, Northwestern University — "Plasmonic Lattices" (host: J. Macosko)

Thurs. Feb. 04, 2021 — Professor Lalit Deshmukh, University of California, San Diego –"ALIX in Wonderland: Multivalency, Phosphorylation-mediated Amyloids, Autoinhibition, and Endosomal Membrane Interactions" (host: S. Cho)

Thurs. Feb. 11, 2021 — Professor Carolyn Bertozzi, Stanford University — "Therapeutic Opportunities in Glycoscience" (host: J. Macosko)

Your questions –

From Gao -- Although all field vectors have the same physical dimensions in Gaussian system, Why do SI units become the current standard? What advantages do they have?

From Nick –

1. On slide 13, is that a double cross product and what does that mean physically?
2. I'm fairly certain I understand B,E but can you explain M,P,D,H?

Note that we will set up the one-on-one meetings on Friday.   Please let me know your schedules and preferences. If any of you prefer to meet with me in small groups, that is also fine.   Please let me know that as well.

http://users.wfu.edu/natalie/s21phy712/

PHY 712 Electrodynamics

MWF 10-10:50 PM	Online	http://www.wfu.edu/~natalie/s21phy712/

Instructor: Natalie Holzwarth	**Office:**300 OPL	**e-mail:**natalie@wfu.edu

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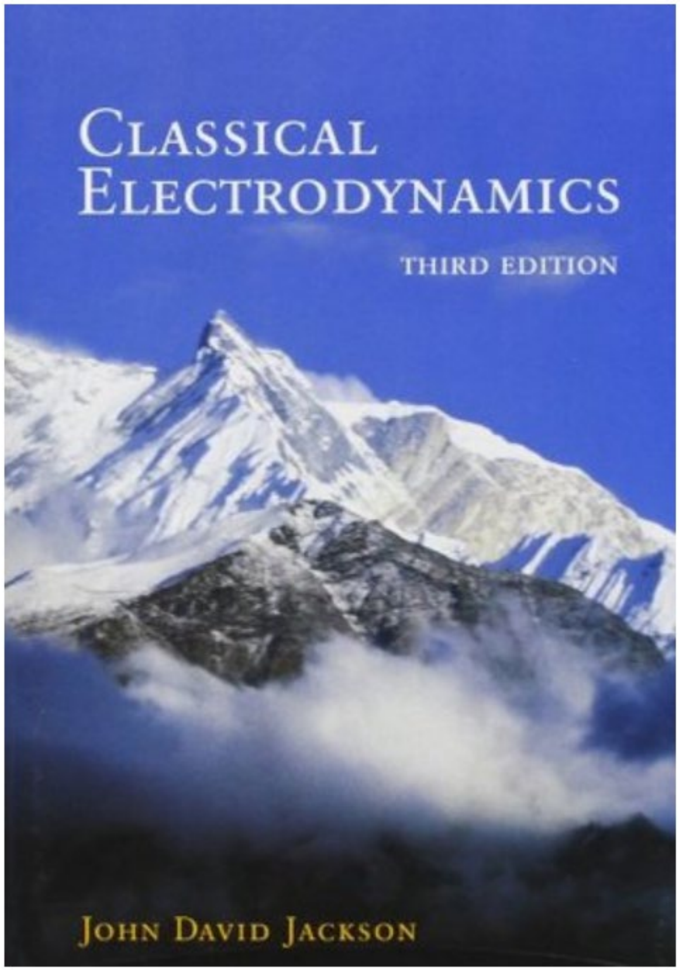
- General information
- Syllabus and homework assignments
- Lecture notes
- Some presentation ideas

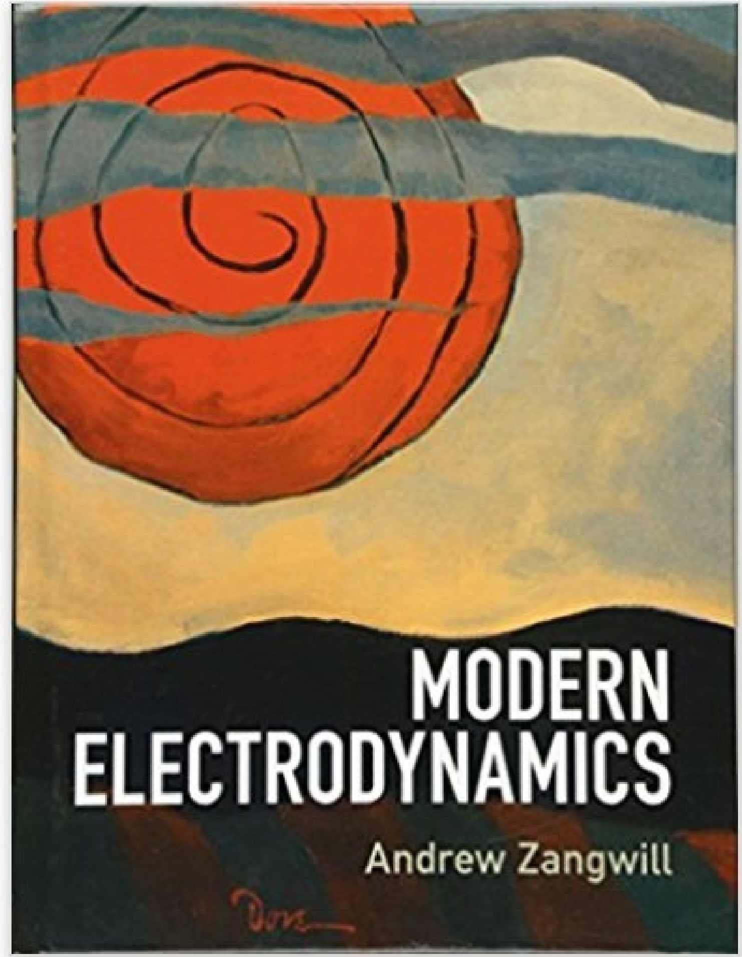
Last modfied: Saturday, 23-Jan-2021 15:36:00 EST

Textbook



Third edition

Optional supplement



http://users.wfu.edu/natalie/s21phy712/info

General Information

This course is a one semester survey of Electrodynamics at the graduate level, using the textbook: **Classical Electrodynamics**, 3rd edition, by John David Jackson (John Wiley & Sons, Inc., 1999) -- "**JDJ**". (link to errata for early printings) Note that it is necessary to get the **third** edition in order to synchronize with the class lectures and homework. The more recent textbook: **Modern Electrodynamics**, by Andrew Zangwill (Cambridge University Press, 2013) will be used as a supplement.

Adapting to the challenges of these unprecedented times, this course is desiged as "online", adjusting to the best recommendations for healthy practices. The course will consist of the following components:

- Synchronous online meetings MWF 10-10:50 AM. (local time in Winston-Salem, NC, USA) Starting with the second meeting, the sessions will focus on discussion of the material, particularly answering your prepared and spontaneous questions.
- Asynchronous review of annotated lecture notes and corresponding textbook sections. Starting with Lecture 2, the annotated lecture notes will be available one day before the corresponding synchronous online discussion. For each class meeting, students will be expected to submit (by email) at least one question for class discussion at least 3 hours before the synchronous online meeting.
- Homework sets. Typically there will be one homework problem associated with each synchronous meeting.
- There will be two take-home exams, one at mid-term and the other during finals week.
- There will be one project on a chosen topic related to electrodynamics.
- There will be weekly one-on-one meetings of each student with the instructor to discuss the course material, homework, and/or projects. These may be face-to-face or online as appropriate.

Links to WFU pandemic policies – Since this class is online, however, it is possible that some of the one-on-one meetings can be face-to-face.

Wake Forest University's Standard Operating Procedure for Class Sessions Following Public Health Guidance (Policy 2.A.02). This is an updated version of our policies and procedures. Please note the clarification on our masking requirement for classes regardless of location (indoors or outdoors), a revision that makes this policy consistent with our current University masking policy.

Spring 2021 College COVID-19 Classrooms FAQs and Syllabus Statement. As in the fall, the Office of the Dean of the College expects the Spring 2021 COVID-19 Syllabus Statement to be on syllabi for any classes taught in-person (face to face or blended) to communicate clear expectations around masking, physical distancing, and illness from the very first day of class. Our COVID-19 Classroom FAQs includes classroom cleaning protocols; guidance on how to reserve space for out-of-class academic activities like masked and physically distanced office hours, small group workshops, film screenings, and language conversation hours through DeaconSpace; and assistance for anyone teaching in a less familiar space this semester.

It is likely that your grade for the course will depend upon the following factors:

Class participation	15%
Problem sets*	35%
Project	15%
Exams	35%

*In general, there will a new assignment after each lecture, so that for optimal learning, it would be best to complete each assignment before the next scheduled lecture. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts.

→ Schedule weekly one-on-one meetings

Some Ideas for Computational Project

The purpose of the "Computational Project" is to provide an opportunity for you to study a topic of your choice in greater depth. The general guideline for your choice of project is that it should have something to do with electrodynamics, and there should be some degree of computation or analysis with the project. The completed project will include a short write-up and a ~20min presentation to the class. You may design your own project or use one of the following list (which will be updated throughout the term).

- Evaluate the Ewald sum of various ionic crystals using Maple or a programing language. (Template available in Fortran code.)
- Work out the details of the finite difference or finite element methods.
- Work out the details of the hyperfine Hamiltonian as discussed in Chapter 5 of Jackson.
- Work out the details of Jackson problem 7.2 and related problems.
- Work out the details of reflection and refraction from birefringent materials.
- Analyze the Kramers-Kronig transform of some optical data or calculations.
- Determine the classical electrodynamics associated with an infrared or optical laser.
- Analyze the radiation intensity and spectrum from an interesting source such as an atomic or molecular transition, a free electron laser, etc.
- Work out the details of Jackson problem 14.15.

http://users.wfu.edu/natalie/s21phy712/homework/

Course schedule for Spring 2021

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Wed: 01/27/2021	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/29/2021
2	Fri: 01/29/2021	Chap. 1	Electrostatic energy calculations		
3	Mon: 02/01/2021	Chap. 1	Electrostatic potentials and fields		
4	Wed: 02/03/2021	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions		
5	Fri: 02/05/2021	Chap. 1 - 3	Brief introduction to numerical methods		
6	Mon: 02/08/2021	Chap. 2 & 3	Image charge constructions		
7	Wed: 02/10/2021	Chap. 2 & 3	Cylindrical and spherical geometries		
8	Fri: 02/12/2021	Chap. 3 & 4	Spherical geometry and multipole moments		

Comment about HW #1: (Jackson problem 1.5)

The time-averaged potential of a neutral hydrogen atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-2r/a_0}}{r} \left(1 + \frac{r}{a_0}\right)$$

where q denotes the magnitude of the elementary charge of an electron or proton and where a_0 denotes the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your results physically.

Be careful to take into account the behavior of the potential for $r \rightarrow 0$.

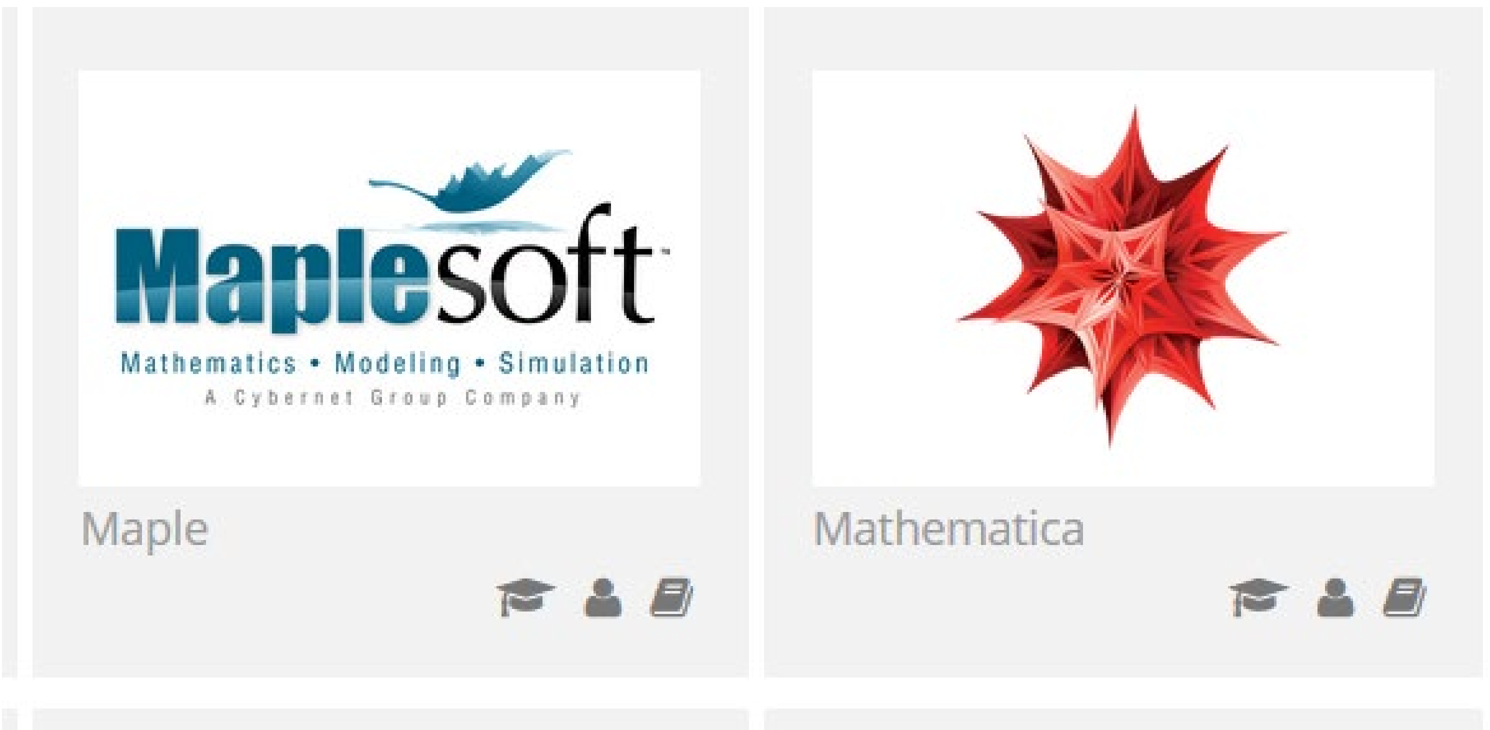
Additional information –

Mid-term exam March 15-19

Presentations for E&M will be in late April and early May

Remember to check your algebraic manipulation software --

https://software.wfu.edu/audience/students/



Material discussed in Appendix of textbook --

Units - SI vs Gaussian

Coulomb's Law

$$F = K_C \frac{q_1 q_2}{r_{12}^2}. \tag{1}$$

Ampere's Law

$$F = K_A \frac{i_1 i_2}{r_{12}^2}\, d\mathbf{s_1} \times d\mathbf{s_2} \times \hat{\mathbf{r}}_{\mathbf{12}}, \tag{2}$$

In the equations above, the current and charge are related by $i_1 = dq_1/dt$ for all unit systems. The two constants K_C and K_A are related so that their ratio K_C/K_A has the units of $(m/s)^2$ and it is *experimentally* known that the ratio has the value $K_C/K_A = c^2$, where c is the speed of light.

Units - SI vs Gaussian – continued

The choices for these constants in the SI and Gaussian units are given below:

	CGS (Gaussian)	SI
K_C	1	$\frac{1}{4\pi\epsilon_0}$
K_A	$\frac{1}{c^2}$	$\frac{\mu_0}{4\pi}$

Here, $\frac{\mu_0}{4\pi} \equiv 10^{-7} N/A^2$ and $\frac{1}{4\pi\epsilon_0} = c^2 \cdot 10^{-7} N/A^2 = 8.98755 \times 10^9 N \cdot m^2/C^2$.

Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m^2/s$	$dyne$	$gm \cdot cm^2/s$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

Rectangular Snip

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

Your questions – Why are SI units the standard?

My (biased) opinion – it was a great debate and the experimentalists won….

Units - SI vs Gaussian – continued

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Your questions – What is the meaning of all of the listed fields?

As we will see throughout the course, the E and B fields represent the basic electric and magnetic fields while the other fields include or represent electric and magnetic effects of matter.

Basic equations of electrodynamics

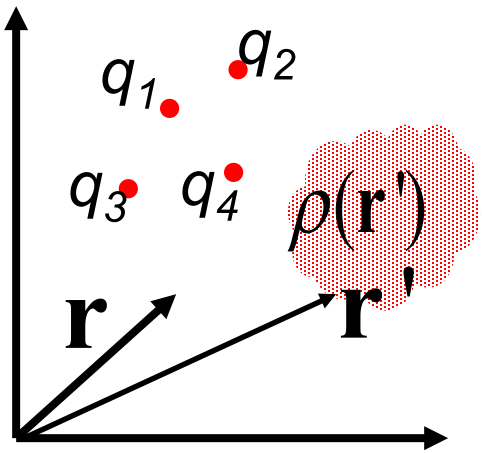
CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$	$u = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

Units choice for this course:

SI units for Jackson in Chapters 1-10

Gaussian units for Jackson in Chapters 11-16

Electrostatics



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_i q_i \frac{\mathbf{r}-\mathbf{r}_i}{\left|\mathbf{r}-\mathbf{r}_i\right|^3}$$

$$= \frac{1}{4\pi\epsilon_0}\int d^3r'\rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^3}$$

Electrostatics

Discrete versus continuous charge distributions

In terms of Dirac delta function:

$$\rho(\mathbf{r}) = \sum_i q_i \ \delta(\mathbf{r} - \mathbf{r}_i)$$

Digression: Note that in cartesian coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \delta(x - x_i)\delta(y - y_i)\delta(z - z_i)$$

in spherical polar coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \frac{1}{r^2}\delta(r - r_i)\delta(\cos\theta - \cos\theta_i)\delta(\phi - \phi_i)$$

Differential equations --

Electrostatics

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{E} = 0$$

Electrostatic potential

$$\mathbf{E} = -\nabla \Phi(r).$$

$$\nabla^2 \Phi(r) = -\rho(r)/\epsilon_0.$$

Relationship between integral and differential forms of electrostatics --

Differential form

$$\nabla^2\Phi(\mathbf{r}) = -\rho(\mathbf{r}) / \epsilon_0$$

Integral form

$$\Phi(\mathbf{r}) =$$

$$\frac{1}{4\pi\epsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Relationship between integral and differential forms of electrostatics --

Need to show: $$\nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -4\pi \delta^3(\mathbf{r} - \mathbf{r}').$$

Noting that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \delta^3(\mathbf{r} - \mathbf{r}') f(\mathbf{r}) = f(\mathbf{r}'),$$

we see that we must show that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) = -4\pi f(\mathbf{r}').$$

We introduce a small radius a such that:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \lim_{a\to 0} \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}}.$$

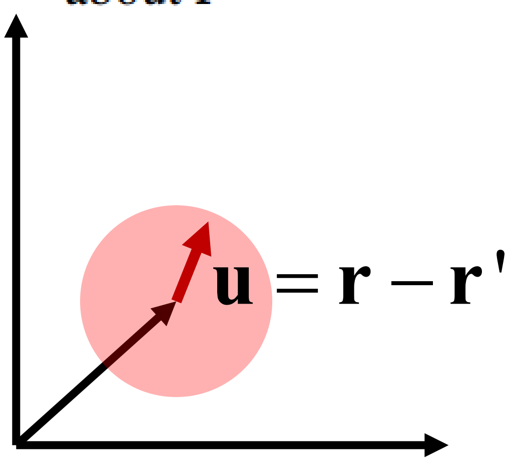
For a fixed value of a,

$$\nabla^2 \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}} = \frac{-3a^2}{(|\mathbf{r}-\mathbf{r}'|^2 + a^2)^{5/2}}.$$

If the function $f(\mathbf{r})$ is continuous, we can make a Taylor expansion of it about the point $\mathbf{r} = \mathbf{r}'$, keeping only the first term. The integral over the small sphere about $\mathbf{r}'$ can be carried out analytically, by changing to a coordinate system centered at $\mathbf{r}'$;

so that

$$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\, \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) \approx f(\mathbf{r}') \lim_{a\to 0} \int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}}.$$



$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du\, \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

For $a \ll R$, $\quad 4\pi \frac{-R^3}{\left(R^2+a^2\right)^{3/2}} \approx -4\pi$

$$\rightarrow \int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r\, \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) \approx f(\mathbf{r}')(-4\pi),$$

$$\rightarrow \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$$

Example in HW1

The electrostatic potential of a neutral H atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right).$$

Find the charge density (both continuous and discrete) for this potential.

Hint #1: For continuous contribution you can use the identity: $\nabla^2\Phi(r) = \frac{1}{r}\frac{\partial^2(r\Phi(r))}{\partial r^2}$

Hint #2: Don't forget to consider possible discrete contributions.