PHY 712 Electrodynamics
10-10:50 AM MWF Online

Discussion for Lecture 30:

Continue reading Chap. 14 –

Radiation by moving charges

1. Recap of results for synchrotron radiation from land based sources
2. Synchrotron radiation from astronomical sources
3. Compton scattering

PHYSICS COLLOQUIUM

4 PM

THURSDAY • APRIL 15, 2021

"Programming with DNA Outside Living Cells: From Gene Circuits to Self-Assembly"

Cell-free transcription-translation (TXTL) has become a highly versatile experimental environment to construct biochemical systems *in vitro* by executing either natural or synthetic gene circuits. In particular, TXTL enables interrogating biochemical systems quantitatively and in isolation far from the complexity of real living cells. I will present several experiments that my lab has done recently using an all-*E. coli* TXTL system. First, I will present this TXTL system, what it is, what it does. In the second part of my talk, I will show examples of dynamical systems directed by gene circuits executed either in test tubes or in microfluidic chips. In the last part of the talk I will show how we construct synthetic cell systems using TXTL and how synthetic cells are convenient to uncover and quantify fundamental aspects of supramolecular assembly.

Dr. Vincent Noireaux
School of Physics and Nanotechnology (PAN)
University of Minnesota
Minneapolis, MN

28	Fri: 04/09/2021	Chap. 14	Radiation from accelerating charged particles	#20	04/12/2021
29	Mon: 04/12/2021	Chap. 14	Synchrotron radiation	#21	04/14/2021
30	Wed: 04/14/2021	Chap. 14	Synchrotron radiation	#22	04/19/2021
31	Fri: 04/16/2021	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/19/2021	Chap. 15	Radiation from collisions of charged particles		
33	Wed: 04/21/2021	Chap. 13	Cherenkov radiation		
34	Fri: 04/23/2021		Special topic: E & M aspects of superconductivity		
35	Mon: 04/26/2021		Special topic: E & M aspects of superconductivity		
36	Wed: 04/28/2021		Review		
37	Fri: 04/30/2021		Review		
	Mon: 05/03/2021		Presentations I		
	Wed: 05/05/2021		Presentations II		

PHY 712 -- Assignment #22

April 12, 2021

Complete reading Chap. 14 in ***Jackson*** *. In class, we showed how the synchotron radiation spectrum is scaled by the critical frequency ω_C or critical energy $E_C = \hbar\omega_C$. Using the intensity formula for radiation in the parallel plane at $\theta=0$, for a beam with E_C=10 GeV, estimate the intensity relative to peak intensity for the following types of radiation (noting your choice of wavelength for each range)*

1. *Infrared*
2. *Visable*
3. *Xray*

Your questions –

From Gao: In my sense, energy level transition radiation is easy to understand for me. From EM, we know that a moving charge also radiates. But I am not sure what mechanism leads to this radiation? Even though we have known how to calculate its intensity already. Could you give some hints?

Comment: While it may not be intuitive, the analysis says that radiation occurs whenever a charged particle accelerates. For quantum systems, other factors occur.

Comment about units

Differential power (cgs Gaussian)

$$\frac{dP_r(t_r)}{d\Omega} = \frac{e^2}{4\pi c} \frac{\left|\hat{\mathbf{r}} \times \left(\left(\hat{\mathbf{r}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right)\right|^2}{\left(1 - \hat{\mathbf{r}} \cdot \boldsymbol{\beta}\right)^5}$$

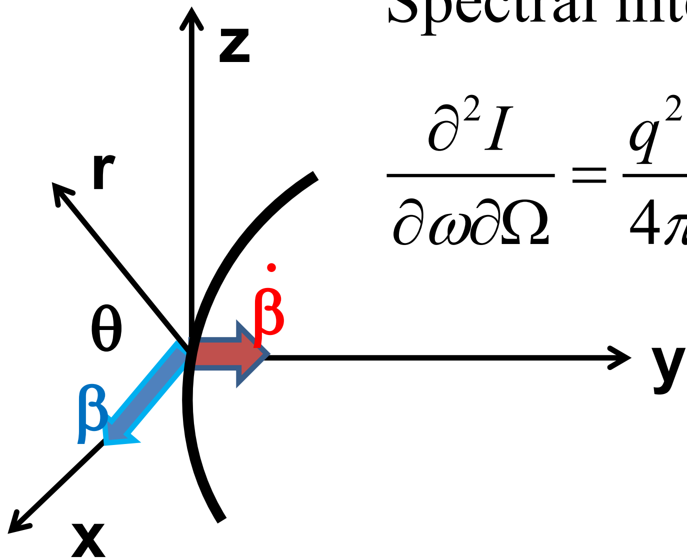
e measured in Statcoulombs
Length measured in cm
Energy measured in ergs

Differential power (SI)

$$\frac{dP_r(t_r)}{d\Omega} = \frac{e^2}{\left(4\pi\epsilon_0\right)4\pi c} \frac{\left|\hat{\mathbf{r}} \times \left(\left(\hat{\mathbf{r}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right)\right|^2}{\left(1 - \hat{\mathbf{r}} \cdot \boldsymbol{\beta}\right)^5}$$

e measured in Coulombs
Length measured in m
Energy measured in joules

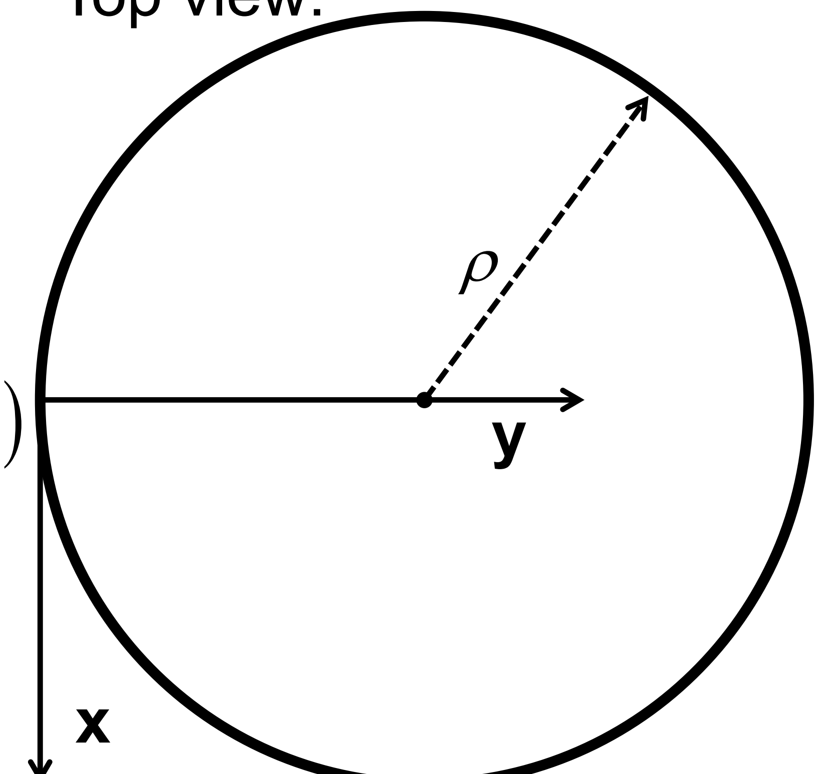
Main results from synchrotron radiation spectra from man made sources --



Spectral intensity relationship:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r\ e^{i\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\ \left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right]\right|^2$$

Top view:

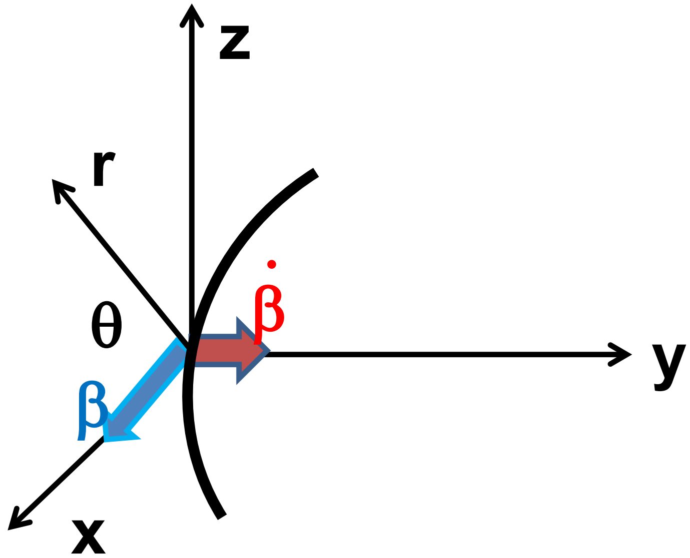


$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho) + \rho\hat{\mathbf{y}}\left(1-\cos(vt_r/\rho)\right)$$

$$\boldsymbol{\beta}(t_r) = \beta\left(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho)\right)$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho) + \rho\hat{\mathbf{y}}\left(1-\cos(vt_r/\rho)\right)$$

$$\boldsymbol{\beta}(t_r) = \beta\left(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho)\right)$$

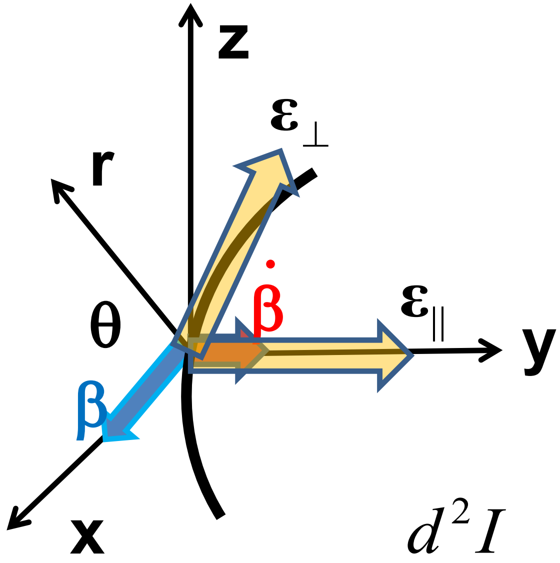
For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Note that we have previous shown that in the radiation zone, the Poynting vector is in the $\hat{\mathbf{r}}$ direction; we can then choose to analyze two orthogonal polarization directions:

$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}) = \beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt_r/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt_r/\rho)\right)$$



$$\boldsymbol{\varepsilon}_{\|} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right) =$$

$$\beta\left(-\boldsymbol{\varepsilon}_{\|}\sin\left(vt_r/\rho\right) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos\left(vt_r/\rho\right)\right)$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\beta)\mathrm{e}^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)}dt\right|^2$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\|}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\}$$

$$C_{\|}(\omega) = \int_{-\infty}^{\infty} dt\,\sin(vt/\rho)\mathrm{e}^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

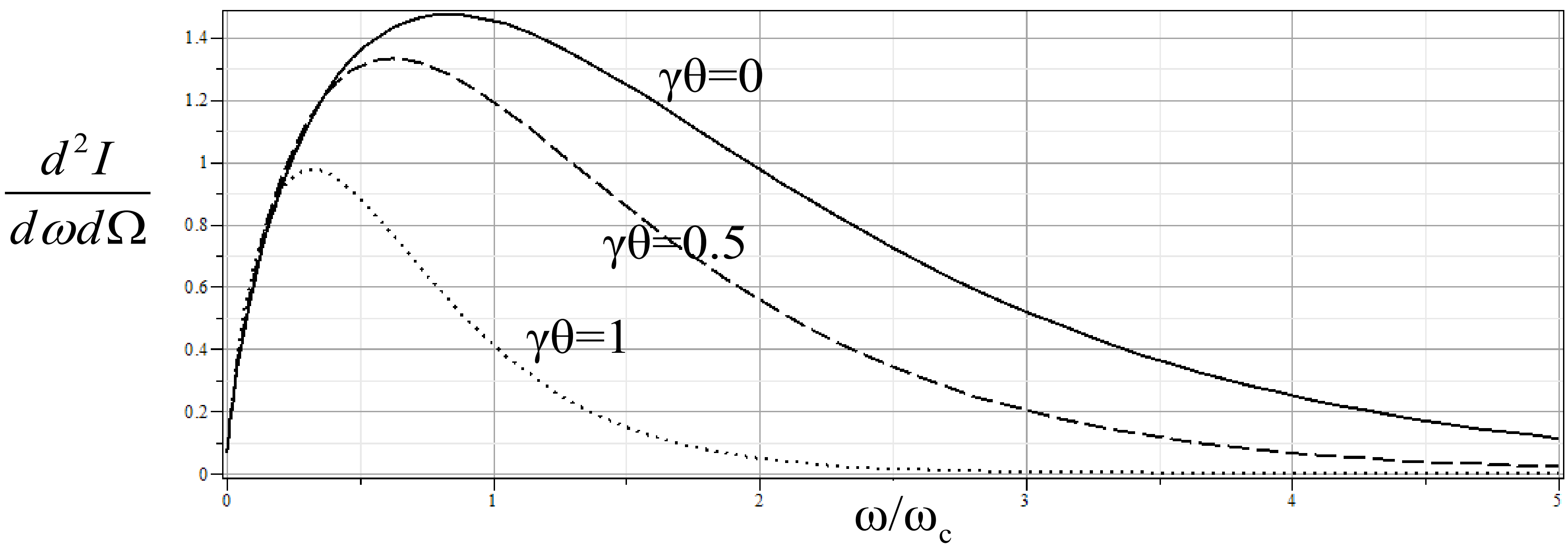
$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt\,\sin\theta\cos(vt/\rho)\mathrm{e}^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

We will analyze this expression for two different cases. The first case, is appropriate for man-made synchrotrons used as light sources. In this case, the light is produced by short bursts of electrons moving close to the speed of light ($v \approx c(1-1/(2\gamma^2))$) passing a beam line port. In addition, because of the design of the radiation ports, $\theta \approx 0$, and the relevant integration times t are close to $t \approx 0$. This results in the form shown in Eq. 14.79 of your text. It is convenient to rewrite this form in terms of a critical frequency $\omega_c \equiv \dfrac{3c\gamma^3}{2\rho}$.

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{ \left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 \right\}$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2\right\}$$

By plotting the intensity as a function of ω, we see that the intensity is largest near $\omega \approx \omega_c$. The plot below shows the intensity as a function of ω/ω_c for $\gamma\theta=0$, 0.5 and 1:

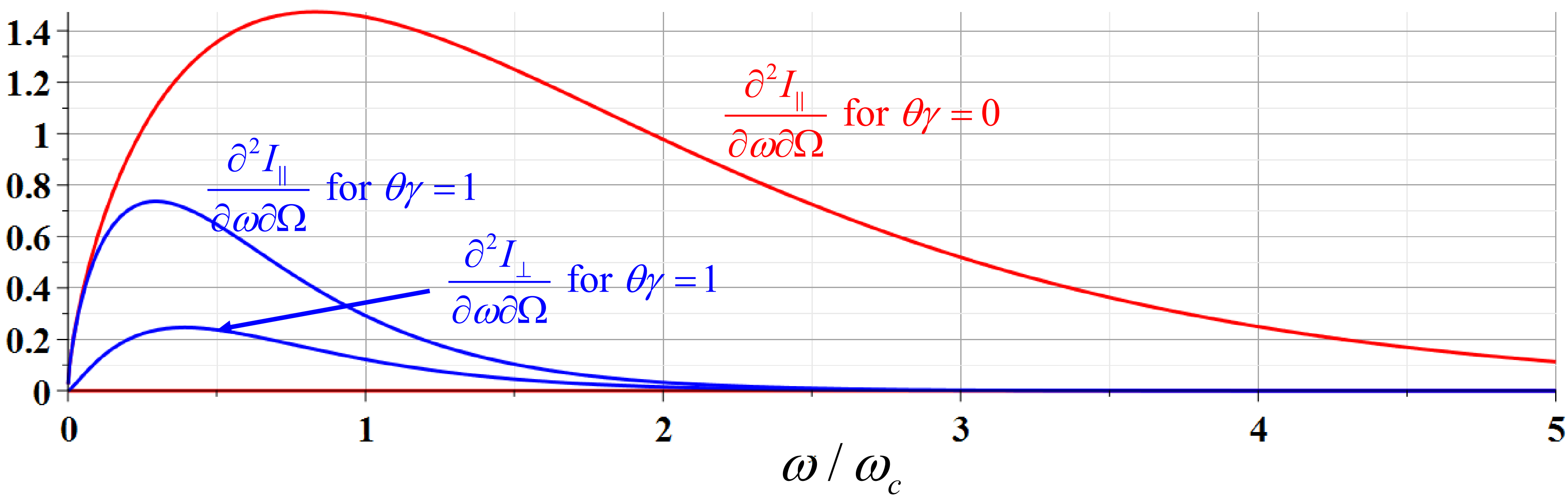


More details

$$\frac{d^2 I}{d\omega d\Omega} = \frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega}$$

$$\frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2$$

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2$$



Comment on light source facilities and the newer free electron laser technology

SCIENCE'S COMPASS • REVIEW

REVIEW: LASER TECHNOLOGY

Free-Electron Lasers: Status and Applications

Patrick G. O'Shea[1] and Henry P. Freund[2]

A free-electron laser consists of an electron beam propagating through a periodic magnetic field. Today such lasers are used for research in materials science, chemical technology, biophysical science, medical applications, surface studies, and solid-state physics. Free-electron lasers with higher average power and shorter wavelengths are under development. Future applications range from industrial processing of materials to light sources for soft and hard x-rays.

tions at wavelengths down to 1 Å, and this is illustrated by the peak brilliance of a wide range of the present-day synchrotrons and the projected performance of FELs (Fig. 3) (7). Consequently, applications in the x-ray region will undergo an upheaval similar to that which followed the invention of the laser at visible wavelengths.

Ultraviolet FEL oscillators using electron

DOI: 10.1126/science.1055718

Free-Electron Lasers: Status and Applications

Patrick G. O'Shea[1] and Henry P. Freund[2]

SCIENCE VOL 292 8 JUNE 2001

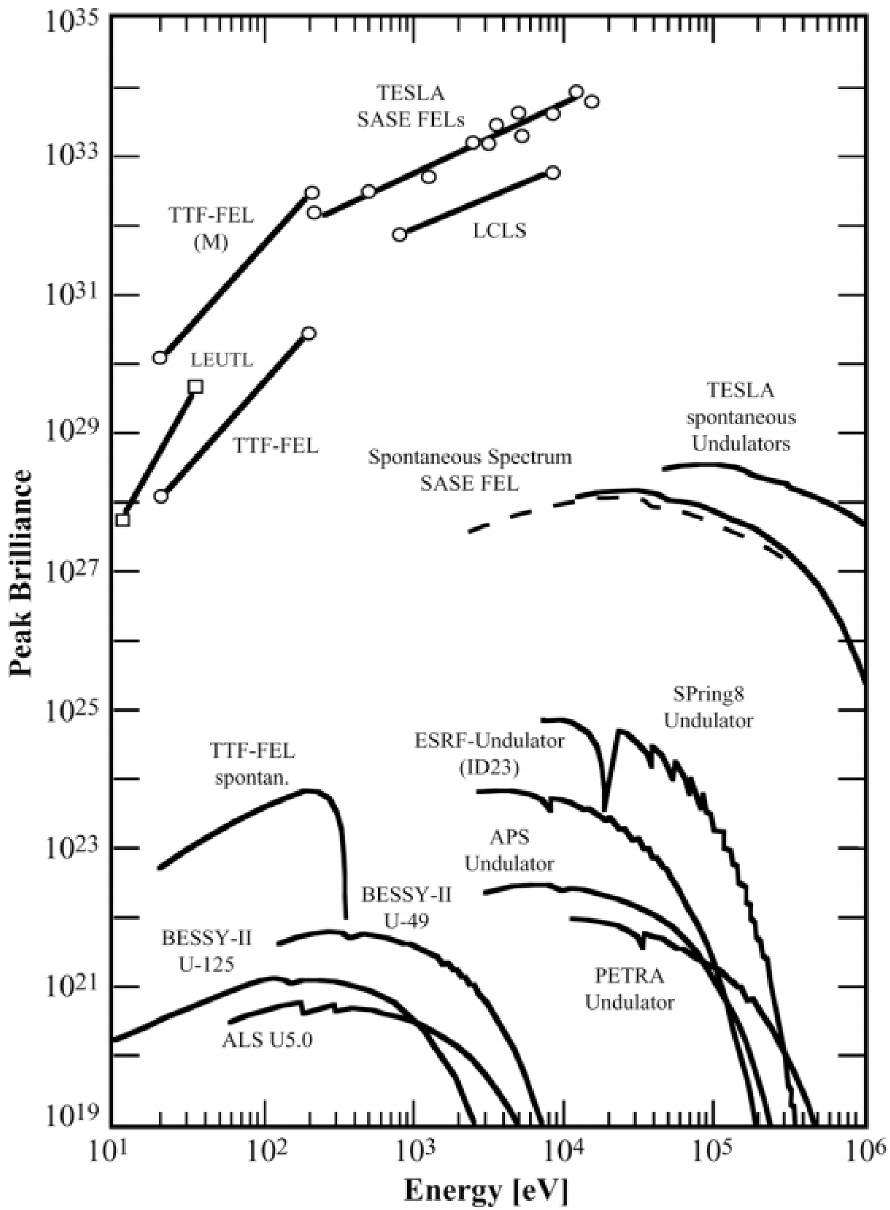


Fig. 3. Peak brilliance of x-ray FELs and undulators for spontaneous radiation at the TESLA Test Facility, in comparison with synchrotron radiation sources. Brilliance is expressed as photons s^{-1} $mrad^{-2}$ mm^{-2} per 0.1% bandwidth. For comparison, the spontaneous spectrum of x-ray FEL undulators is also shown. The label TTF-FEL indicates design values for the FEL at the TESLA Test Facility, with (M) for the planned seeded version (*28*).

04/13/2020

The above analysis applies to a class of man-made facilities dedicated to producing intense radiation in the continuous spectrum. For more specific information on man-made synchrotron sources, the following web page is useful: http://www.als.lbl.gov/als/synchrotron_sources.html.

Synchrotron radiation is also produced by astronomical sources as analyzed by Julian Schwinger –

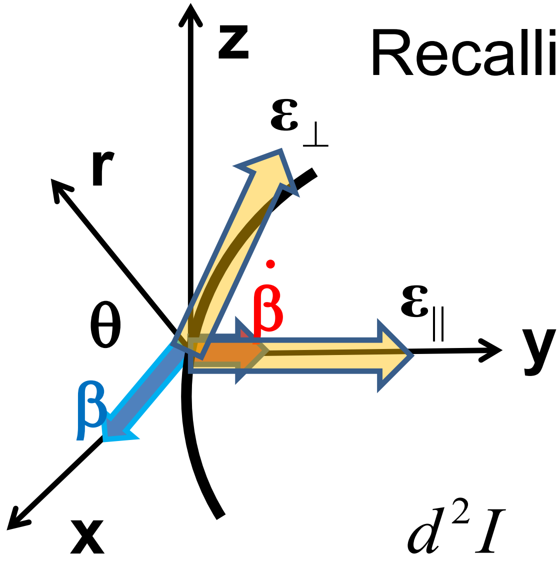
PHYSICAL REVIEW VOLUME 75, NUMBER 12 JUNE 15, 1949

On the Classical Radiation of Accelerated Electrons

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
(Received March 8, 1949)

This paper is concerned with the properties of the radiation from a high energy accelerated electron, as recently observed in the General Electric synchrotron. An elementary derivation of the total rate of radiation is first presented, based on Larmor's formula for a slowly moving electron, and arguments of relativistic invariance. We then construct an expression for the instantaneous power radiated by an electron moving along an arbitrary, prescribed path. By casting this result into various forms, one obtains the angular distribution, the spectral distribution, or the combined angular and spectral distributions of the radiation. The method is based on an examination of the rate at which the electron irreversibly transfers energy to the electromagnetic field, as determined by half the difference of retarded and advanced electric field intensities. Formulas are obtained for an arbitrary charge-current distribution and then specialized to a point charge. The total radiated power and its angular distribution are obtained for an arbitrary trajectory. It is found that the direction of motion is a strongly preferred direction of emission at high energies. The spectral distribution of the radiation depends upon the detailed motion over a time interval large compared to the period of the radiation. However, the narrow cone of radiation generated by an energetic electron indicates that only a small part of the trajectory is effective in producing radiation observed in a given direction, which also implies that very high frequencies are emitted. Accordingly, we evaluate the spectral and angular distributions of the high frequency radiation by an energetic electron, in their dependence upon the parameters characterizing the instantaneous orbit. The average spectral distribution, as observed in the synchrotron measurements, is obtained by averaging the electron energy over an acceleration cycle. The entire spectrum emitted by an electron moving with constant speed in a circular path is also discussed. Finally, it is observed that quantum effects will modify the classical results here obtained only at extraordinarily large energies.

DOI:https://doi.org/10.1103/PhysRev.75.1912



Recalling general results of analysis --

$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\beta}\right) =$$

$$\beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin\left(vt_r/\rho\right) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos\left(vt_r/\rho\right)\right)$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \beta) e^{i\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} dt\right|^2$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\}$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

Useful identity involving Bessel functions

$$e^{-iA\sin\alpha} = \sum_{m=-\infty}^{\infty} J_m(A) e^{-im\alpha}$$ Here $J_m(A)$ is a Bessel function of integer order m.

In our case, $A = \frac{\omega\rho}{c}\cos\theta$ and $\alpha = \frac{vt}{\rho}$.

$$C_{||}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta \sin(vt/\rho))}$$

$$= \frac{c}{-i\omega\rho} \frac{\partial}{\partial \cos\theta} \int_{-\infty}^{\infty} dt e^{i\omega(t - \frac{\rho}{c}\cos\theta \sin(vt/\rho))}$$

$$= \frac{c}{-i\omega\rho} \frac{\partial}{\partial \cos\theta} \sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right) 2\pi\delta(\omega - m\frac{v}{\rho}).$$

Some details for last step --

$$e^{-iA\sin\alpha} = \sum_{m=-\infty}^{\infty} J_m(A) e^{-im\alpha}$$ Here $J_m(A)$ is a Bessel function of integer order m.

In our case, $A = \frac{\omega\rho}{c}\cos\theta$ and $\alpha = \frac{vt}{\rho}$.

$$\int_{-\infty}^{\infty} dt e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))} = \sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\int_{-\infty}^{\infty} dt e^{i\omega(t - m\frac{v}{c}t))}$$

$$= \sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right) 2\pi\delta(\omega - m\frac{v}{\rho}).$$

Astronomical synchrotron radiation -- continued:

Note that:

$$\int_{-\infty}^{\infty} dt \mathrm{e}^{i(\omega - m\frac{v}{\rho})t} = 2\pi\delta(\omega - m\frac{v}{\rho}).$$

$$\Rightarrow C_{\|}(\omega) = 2\pi i \sum_{m=-\infty}^{\infty} J_m^{'}\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}),$$

where $J_m^{'}(A) \equiv \dfrac{dJ_m(A)}{dA}$

Similarly:

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta \cos(vt/\rho) \mathrm{e}^{i\omega(t - \frac{\rho}{c}\cos\theta \sin(vt/\rho))}$$

$$= 2\pi \frac{\tan\theta}{v/c} \sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}).$$

Astronomical synchrotron radiation -- continued:

In both of the expressions, the sum over *m* includes both negative and positive values. However, only the positive values of ω and therefore positive values of *m* are of interest. Using the identity: $J_{-m}(A) = (-1)^m J_m(A),$ the result becomes:

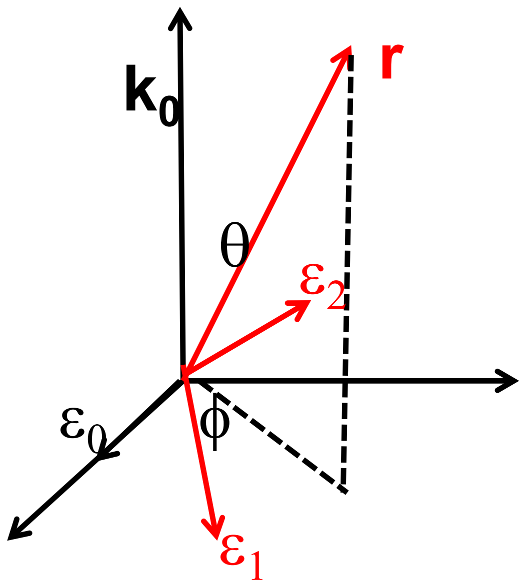
$$\frac{d^2 I}{d\omega d\Omega} =$$

$$\frac{q^2\omega^2\beta^2}{c}\sum_{m=0}^{\infty}\delta(\omega - m\frac{v}{\rho})\left\{\left[J_m'\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2 + \frac{\tan^2\theta}{v^2/c^2}\left[J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2\right\}.$$

These results were derived by Julian Schwinger (Phys. Rev. **75**, 1912-1925 (1949)). The discrete case is similar to the result quoted in Problem 14.15 in Jackson's text.

Back to fundamental processes – Thompson and Compton scattering (see section 14.8 in Jackson)

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



$$\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right)$$