PHY 712 Electrodynamics
10-10:50 AM MWF online

Plan for Lecture 31:

Finish reading Chap. 14 and start Chap. 15 –

Radiation from scattering charged particles

1. **Thompson and Compton scattering**
2. **Radiation from particle collisions**

In this lecture we will discuss radiation due to collions.

28	Fri: 04/09/2021	Chap. 14	Radiation from accelerating charged particles	#20	04/12/2021
29	Mon: 04/12/2021	Chap. 14	Synchrotron radiation	#21	04/14/2021
30	Wed: 04/14/2021	Chap. 14	Synchrotron radiation	#22	04/19/2021
31	Fri: 04/16/2021	Chap. 15	Radiation from collisions of charged particles	#23	04/21/2021
32	Mon: 04/19/2021	Chap. 15	Radiation from collisions of charged particles		
33	Wed: 04/21/2021	Chap. 13	Cherenkov radiation		
34	Fri: 04/23/2021		Special topic: E & M aspects of superconductivity		
35	Mon: 04/26/2021		Special topic: E & M aspects of superconductivity		
36	Wed: 04/28/2021		Review		
37	Fri: 04/30/2021		Review		
	Mon: 05/03/2021		Presentations I		
	Wed: 05/05/2021		Presentations II		

PHY 712 -- Assignment #23

April 16, 2021

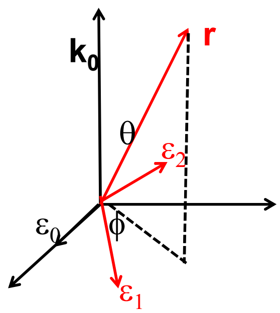
*Finish reading Chap. 14 and start Chap. 15 in **Jackson** .*

1. *Derive the Compton formula given on page 696 of **Jackson**.*

Last homework set for the semester.

Thompson scattering --

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



Incident electomagnetic wave:

$\mathbf{k}_0$ propagation direction

$\boldsymbol{\varepsilon}_0$ polarization direction

$$\mathbf{E}(\mathbf{r}',t') = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}'-i\omega t'}\right)$$

Scattered radiation:

$\mathbf{r}$ observed position

$\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2$ polarization directions

Consider the effects of a charged particle encountering an electromagnetic wave.

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \dot{\mathbf{v}}\right)\right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q) is caused by an electric field: $\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$

$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Time averaged power: $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 \left|E_0\right|^2 \left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0\right)\right|^2$

The power generated in the far field approximated by the non-relativistic limit, is given here. Now we consider that the particle acceleration is due to the oscillating electric field. The power of the radiation of the particle is then given here.

What assumptions are made to conclude that

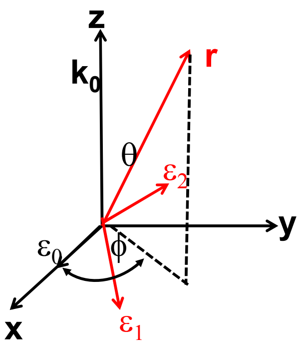
$$\dot{\mathbf{v}} = \frac{q}{m_q} \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right) \quad ?$$

Is it always true?

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\boldsymbol{\varepsilon}_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

Are these polarizations unique?

By evaluating the radiation geometry, we can determine the power distribution.

Note that we are associating the vector $\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\dot{\mathbf{v}}\right)$ with the polarization of the light. Why?

Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{\left(R-\frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left[\left(\mathbf{R}-\frac{\mathbf{v}R}{c}\right)\left(1-\frac{v^2}{c^2}\right)+\left(\mathbf{R}\times\left\{\left(\mathbf{R}-\frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right\}\right)\right]. \tag{19}$$

$$\mathbf{B}(\mathbf{r},t)=\frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{\left(R-\frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left(1-\frac{v^2}{c^2}+\frac{\dot{\mathbf{v}}\cdot\mathbf{R}}{c^2}\right)-\frac{\mathbf{R}\times\dot{\mathbf{v}}/c}{\left(R-\frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^2}\right]. \tag{20}$$

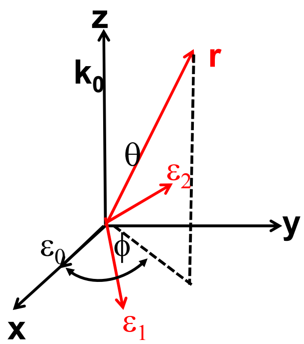
In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r},t)=\frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}. \tag{21}$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0)\right|^2$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0) = \hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\boldsymbol{\varepsilon}_0) - \boldsymbol{\varepsilon}_0$ (perpendicular to $\hat{\mathbf{r}}$)

denote scattered light polarization by $\boldsymbol{\varepsilon}^*$

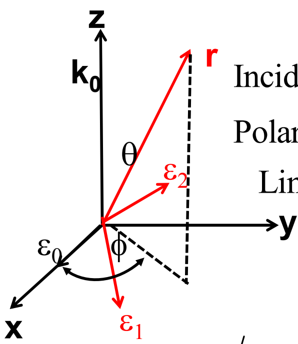
$\boldsymbol{\varepsilon}^*\cdot\left(\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0)\right) = -\boldsymbol{\varepsilon}^*\cdot\boldsymbol{\varepsilon}_0$

The radiated field is polarized perpendicularly to the propagation direction so that the power depends on the dot product of radiation polarization and the incident polarization.

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0\right)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light: $\boldsymbol{\varepsilon}^*$

Linear combination of

$$\boldsymbol{\varepsilon}_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

$$\left\langle \left|\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle = \left\langle \left|\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle + \left\langle \left|\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle = \left\langle \cos^2\theta \cos^2\phi \right\rangle + \left\langle \sin^2\phi \right\rangle$$

Evaluating the dot products for this geometry.

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization $\boldsymbol{\varepsilon}^*$:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 |E_0|^2 |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2$$

Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

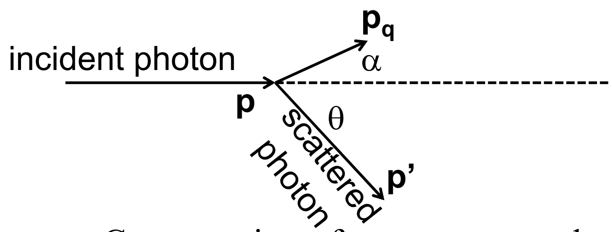
$$\left\langle |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \left\langle |\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi + \left\langle |\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \frac{1}{2}\cos^2\theta + \frac{1}{2}$$

Averaged cross section: $\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \frac{1}{2}\left(1 + \cos^2\theta\right)$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Summary of results in the non-relativistic limit.

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy:

$$p = p'\cos\theta + p_q \cos\alpha \qquad pc = \hbar\omega$$

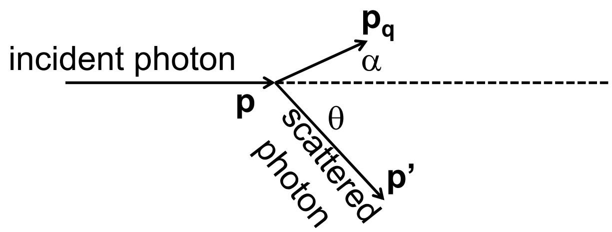
$$0 = p'\sin\theta - p_q \sin\alpha \qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p'^{\,2}_q c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$

When the photon energy is comparable to the rest mass energy of the charged particle, we have to consider momentum and energy conservation. This was ferst analyzed by Compton. Your homework asks you to examine the Compton scattering relationships.

Thompson scattering – relativistic and quantum modifications



Relativistic and quantum modifications to averaged cross section:

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

$$\frac{p'}{p} = \frac{1}{1+\frac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$

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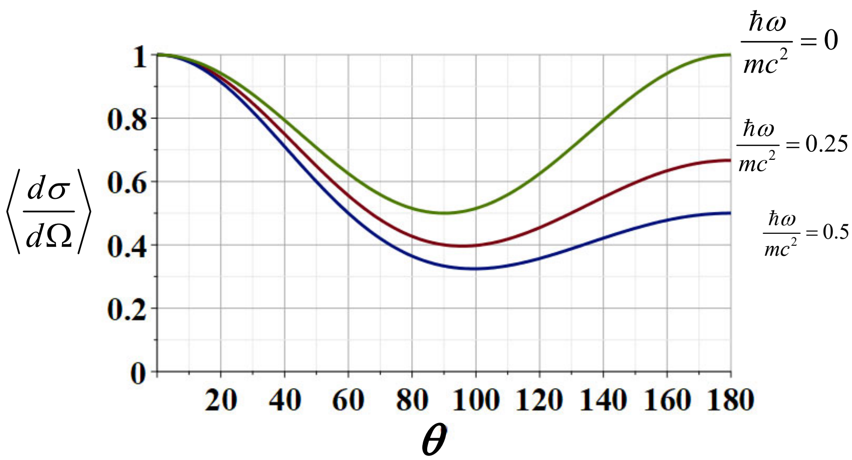
The effects of the Compton scattering change the cross section.

Modified Thompson scattering cross section



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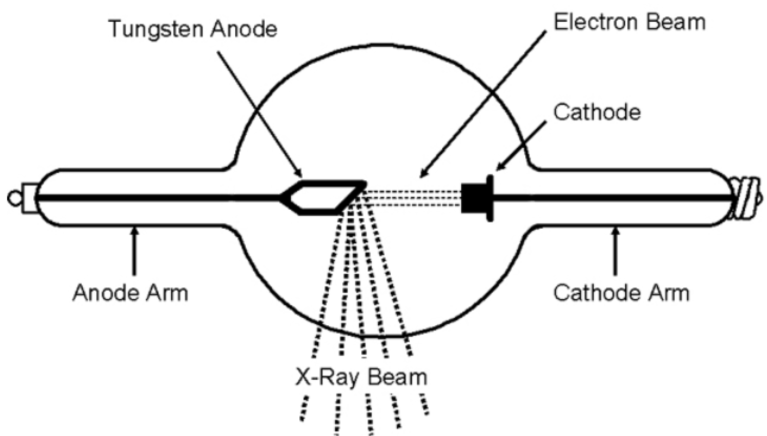
Plot of the changes to the Thompson cross section due to the Compton effect.

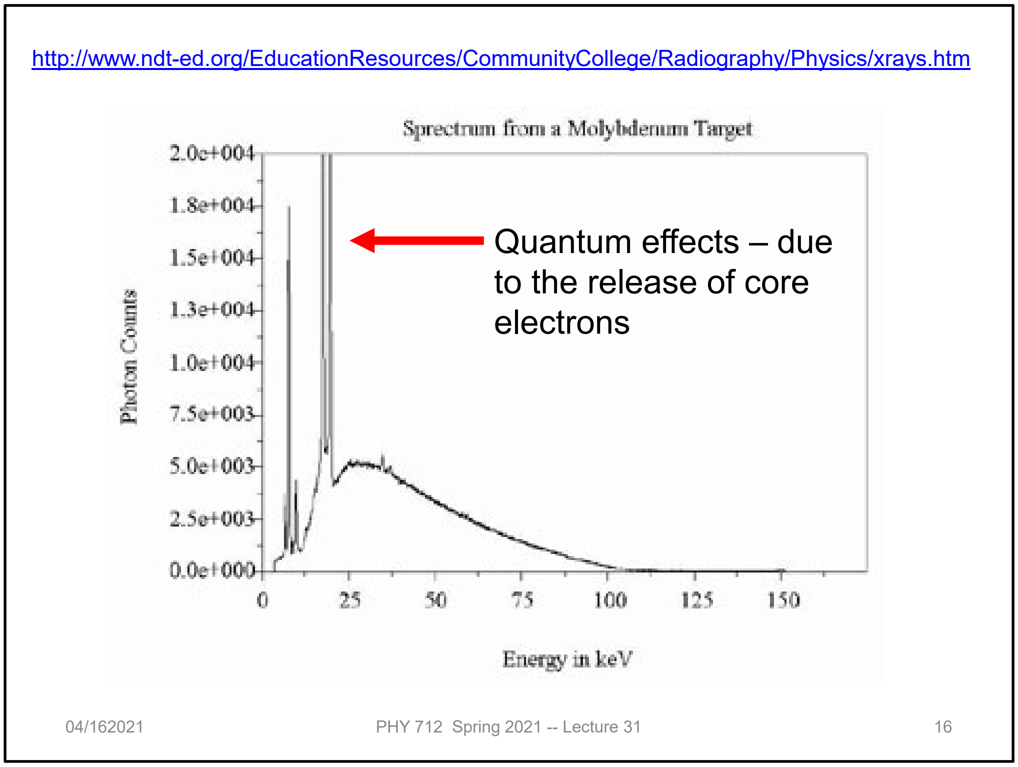
Up to now, we have been considering (re)radiation due to a charged particle interacting with an electromagnetic field. Next time we will consider radiation due to interactions (collisions) of charged particles themselves.

Radiation produced by collisions of charged particles

Generation of X-rays in a Coolidge tube
https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm



Next time we will consider more generally the properties of radiation produced by charged particle collisions.



Example of x-ray spectrum .