PHY 712 Electrodynamics
10-10:50 AM MWF online

Discussion for Lecture 31:

Finish reading Chap. 14 and start Chap. 15 –

Radiation from scattering charged particles

1. **Thompson and Compton scattering**
2. **Radiation from particle collisions**

28	Fri: 04/09/2021	Chap. 14	Radiation from accelerating charged particles	#20	04/12/2021
29	Mon: 04/12/2021	Chap. 14	Synchrotron radiation	#21	04/14/2021
30	Wed: 04/14/2021	Chap. 14	Synchrotron radiation	#22	04/19/2021
31	Fri: 04/16/2021	Chap. 15	Radiation from collisions of charged particles	#23	04/21/2021
32	Mon: 04/19/2021	Chap. 15	Radiation from collisions of charged particles		
33	Wed: 04/21/2021	Chap. 13	Cherenkov radiation		
34	Fri: 04/23/2021		Special topic: E & M aspects of superconductivity		
35	Mon: 04/26/2021		Special topic: E & M aspects of superconductivity		
36	Wed: 04/28/2021		Review		
37	Fri: 04/30/2021		Review		
	Mon: 05/03/2021		Presentations I		
	Wed: 05/05/2021		Presentations II		

PHY 712 -- Assignment #23

April 16, 2021

*Finish reading Chap. 14 and start Chap. 15 in **Jackson** .*

1. *Derive the Compton formula given on page 696 of **Jackson**.*

Your questions –

From Nick -- Can you review how an electric field (or magnetic) can accelerate a particle. What are the fields actually doing (beyond the mathematics)? Where do the forces come into play? My undergraduate E&M is rusty.

From Gao -- How is the Compton effect added into Thompson scattering?

Also comment on typo from Lecture 25

Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$\cancel{kg \cdot m^2/s}$	$dyne$	$\cancel{gm \cdot cm^2/s}$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

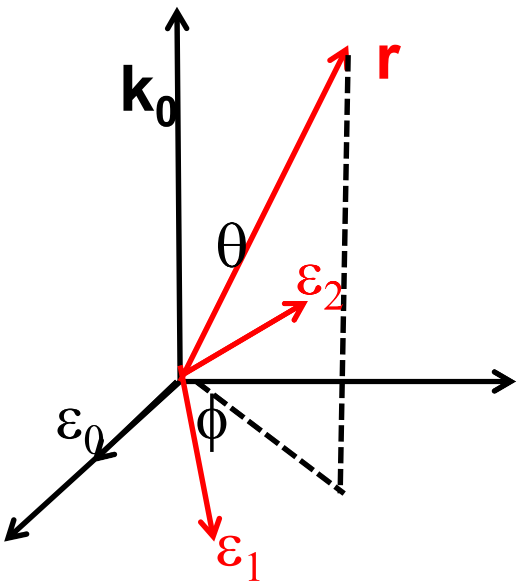
Units - SI vs Gaussian – Corrected

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m\ /s^2$	$dyne$	$gm \cdot cm\ /s^2$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

Thompson scattering --

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



Incident electomagnetic wave:

$\mathbf{k}_0$ propagation direction

$\boldsymbol{\varepsilon}_0$ polarization direction

$$\mathbf{E}(\mathbf{r}',t') = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}'-i\omega t'}\right)$$

Scattered radiation:

$\mathbf{r}$ observed position

$\boldsymbol{\varepsilon}_1,\boldsymbol{\varepsilon}_2$ polarization directions

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\dot{\mathbf{v}}\right)\right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q)

is caused by an electric field: $\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right)$

$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right)$$

Time averaged power: $\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 \left|E_0\right|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$

What assumptions are made to conclude that

$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right) \quad ?$$

Is it always true?

Comment on acceleration

Lorentz force: $$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$$

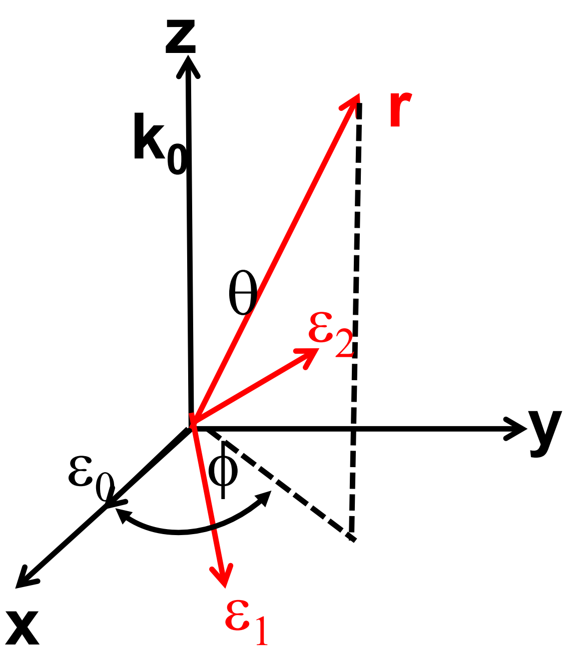
For $v << c$, the dominate force on a charged particle is from the electric field. According to Newton:

$$m_q \frac{d\mathbf{v}}{dt} \equiv m_q \dot{\mathbf{v}} = q\mathbf{E}(\mathbf{r},t) = q\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\boldsymbol{\varepsilon}_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

Are these polarizations unique?

Note that we are associating the vector $\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\dot{\mathbf{v}})$ with the polarization of the light. Why?

Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R-\frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left[\left(\mathbf{R}-\frac{\mathbf{v}R}{c}\right)\left(1-\frac{v^2}{c^2}\right)+\left(\mathbf{R}\times\left\{\left(\mathbf{R}-\frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right\}\right)\right]. \tag{19}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{\left(R-\frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left(1-\frac{v^2}{c^2}+\frac{\dot{\mathbf{v}}\cdot\mathbf{R}}{c^2}\right)-\frac{\mathbf{R}\times\dot{\mathbf{v}}/c}{\left(R-\frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^2}\right]. \tag{20}$$

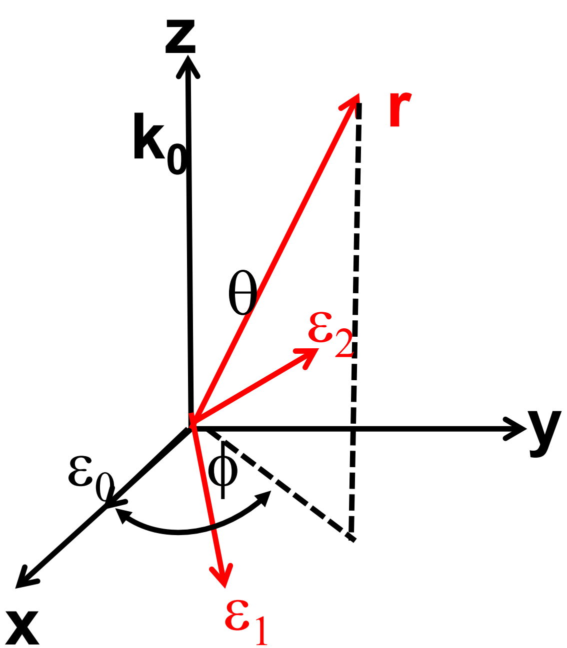
In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}. \tag{21}$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0) = \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \boldsymbol{\varepsilon}_0) - \boldsymbol{\varepsilon}_0 \quad \text{(perpendicular to } \hat{\mathbf{r}}\text{)}$$

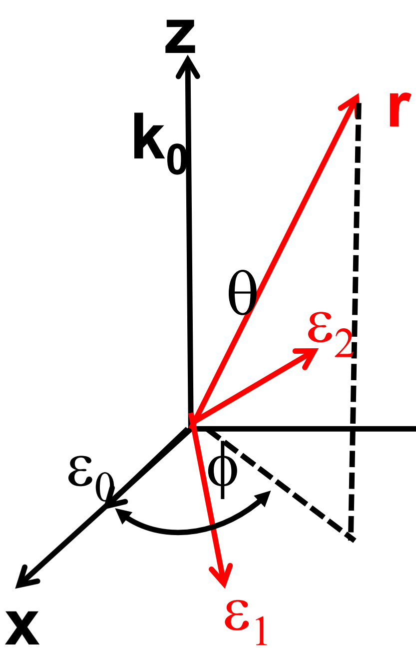
denote scattered light polarization by $\boldsymbol{\varepsilon}^*$

$$\boldsymbol{\varepsilon}^* \cdot \left(\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0)\right) = -\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light: $\boldsymbol{\varepsilon}^*$

Linear combination of

$$\boldsymbol{\varepsilon}_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

$$\left\langle \left|\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle = \left\langle \left|\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle + \left\langle \left|\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle = \left\langle \cos^2\theta\cos^2\phi \right\rangle + \left\langle \sin^2\phi \right\rangle$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization $\boldsymbol{\varepsilon}^*$:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 |E_0|^2 |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2$$

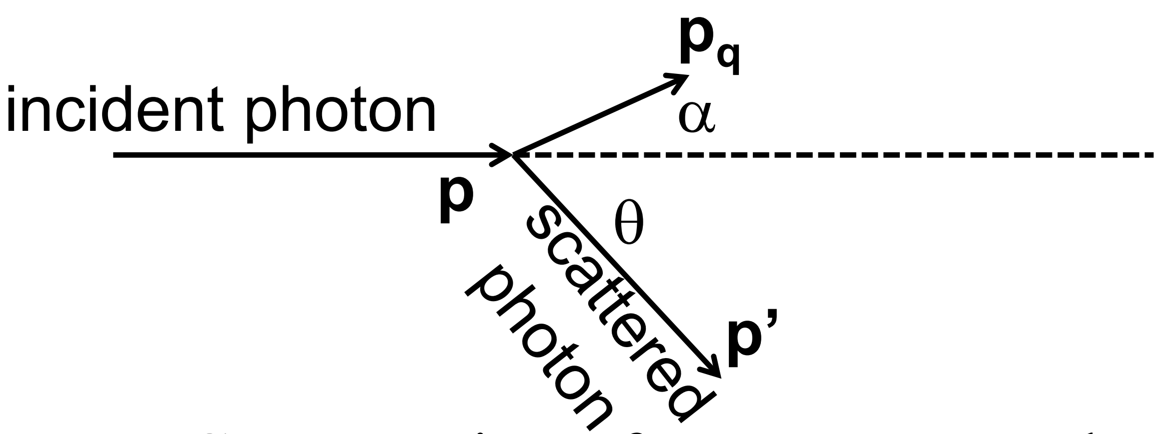
Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

$$\left\langle |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \left\langle |\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi + \left\langle |\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \frac{1}{2}\cos^2\theta + \frac{1}{2}$$

Averaged cross section: $$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \frac{1}{2}\left(1 + \cos^2\theta\right)$$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy:

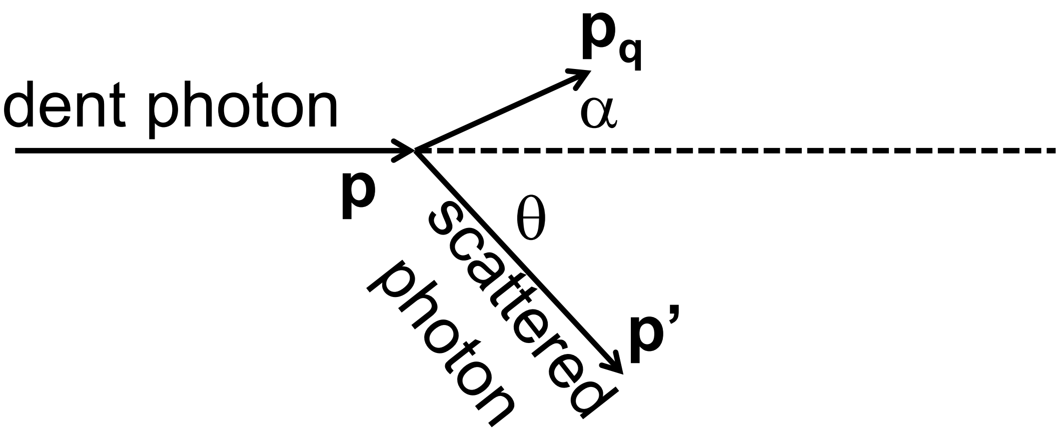
$$p = p'\cos\theta + p_q \cos\alpha \qquad\qquad pc = \hbar\omega$$

$$0 = p'\sin\theta - p_q \sin\alpha \qquad\qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p'^2_q c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$

Thompson scattering – relativistic and quantum modifications

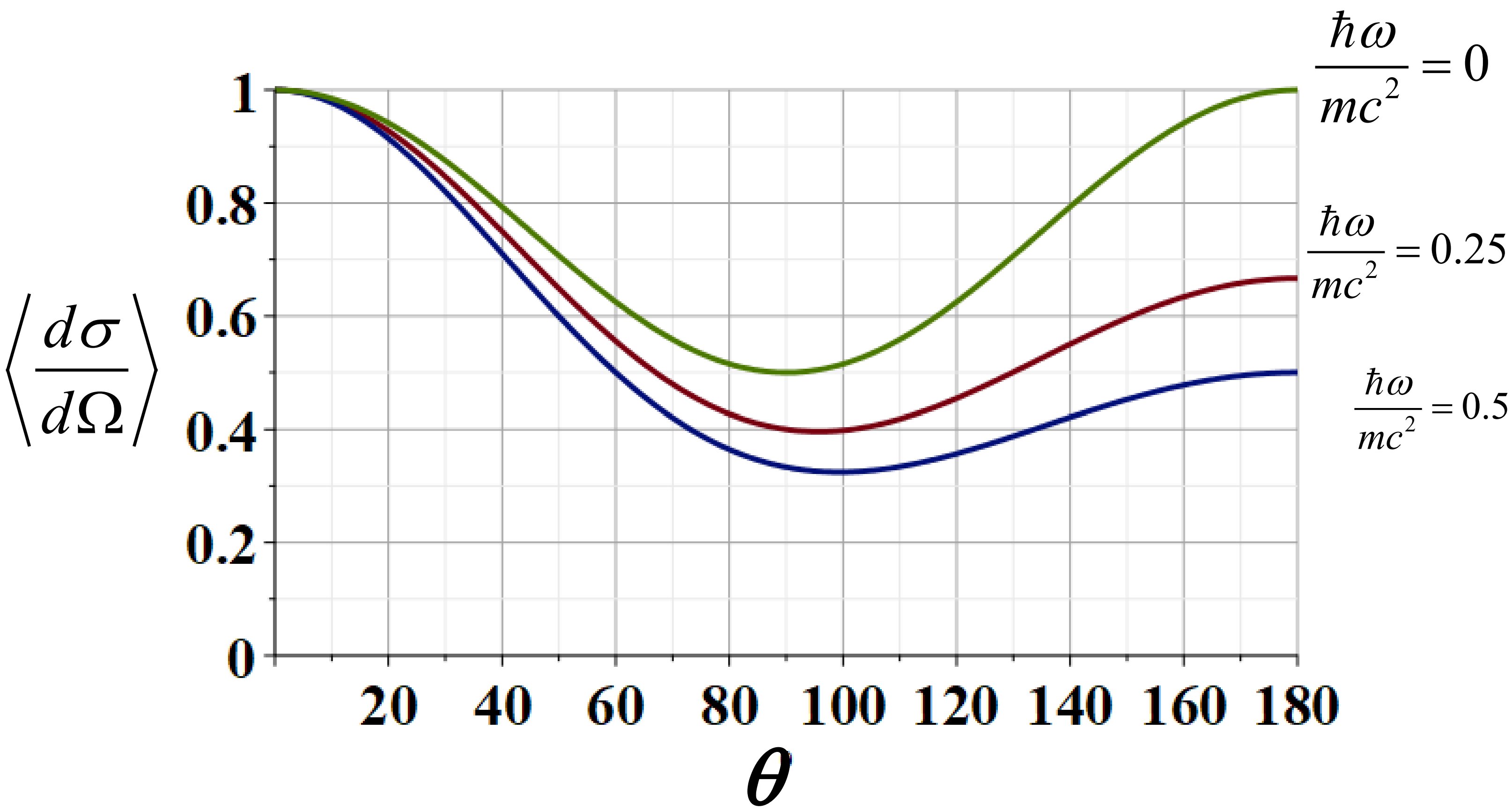


Relativistic and quantum modifications to averaged cross section:

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

$$\frac{p'}{p} = \frac{1}{1+\dfrac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$

Modified Thompson scattering cross section



In fact, the more accurate treatment by Klein and Nishina gives

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$

Klein-Nishina formula

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2} \left(\frac{p'}{p} + \frac{p}{p'} - \sin^2\theta \right)$$

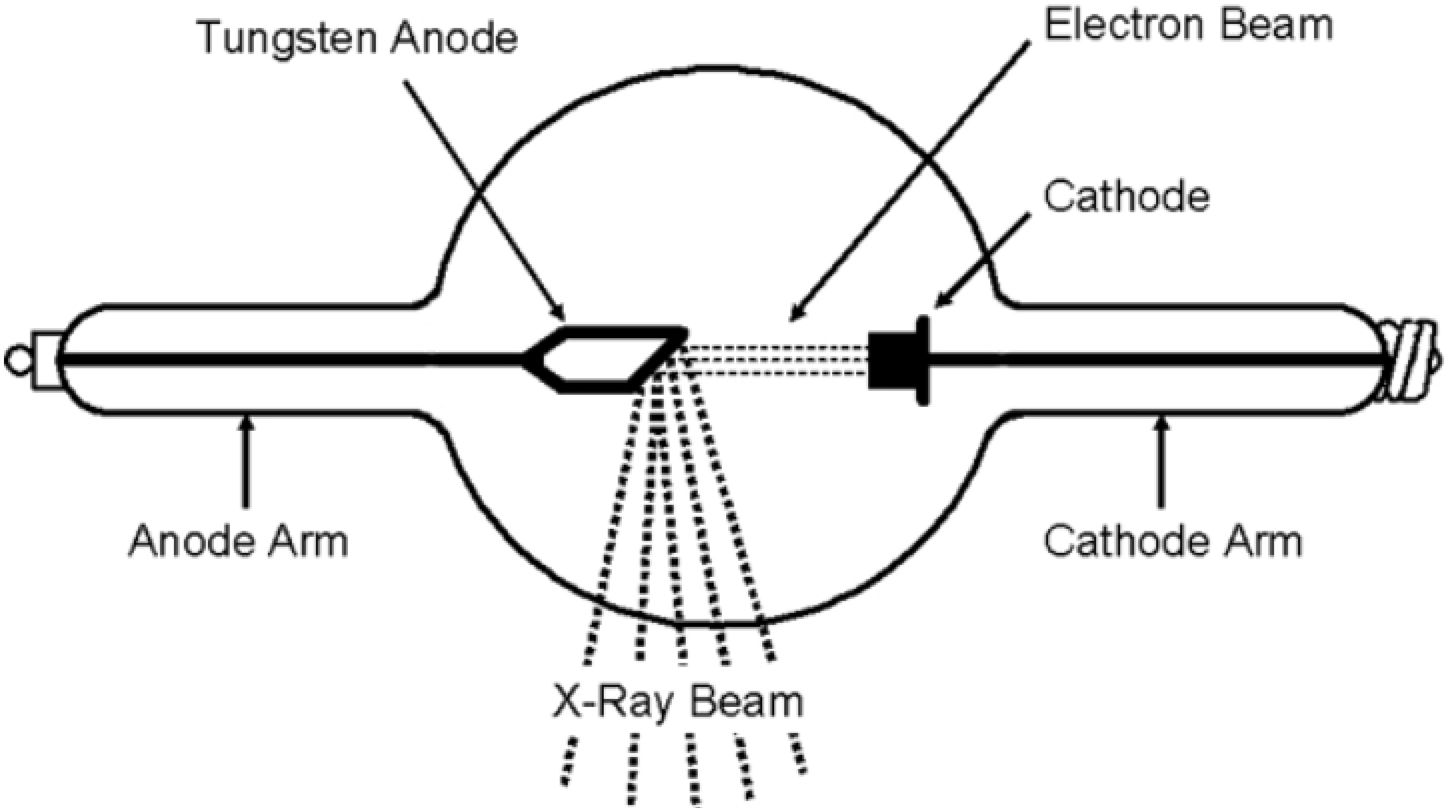
Note that for $\frac{\hbar\omega}{m_q c^2} << 1$ all results are consistent

Up to now, we have been considering (re)radiation due to a charged particle interacting with an electromagnetic field. Next time we will consider radiation due to interactions (collisions) of charged particles themselves.

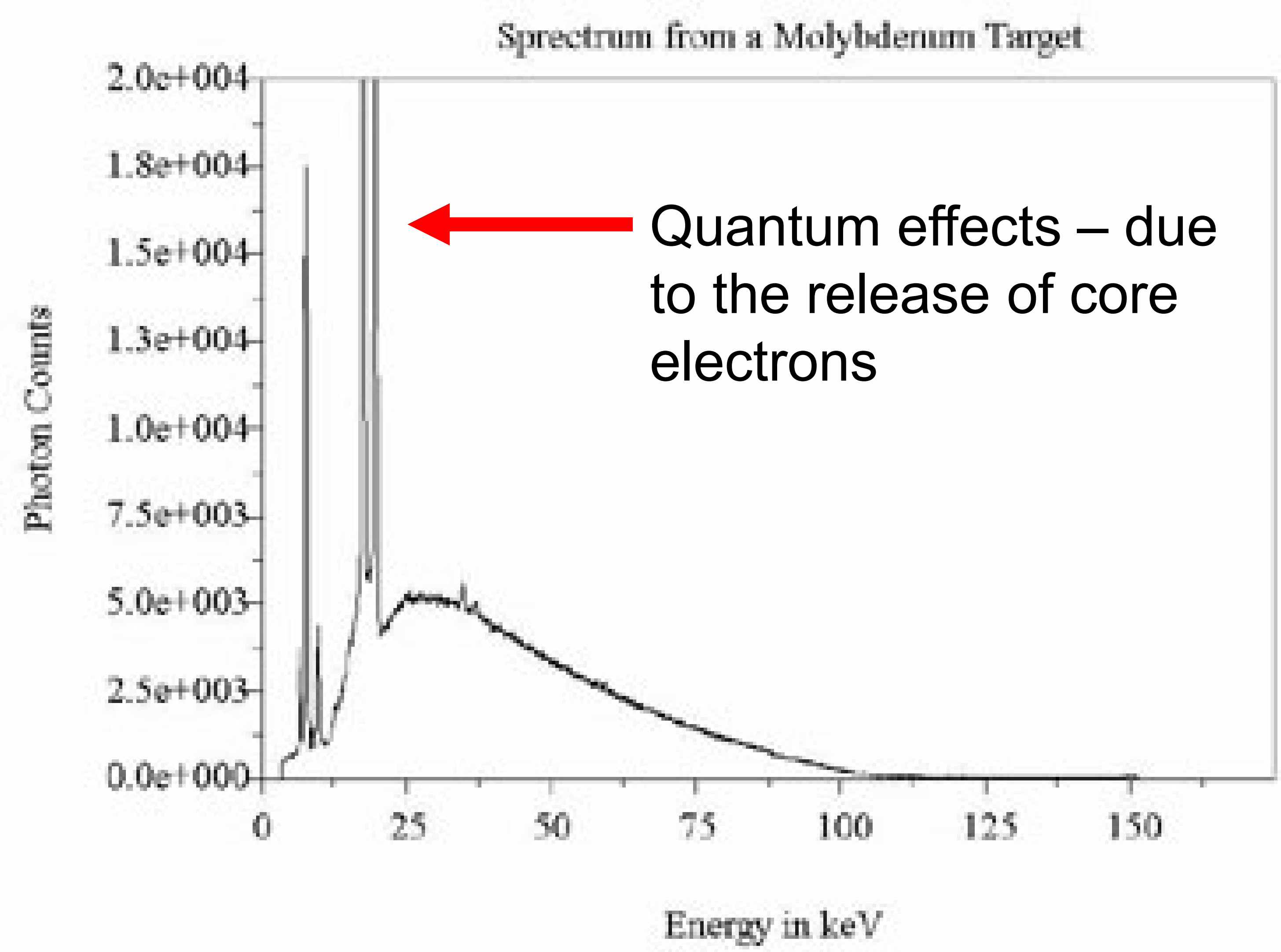
Radiation produced by collisions of charged particles

Generation of X-rays in a Coolidge tube

https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm



http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Quantum effects – due to the release of core electrons