PHY 712 Electrodynamics
10-10:50 AM MWF Online

Discussion for Lecture 33:

Special Topics in Electrodynamics:

Cherenkov radiation

References: Jackson Chapter 13.4

Zangwill Chapter 23.7

Smith Chapter 6.4

28	Fri: 04/09/2021	Chap. 14	Radiation from accelerating charged particles	#20	04/12/2021
29	Mon: 04/12/2021	Chap. 14	Synchrotron radiation	#21	04/14/2021
30	Wed: 04/14/2021	Chap. 14	Synchrotron radiation	#22	04/19/2021
31	Fri: 04/16/2021	Chap. 15	Radiation from collisions of charged particles	#23	04/21/2021
32	Mon: 04/19/2021	Chap. 15	Radiation from collisions of charged particles		
33	Wed: 04/21/2021	Chap. 13	Cherenkov radiation		
34	Fri: 04/23/2021		Special topic: E & M aspects of superconductivity		
35	Mon: 04/26/2021		Special topic: E & M aspects of superconductivity		
36	Wed: 04/28/2021		Review		
37	Fri: 04/30/2021		Review		
	Mon: 05/03/2021		Presentations I		
	Wed: 05/05/2021		Presentations II		

PHYSICS COLLOQUIUM

4 PM

THURSDAY

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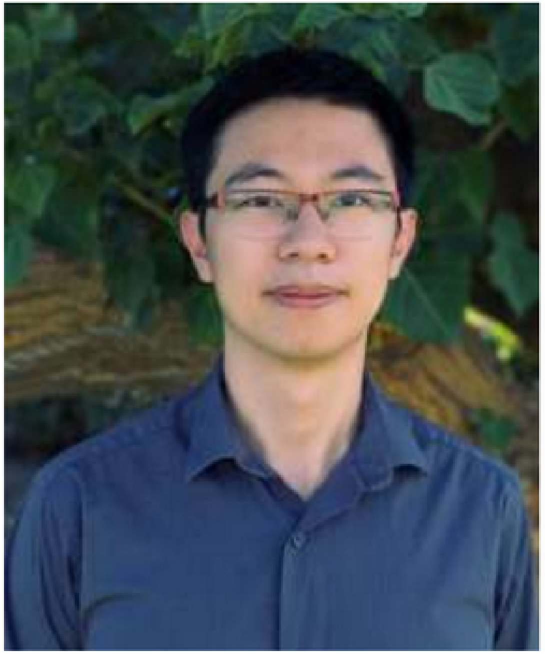
APRIL 22, 2021

"Quantum Computing with Superconducting Qubits"



Dr. Jimmy Chen
Research Scientist
Google AI Quantum

Quantum computers promise to perform certain computational tasks exponentially faster than classical processors. However, achieving these speed-ups requires sufficiently low error rates in the quantum processor. In this talk, I will discuss the development of a programmable quantum processor named Sycamore, which consists of 53 superconducting qubits with state of the art operational fidelities. We benchmark the performance of Sycamore on randomly generated quantum circuits which are significantly more complex than any previous quantum computation. We also show

Ph.D. DEFENSE

THURSDAY • APRIL 22, 2021

"TUNABLE COLLAGEN I MATRICES FOR CELLULAR MIGRATION ASSAY AND MECHANICAL PROPERTIES OF MITOTIC MAMMARY CELLS"

Melissa Pashayan
Dr. Jed Macosko, Advisor
Department of Physics
Wake Forest University

The mechanical properties of cells influence their function in the way they differentiate, proliferate, migrate, adhere, and sense their local microenvironment. In disease, these properties are significantly altered and are hallmarks of disease onset and progression. Cancer cells, particularly, have been shown to be softer than normal tissue, and the changes in mechanical properties are linked to tumor formation and metastatic potential. The study of mechanical properties of different cells and their diseased counterparts can therefore provide insight into both the normal development of tissue and the pathophysiology of disease and may aid in the development of novel diagnostic techniques and treatments.

Oral Defense
1:00 pm EST
via Video Conference

Note: For additional information on the defense or to obtain the video conference links, contact wfuphys@wfu.edu.

Your questions –
From Gao -- What will happen if the material has a negative refractive index?

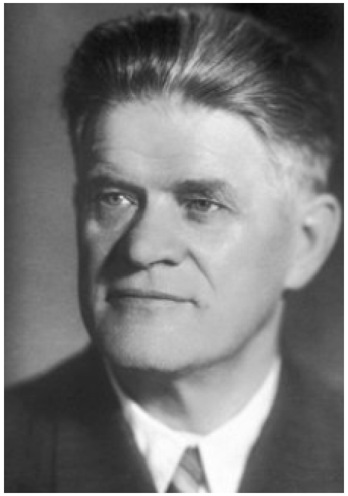
Comment – This is a very interesting question that has been topic of recent research, usually associated with radar and the notion of “cloaking”. The equations are even more complicated….

Cherenkov radiation

Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013. http://www.britannica.com/EBchecked/media/174732

The Nobel Prize in Physics 1958

Pavel A. Cherenkov
Il´ja M. Frank
Igor Y. Tamm



Affiliation at the time of the award: P.N. Lebedev Physical Institute, Moscow, USSR

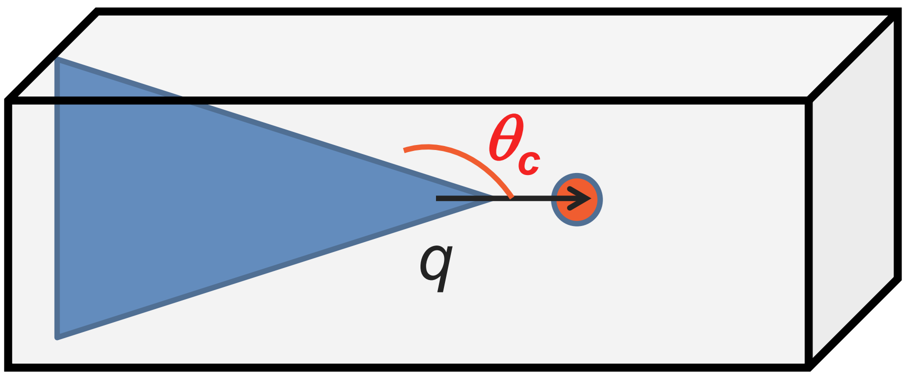
Prize motivation: "for the discovery and the interpretation of the Cherenkov effect."

https://www.nobelprize.org/prizes/physics/1958/ceremony-speech/

References for notes: Glenn S. Smith, *An Introduction to Electromagnetic Radiation* (Cambridge UP, 1997), Andrew Zangwill, Modern Electrodynamics (Cambridge UP, 2013)

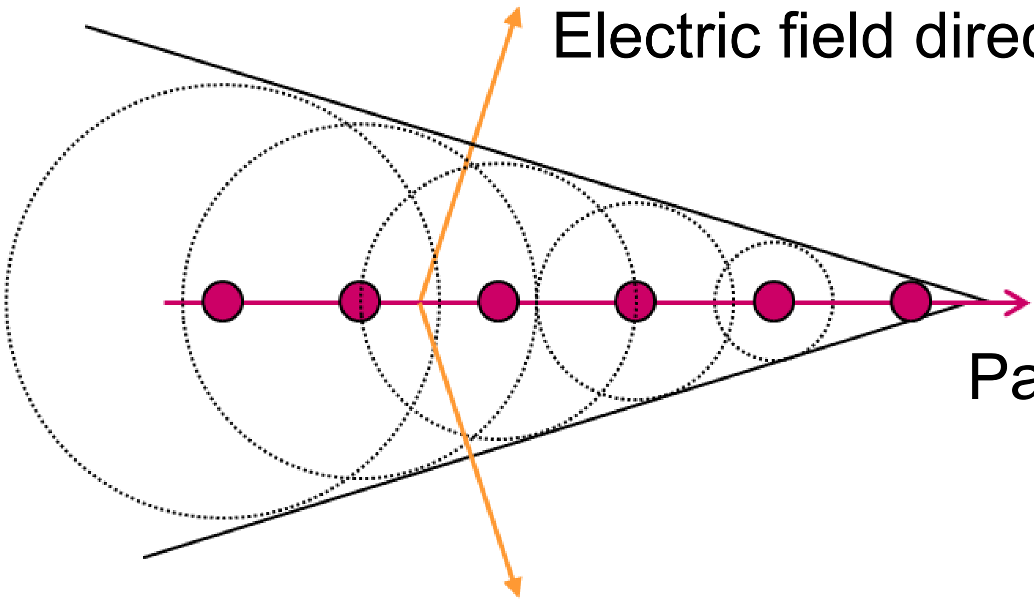
Cherenkov radiation

Discovered ~1930; bluish light emitted by energetic charged particles traveling within dielectric materials



Note that some treatments give the critical angle as θ_c-π/2.

From: http://large.stanford.edu/courses/2014/ph241/alaeian2/



Maxwell's potential equations within a material having permittivity and permeability (Lorentz gauge; cgs Gaussian units)

$$\nabla^2\Phi - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\rho$$

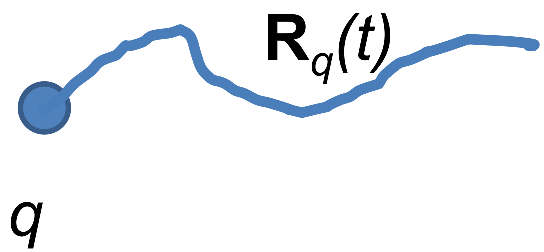
$$\nabla^2\mathbf{A} - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\mathbf{J}$$

Here the values of μ and ε depend on the material and on frequency.

Source: charged particle moving on trajectory $\mathbf{R}_q(t)$:

$$\rho(\mathbf{r},t) = q\delta(\mathbf{r}-\mathbf{R}_q(t))$$

$$\mathbf{J}(\mathbf{r},t) = q\dot{\mathbf{R}}_q(t)\delta(\mathbf{r}-\mathbf{R}_q(t))$$



Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon} \frac{1}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

$$\mathbf{A}(\mathbf{r},t) = q\mu \frac{\boldsymbol{\beta}_n}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r)$$

$$\boldsymbol{\beta}_n(t_r) \equiv \frac{\dot{\mathbf{R}}_q(t_r)}{c_n} \qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

$$t_r = t - \frac{R(t_r)}{c_n}$$

Example --

$$\beta_n \equiv \frac{v}{c_n} \qquad\qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

Consider water with $n \approx 1.3$

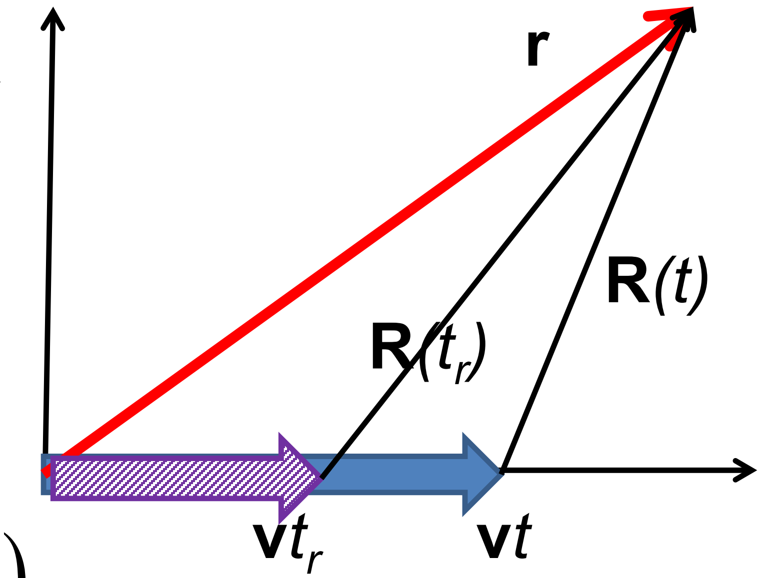
Which of these particles could produce Cherenkov radiation?

1. A neutron with speed c?
2. An electron with speed 0.6c?
3. A proton with speed 0.6c?
4. An electron with speed 0.8c?
5. An alpha particle with speed 0.8c?
6. None of these?

Further comment –

As discussed particularly in Chap. 13 of Jackson, a particle moving within a medium is likely to be slowed down so that the Cherenkov effect will only happen while $\beta_n > 1$.

Consider a particle moving at constant velocity **v**; $v > c_n$



Some algebra

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r) = \left|\mathbf{R}(t) + \mathbf{v}(t - t_r)\right|$$

Quadratic equation for $(t - t_r)c_n$:

$$\left((t - t_r)c_n\right)^2 = R^2(t) + 2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t - t_r)c_n + \beta_n^{\ 2}\left((t - t_r)c_n\right.$$

$$\left(\beta_n^{\ 2} - 1\right)\left((t - t_r)c_n\right)^2 + 2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t - t_r)c_n + R^2(t) = 0$$

Quadratic equation for $(t-t_r)c_n$:

$$\left(\beta_n^{\ 2}-1\right)\left(\left(t-t_r\right)c_n\right)^2+2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\left(t-t_r\right)c_n+R^2(t)=0$$

For β_n>1, how can the equality be satisfied?

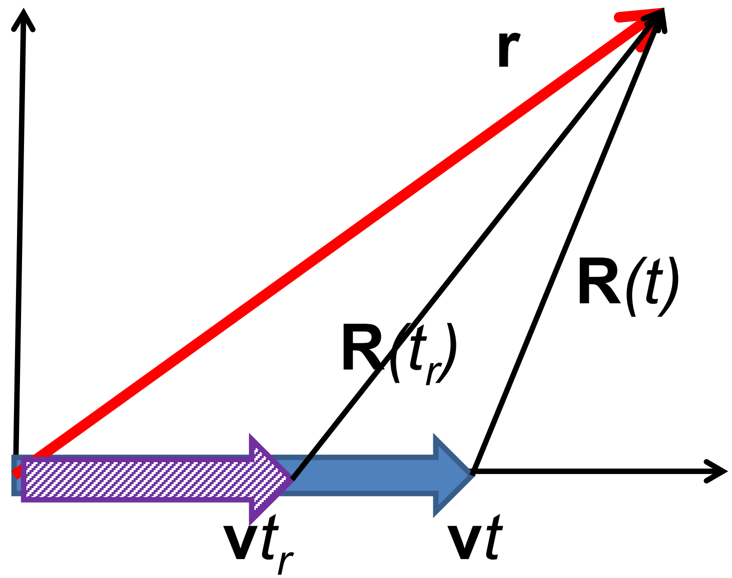
1. No problem
2. It cannot be satisfied.
3. It can only be satisfied for special conditions

From solution of quadratic equation:

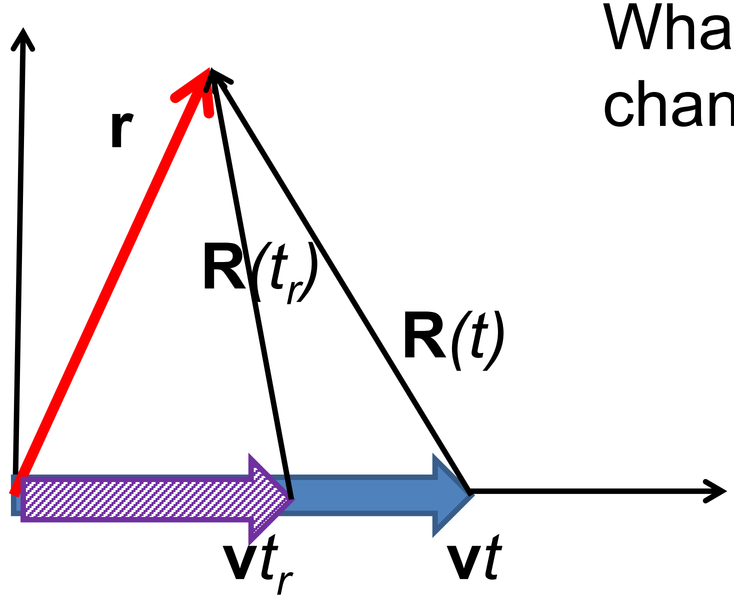
$$\left(t-t_r\right)c_n=\frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\pm\sqrt{\left(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\right)^2-\left(\beta_n^{\ 2}-1\right)R^2(t)}}{\beta_n^{\ 2}-1}$$

$\Rightarrow \mathbf{R}(t)\cdot\boldsymbol{\beta}_n<0$ (initial diagram is incorrect!)

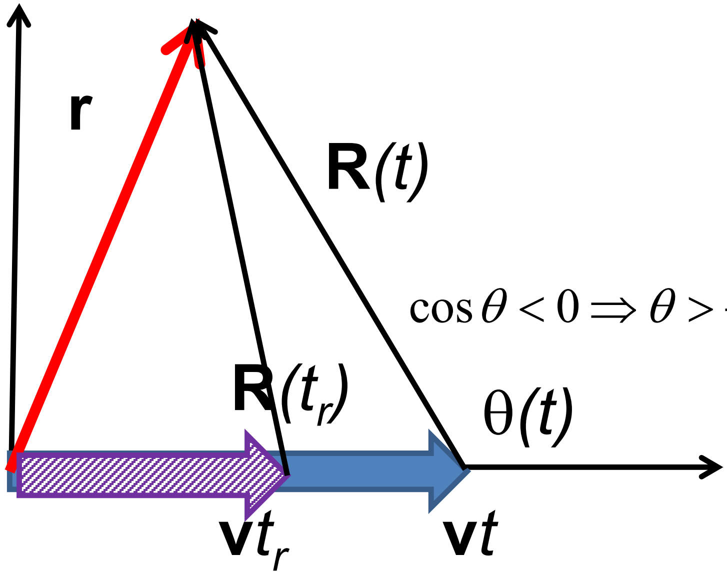
Original diagram:



New diagram:



What is the significance of changing the diagram?



$$\cos\theta < 0 \Rightarrow \theta > \frac{\pi}{2}$$

$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r)$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n =$$

$$(t - t_r)c_n\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$= R(t_r)\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$R(t_r) = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm \sqrt{\left(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\right)^2 - \left(\beta_n^{\,2} - 1\right)R^2(t)}}{\beta_n^{\,2} - 1}$$

$$R(t_r) = \frac{R(t)}{\beta_n^{\,2} - 1}\left(-\beta_n\cos\theta \pm \sqrt{1 - \beta_n^{\,2}\sin^2\theta}\right) = (t - t_r)c_n$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n = \mp R(t)\sqrt{1 - \beta_n^{\,2}\sin^2\theta}$$

Recall the Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon} \frac{1}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

$$\mathbf{A}(\mathbf{r},t) = q\mu \frac{\boldsymbol{\beta}_n}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r)$$

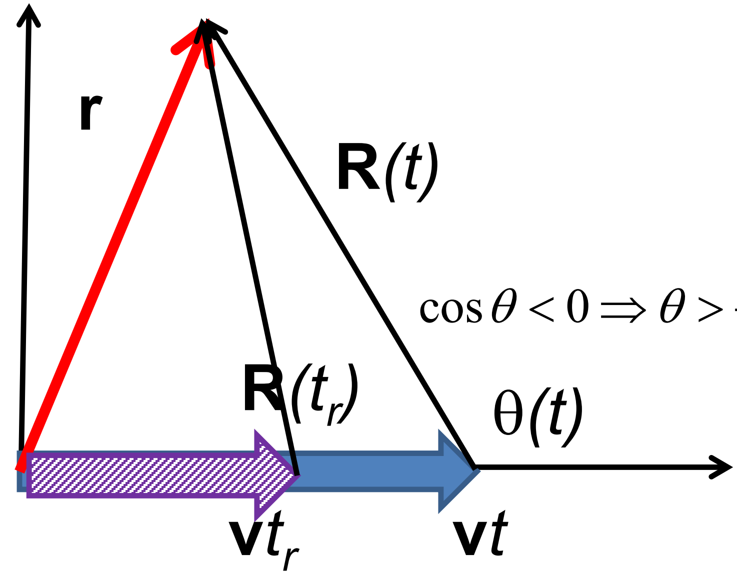
$$\boldsymbol{\beta}_n(t_r) \equiv \frac{\dot{\mathbf{R}}_q(t_r)}{c_n} \qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

$$t_r = t - \frac{R(t_r)}{c_n}$$

Liénard-Wiechert potentials for two solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon}\frac{1}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$

$$\mathbf{A}(\mathbf{r},t) = q\mu\frac{\boldsymbol{\beta}_n}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$



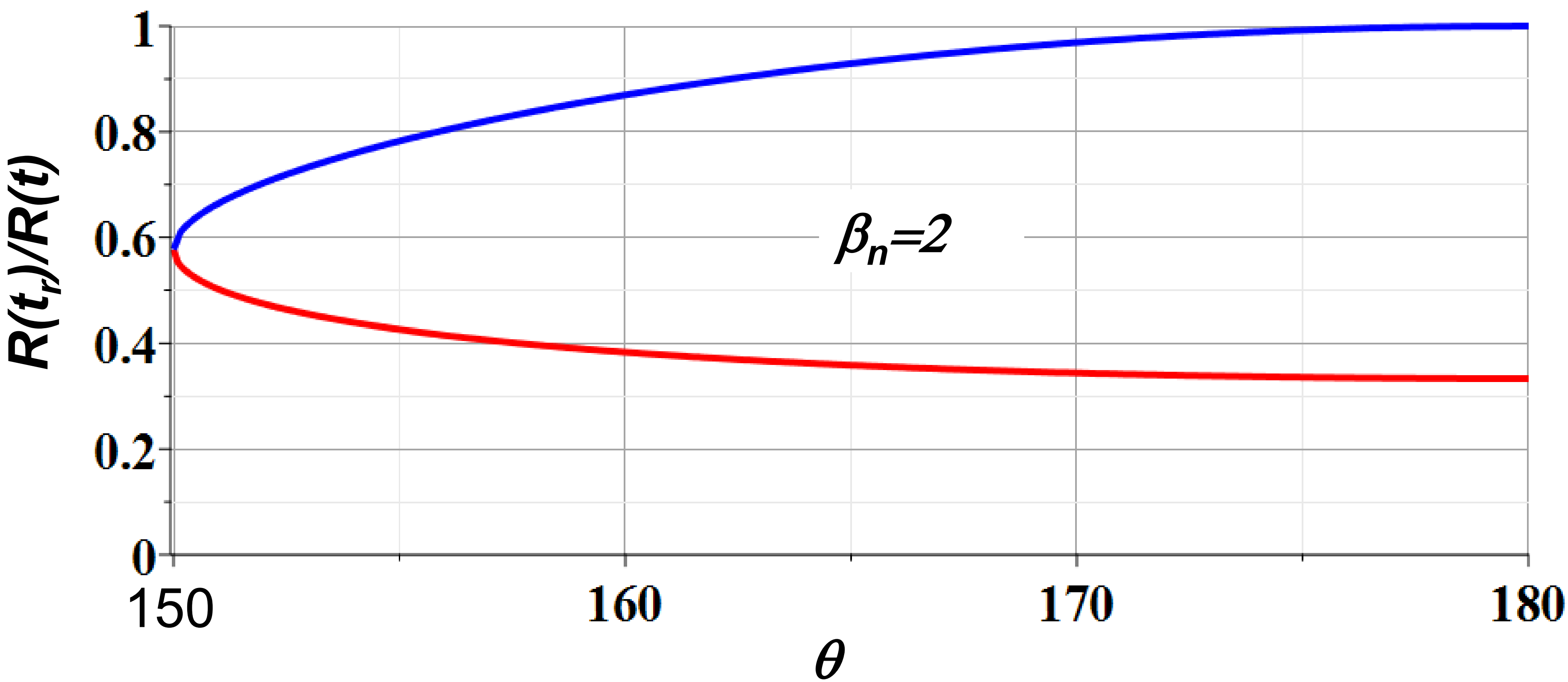
For $\beta_n > 1$, the range of θ is limited further:

$$R(t_r) = \frac{R(t)}{\beta_n{}^2 - 1}\left(-\beta_n\cos\theta \pm \sqrt{1-\beta_n{}^2\sin^2\theta}\right) \geq 0$$

$$\Rightarrow |\sin\theta| \leq \frac{1}{\beta_n} \equiv |\sin\theta_c| \quad \text{and} \quad \pi \geq \theta_c \geq \pi/2 \qquad \cos\,\theta_c = -\sqrt{1-\frac{1}{\beta_n^2}}$$

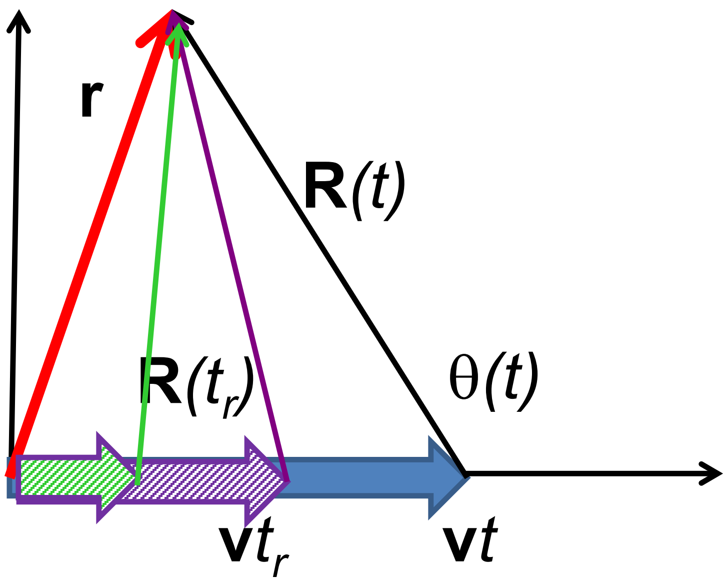
In this range, $\theta \geq \theta_c$

$$R(t_r) = \frac{R(t)}{\beta_n{}^2 - 1}\left(-\beta_n \cos\theta \pm \sqrt{1-\beta_n{}^2 \sin^2\theta}\right)$$



$$\theta_c = 150^o \text{ for this case}$$

Physical fields for $\beta_n > 1$ -- two retarded solutions contribute



$$\theta \le \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Define $\cos\theta_C \equiv -\sqrt{1-\frac{1}{\beta_n^{\,2}}}$

$$\Rightarrow \cos\theta \le \cos\theta_C$$

Adding two solutions; in terms of Heaviside $\Theta(x)$:

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

Physical fields for $\beta_n > 1$

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

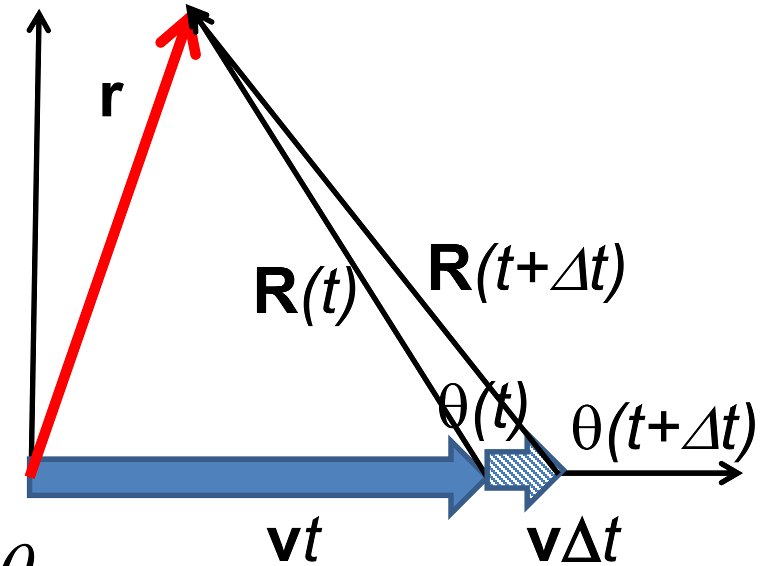
$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi - \frac{1}{c_n}\frac{\partial\mathbf{A}}{\partial t} \qquad\qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{\left(R(t)\right)^2\sqrt{1-\beta_n{}^2\sin^2\theta}} \quad \times$$

$$\left(-\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n{}^2-1}\;\delta(\cos\theta_C - \cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Intermediate steps:



$$\frac{d\theta}{dt} = \frac{v\sin\theta}{R} \qquad \frac{dR}{dt} = -v\cos\theta$$

Using instantaneous polar coordinates: $\nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial \theta}$

$$\nabla\Theta\left(\cos\theta_C - \cos\theta(t)\right) = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta\left(\cos\theta_C - \cos\theta(t)\right)}{\partial t} = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{v\sin^2\theta(t)}{R(t)}$$

Power radiated:

$$\frac{dP(t)}{d\Omega} = \left(R(t)\right)^2 \hat{\mathbf{R}}\cdot\mathbf{S}(t) = \left(R(t)\right)^2 \frac{c_n}{4\pi}\left|\mathbf{E}\left(\mathbf{R}(t),t\right)\right|^2 = \left|\boldsymbol{a}(t)\right|^2$$

where

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{\left(R(t)\right)^2\sqrt{1-\beta_n{}^2\sin^2\theta}} \quad \times$$

$$\left(\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C-\cos\theta(t)) + \sqrt{\beta_n{}^2-1}\;\delta(\cos\theta_C-\cos\theta(t))\right)$$

Spectral analysis using Parseval's theorem:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty}\left|\boldsymbol{a}(t)\right|^2 dt = \int_{-\infty}^{\infty}\left|\tilde{\boldsymbol{a}}(\omega)\right|^2 d\omega = \int_0^{\infty}\left(\left|\tilde{\boldsymbol{a}}(\omega)\right|^2 + \left|\tilde{\boldsymbol{a}}(-\omega)\right|^2\right)d\omega = \int_0^{\infty}\frac{\partial^2 I}{\partial\Omega\partial\omega}d\omega$$

$$\tilde{\boldsymbol{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt\;\boldsymbol{a}(t)\;e^{i\omega t}$$

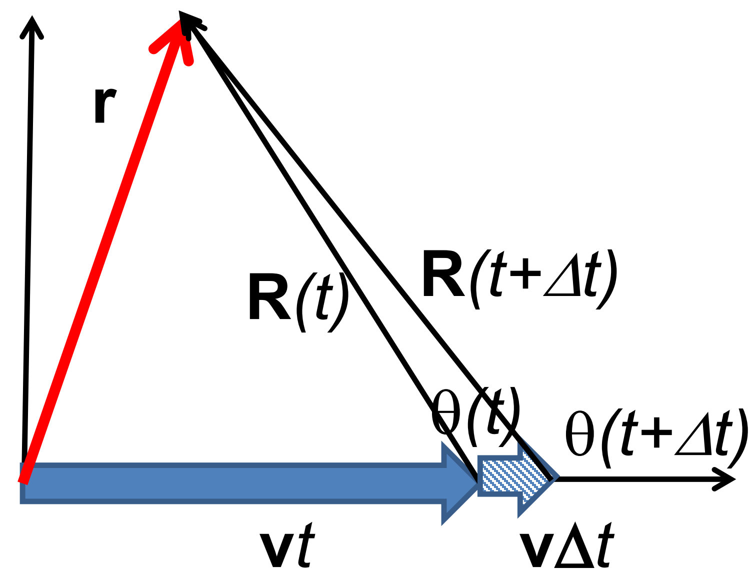
$$\boldsymbol{a}(t)=\frac{K}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\left(-\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C-\cos\theta(t))+\sqrt{\beta_n{}^2-1}\;\delta(\cos\theta_C-\cos\theta(t))\right)$$

Denote $t=0$ corresponding the angle θ_c

$$\theta(t)=\theta_c+\Delta\theta(t) \text{ where } \Delta\theta(t)\approx vt\frac{\sin\theta_c}{R(0)}$$

$$\cos\theta_c-\cos\theta(t)\approx\frac{c_n t}{\beta_n R(0)}$$

$$1-\beta_n^2\sin^2\theta(t)\approx-\frac{2c_n t\sqrt{\beta_n^2-1}}{R(0)}$$



Approximate amplitude near $t\approx 0$:

$$\boldsymbol{a}(t)\approx K\frac{\left(\beta_n^2-1\right)^{1/4}}{(2c_n)^{3/2}\sqrt{R(0)}}\left(\frac{\delta(t)}{\sqrt{t}}-\frac{\Theta(t)}{2\sqrt{t^3}}\right)$$

Approximate Fourier amplitude:

$$\tilde{\boldsymbol{a}}(\omega)\approx K\sqrt{\frac{\pi}{2}}\frac{\left(\beta_n^2-1\right)^{1/4}(1-i)}{(c_n)^{3/2}\sqrt{R(0)}}\sqrt{\omega}$$

Noting that $\beta_n = \frac{c}{n(\omega)} = \frac{c}{\sqrt{\epsilon(\omega)}}$

$$\frac{\partial^2 I}{\partial\Omega\partial\omega} \propto \omega\left(1 - \frac{c^2}{v^2\epsilon(\omega)}\right)$$

When the dust clears --

Frequency dependence of intensity:

$$\frac{dI}{d\omega} \approx \frac{q^2}{c^2}\omega\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)$$

From this expression, how would you explain that Cherenkov radiation is typically observed as a blue glow?

1. It is still a mystery.
2. It is obvious from the result.