PHY 712 Electrodynamics
10-10:50 AM MWF Online

Discussion for Lecture 34:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity

28	Fri: 04/09/2021	Chap. 14	Radiation from accelerating charged particles	#20	04/12/2021
29	Mon: 04/12/2021	Chap. 14	Synchrotron radiation	#21	04/14/2021
30	Wed: 04/14/2021	Chap. 14	Synchrotron radiation	#22	04/19/2021
31	Fri: 04/16/2021	Chap. 15	Radiation from collisions of charged particles	#23	04/21/2021
32	Mon: 04/19/2021	Chap. 15	Radiation from collisions of charged particles		
33	Wed: 04/21/2021	Chap. 13	Cherenkov radiation		
34	Fri: 04/23/2021		Special topic: E & M aspects of superconductivity		
35	Mon: 04/26/2021		Special topic: E & M aspects of superconductivity		
36	Wed: 04/28/2021		Review		
37	Fri: 04/30/2021		Review		
	Mon: 05/03/2021		Presentations I		
	Wed: 05/05/2021		Presentations II		

Important dates: Final exams available May 6; due May 14
Outstanding work due May 14

Advice about presentations

Each presentation should be roughly ~ 10 minutes using power point or the equivalent

It should contain the following

1. Introduction and motivation
2. Some detailed derivation and/or numerical work
3. Conclusions and summary of what you learned
4. Bibliography including any possible online sources.

Materials to turn in

1. Presentation slides (or pdf version)
2. If you have chosen to review a literature paper, please include its pdf file if possible.
3. Maple, Mathematica, or other software files that were used in the project

What will you do after May 14?

Relax a minute or two

Several of you will want to start preparing for the Qualifier Exams which will be administered (tentative dates):
Monday, June 21 to Thursday, June 24 during the hours 9:00 am - 12 pm.

Your questions –

From Tim -- How come on slide 11 the magnetic field is only changing in the z direction? Is that just how they prepared the magnetic field in the superconducting material?

From Gao -- You gave some comparison between type 1 and 2 superconductors in lecture notes. Could you introduce the mechanism leads to these two types of superconductors.

Special topic: Electromagnetic properties of superconductors

Ref:D. Teplitz, editor, Electromagnetism – paths to research, Plenum Press (1982); Chapter 1 written by Brian Schwartz and Sonia Frota-Pessoa

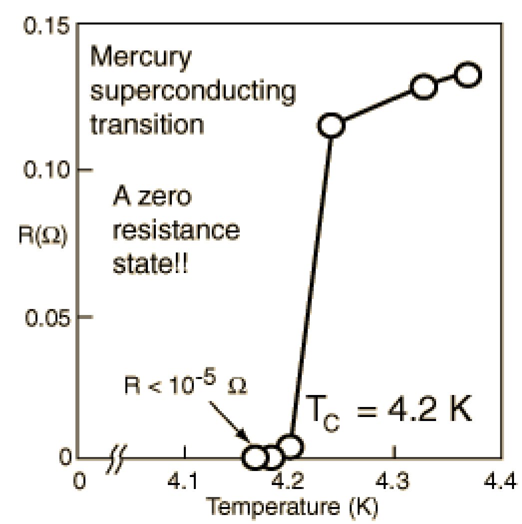
History:

1908 H. Kamerlingh Onnes successfully liquified He

1911 H. Kamerlingh Onnes discovered that Hg at 4.2 K has vanishing resistance

1957 Theory of superconductivity by Bardeen, Cooper, and Schrieffer

The surprising observation was that electrical resistivity abruptly dropped when the temperature of the material was lowered below a critical temperature T_c.



Fritz London 1900-1954

Fritz London, 1947, photo: Lotte Meitner-Graf

Fritz London, one of the most distinguished scientists on the Duke University faculty, was an internationally recognized theorist in Chemistry, Physics and the Philosophy of Science. He was born in Breslau, Germany (now Wroclaw, Poland) in 1900. In 1933 he was

He immigrated to the United States in 1939, and came to Duke University, first as a Professor of Chemistry. In 1949 he received a joint appointment in Physics and Chemistry and became a James B. Duke Professor. In 1953 he became the 5th recipient of the Lorentz medal, awarded by the Royal Netherlands Academy of Sciences, and was the first American citizen to receive this honor. He died in Durham in 1954.

https://phy.duke.edu/about/history/historical-faculty/fritz-london

Some phenomenological theories < 1957 thanks to F. London

Drude model of conductivity in "normal" materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - m\frac{\mathbf{v}}{\tau}$$

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-t/\tau} - \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{J} = -ne\mathbf{v}; \qquad \text{for } t >> \tau \qquad \Rightarrow \qquad \mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} \equiv \sigma\mathbf{E}$$

Note: Equations are in cgs Gaussian units.

London model of conductivity in superconducting materials; $\tau \to \infty$

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{e\mathbf{E}}{m} \qquad \frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Properties of a normal metal

Drude model of conductivity in "normal" materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - m\frac{\mathbf{v}}{\tau}$$

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-t/\tau} - \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{J} = -ne\mathbf{v}; \qquad \text{for } t >> \tau \qquad \Rightarrow \qquad \mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} \equiv \sigma\mathbf{E}$$

Does this model allow for any temperature dependence on the resistivity?

1. No.
2. Yes.
3. Maybe.

London model of conductivity in superconducting materials; $\tau \to \infty$

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{e\mathbf{E}}{m} \qquad \frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

How is the London model different from the Drude model?

1. Subtle difference.
2. Big difference.

Some phenomenological theories < 1957

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times (\nabla \times \mathbf{B}) = -\nabla^2\mathbf{B} = \frac{4\pi}{c}\nabla \times \mathbf{J} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$-\nabla^2\frac{\partial \mathbf{B}}{\partial t} = \frac{4\pi}{c}\nabla \times \frac{\partial \mathbf{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^3 \mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial \mathbf{B}}{\partial t} = \frac{4\pi ne^2}{mc}\nabla \times \mathbf{E} - \frac{1}{c^2}\frac{\partial^3 \mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial \mathbf{B}}{\partial t} = -\frac{4\pi ne^2}{mc^2}\frac{\partial \mathbf{B}}{\partial t} - \frac{1}{c^2}\frac{\partial^3 \mathbf{B}}{\partial t^3}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

Are these equations

1. Exact?
2. Approximate?
3. Wrong?

London model – continued

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

For slowly varying solution:

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2}\right)\mathbf{B} = 0 \qquad \text{for } \frac{\partial \mathbf{B}}{\partial t} = \hat{\mathbf{z}}\frac{\partial B_z(x,t)}{\partial t}:$$

$$\Rightarrow \frac{\partial B_z(x,t)}{\partial t} = \frac{\partial B_z(0,t)}{\partial t}e^{-x/\lambda_L}$$

Here we assume we know the boundary value at *x=0*.

London's leap: $B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Consistent results for current density:

$$\frac{4\pi}{c}\nabla \times \mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \mathbf{J} = \hat{\mathbf{y}}J_y(x) \qquad \Rightarrow J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$$

London model – continued

Penetration length for superconductor: $\lambda_L^2 \equiv \dfrac{mc^2}{4\pi ne^2}$ Typically, $\lambda_L \approx 10^{-7}\, m$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Vector potential for $\mathbf{B} = \nabla \times \mathbf{A}$ and $\nabla \cdot \mathbf{A} = 0$:

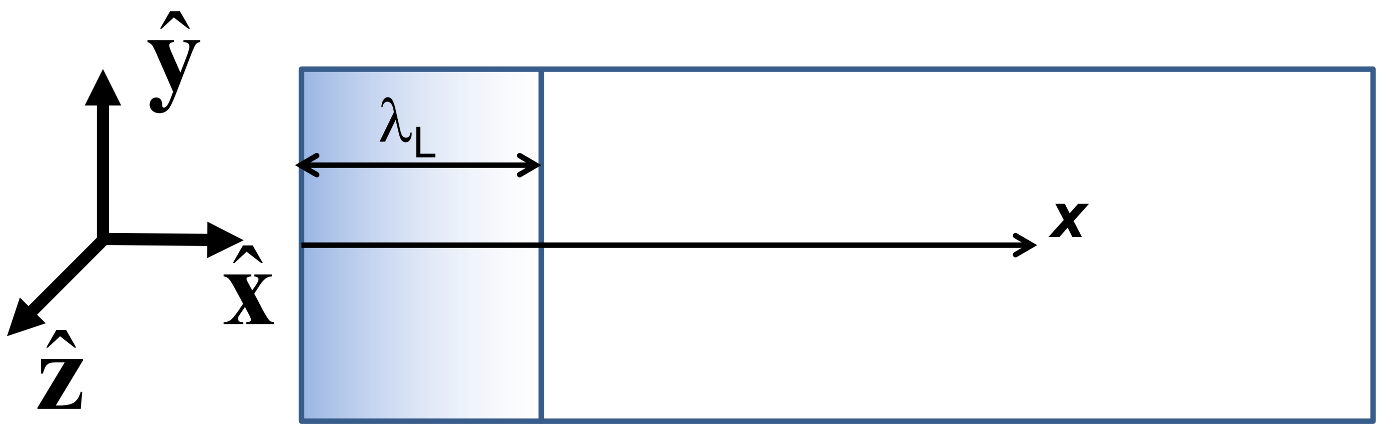
$\mathbf{A} = \hat{\mathbf{y}} A_y(x)$ $\qquad A_y(x) = -\lambda_L B_z(0) e^{-x/\lambda_L}$

Note that: $\nabla \times \mathbf{B} = \dfrac{4\pi}{c}\mathbf{J}$

$-\nabla^2 \mathbf{A} = \dfrac{4\pi}{c}\mathbf{J} \Rightarrow \nabla^2 \mathbf{A} + \dfrac{4\pi}{c}\mathbf{J} = 0$

Recall form for current density: $J_y(x) = \lambda_L \dfrac{ne^2}{mc} \mathrm{B}_z(0)\,\mathrm{e}^{-\mathrm{x}/\lambda_L}$

$$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A} \right) = 0$$



Behavior of superconducting material – exclusion of magnetic field according to the London model

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi n e^2}$

$$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$$

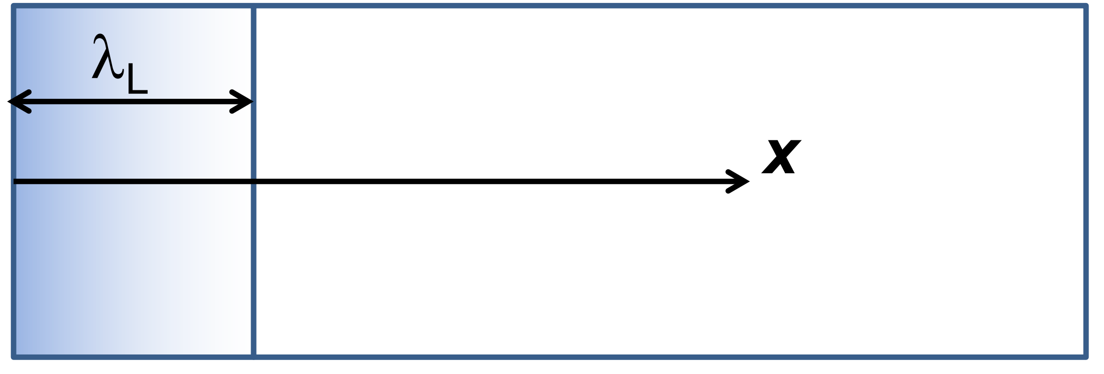
Vector potential for $\nabla \cdot \mathbf{A} = 0$:

$$\mathbf{A} = \hat{\mathbf{y}} A_y(x) \qquad A_y(x) = -\lambda_L B_z(0)e^{-x/\lambda_L}$$

Current density: $J_y(x) = \lambda_L \frac{ne^2}{mc} \mathrm{B}_z(0)\,\mathrm{e}^{-\mathrm{x}/\lambda_L}$

$$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A} \right) = 0$$

Typically, $\lambda_L \approx 10^{-7} m$



Behavior of magnetic field lines near superconductor

normal state: superconducting state:

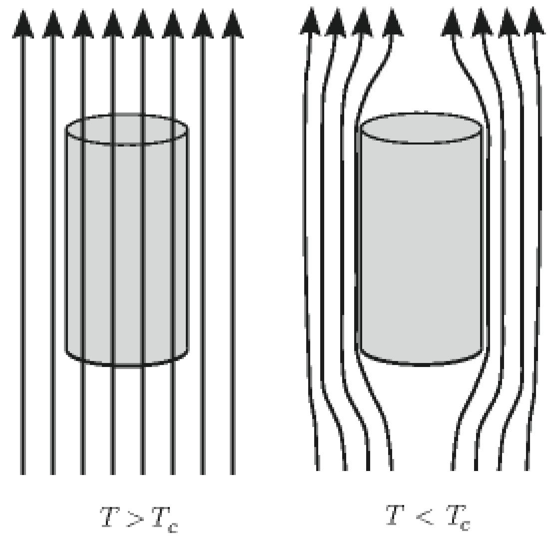
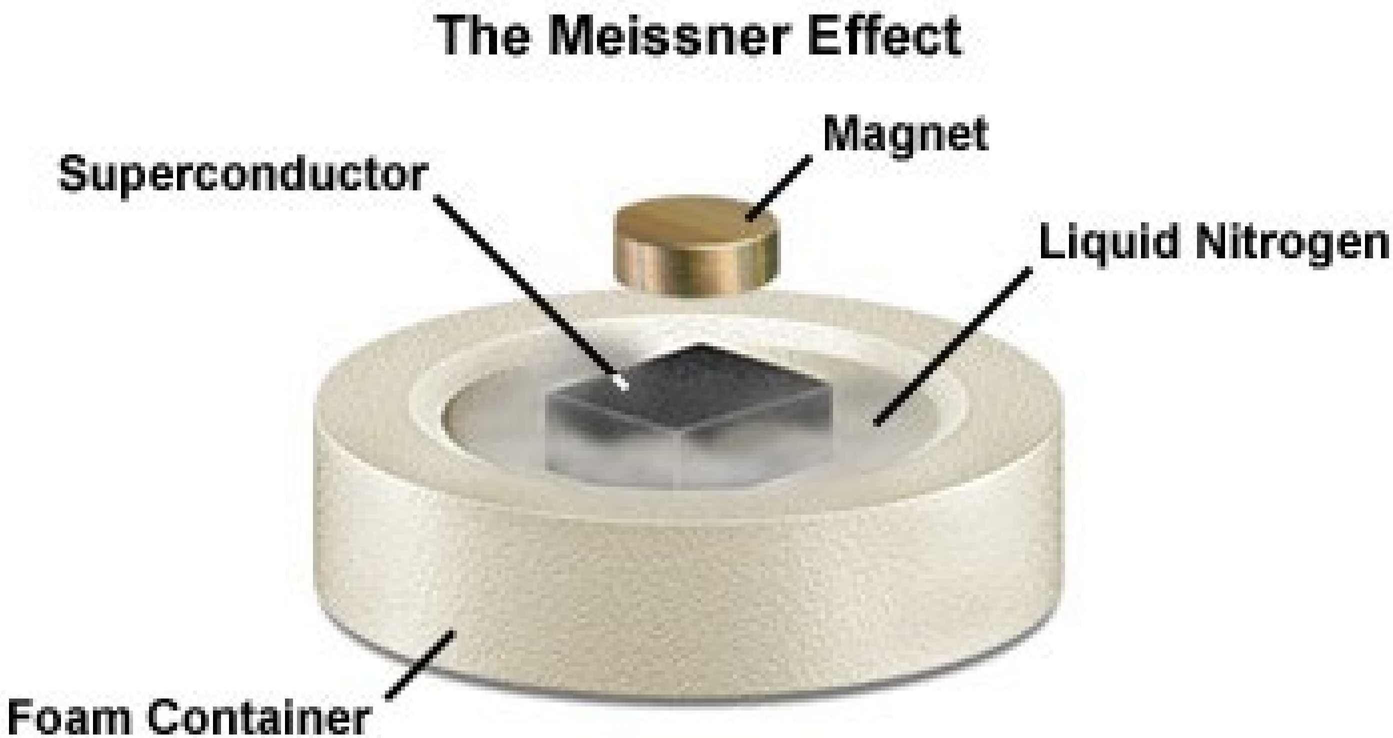


Figure 18.2 Exclusion of a weak external magnetic field from the interior of a superconductor.


The Meissner Effect
Magnet
Superconductor
Liquid Nitrogen
Foam Container

Need to consider phase equilibria between "normal" and superconducting state as a function of temperature and applied magnetic fields.

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$$

Within the superconductor, if $\mathbf{B} = 0$

$$\text{then} \quad \mathbf{H} + 4\pi\mathbf{M} = 0 \quad \text{or} \quad \mathbf{M} = -\frac{\mathbf{H}}{4\pi}$$

Magnetization field

Treating London current in terms of corresponding magnetization field **M**:

$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$

$\Rightarrow$ For $x >> \lambda_L$, $\quad \mathbf{H}=-4\pi\mathbf{M}, \qquad \mathbf{M}(\mathbf{H})=-\frac{\mathbf{H}}{4\pi}$

Here H is thought of in terms of an applied field.

Gibbs free energy associated with magnetization for superconductor:

$$G_S(H_a)=G_S(H=0)-\int_0^{H_a} dH M(H)=G_S(0)-\int_0^{H_a} dH\left(\frac{-H}{4\pi}\right)=G_S(0)+\frac{1}{8\pi}H_a^2$$

This relation is true for an applied field $\mathrm{H}_a \leq H_C$ when the superconducting and normal Gibbs free energies are equal:

$$G_S(H_C)=G_N(H_C)\approx G_N(H=0)$$

Condition at phase boundary between normal and superconducting states:

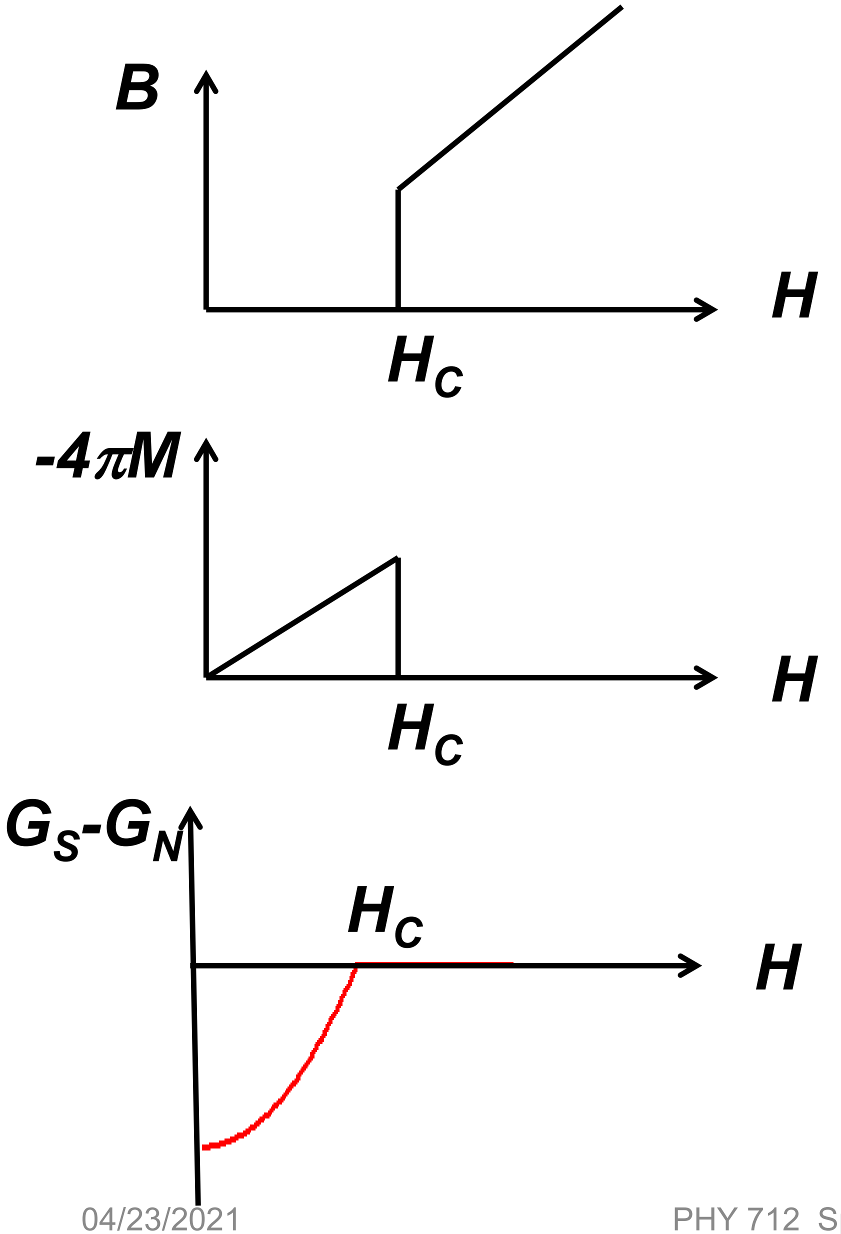
$$G_N(H_C)\approx G_N(0)=G_S(H_C)=G_S(0)+\frac{1}{8\pi}H_C^2$$

At $T=0K$

$$\Rightarrow G_S(0)-G_N(0)=-\frac{1}{8\pi}H_C^2$$

$$G_S(H_a)-G_N(H_a)=\begin{cases}-\frac{1}{8\pi}\left(H_C^2-H_a^2\right) & \text{for } H_a<H_C \\ 0 & \text{for } H_a>H_C\end{cases}$$

Magnetization field (for “type I” superconductor)



Inside superconductor

$$\mathbf{B}=0=\mathbf{H}+4\pi\mathbf{M} \text{ for } H<H_C$$

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

$$G_S(0) - G_N(0) = -\frac{H_C^2}{8\pi} \approx -2N(E_F)(\hbar\omega)^2 e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Temperature dependence of critical field

$$H_c(T) \approx H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

From PR **108**, 1175 (1957) Bardeen, Cooper, and Schrieffer, "Theory of Superconductivity"

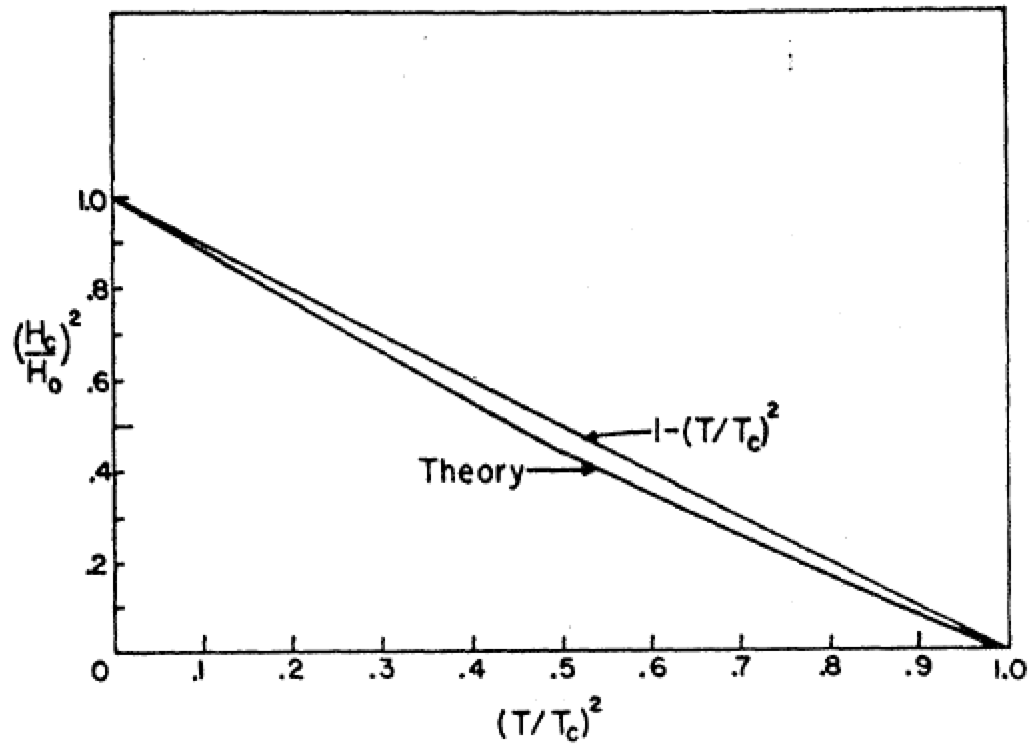


FIG. 2. Ratio of the critical field to its value at $T=0°K$ *vs* $(T/T_c)^2$. The upper curve is the $1-(T/T_c)^2$ law of the Gorter-Casimir theory and the lower curve is the law predicted by the theory in the weak-coupling limit. Experimental values generally lie between the two curves.

$$T_c \approx \frac{\hbar\omega}{k} e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Type I elemental superconductors

http://wuphys.wustl.edu/~jss/NewPeriodicTable.pdf

Periodic Table of Superconductivity

(dedicated to the memory of Bernd Matthias; compiled by James S. Schilling)

30 elements superconduct at ambient pressure, 23 more superconduct at high pressure.

ambient pressure superconductor: T_c(K) / T_c^{max}(K) / P(GPa)

high pressure superconductor: T_c^{max}(K) / P(GPa)

H																	He
Li 0.0004 14 30	Be 0.026 3.7 30											B 11 250	C	N	O 0.6 100	F	Ne
Na	Mg											Al 1.14	Si 8.2 15.2	P 13 30	S 17.3 190	Cl	Ar
K	Ca 29 217	Sc 19.6 106	Ti 0.39 3.35 56.0	V 5.38 16.5 120	Cr	Mn	Fe 2.1 21	Co	Ni	Cu	Zn 0.875	Ga 1.091 7 1.4	Ge 5.35 11.5	As 2.4 32	Se 8 150	Br 1.4 100	Kr
Rb	Sr 7 50	Y 19.5 115	Zr 0.546 11 30	Nb 9.50 9.9 10	Mo 0.92	Tc 7.77	Ru 0.51	Rh .00033	Pd	Ag	Cd 0.56	In 3.404	Sn 3.722 5.3 11.3	Sb 3.9 25	Te 7.5 35	I 1.2 25	Xe
Cs 1.3 12	Ba 5 18	insert La-Lu	Hf 0.12 8.6 62	Ta 4.483 4.5 43	W 0.012	Re 1.4	Os 0.655	Ir 0.14	Pt	Au	Hg-α 4.153	Tl 2.39	Pb 7.193	Bi 8.5 9.1	Po	At	Rn
Fr	Ra	insert Ac-Lr	Rf	Ha													

La-fcc 6.00 13 15	Ce 1.7 5	Pr	Nd	Pm	Sm	Eu 2.75 142	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 12.4 174
Ac	Th 1.368	Pa 1.4	U 0.8(β) 2.4(α) 1.2	Np	Pu	Am 0.79 2.2 6	Cm	Bk	Cf	Es	Fm	Md	No	Lr

M. Debessai, T. Matsuoka, J.J. Hamlin, W. Bi, Y. Meng, K. Shimizu, and J.S. Schilling, J. Phys.: Conf. Series **215**, 012034 (2010).
High pressure data for Ca and Be: K. Shimizu email from 9 Dec 2013.

Type I superconductors:

$$H_c(T) = H_c(0)\left(1 - \frac{T^2}{T_c^2}\right)$$

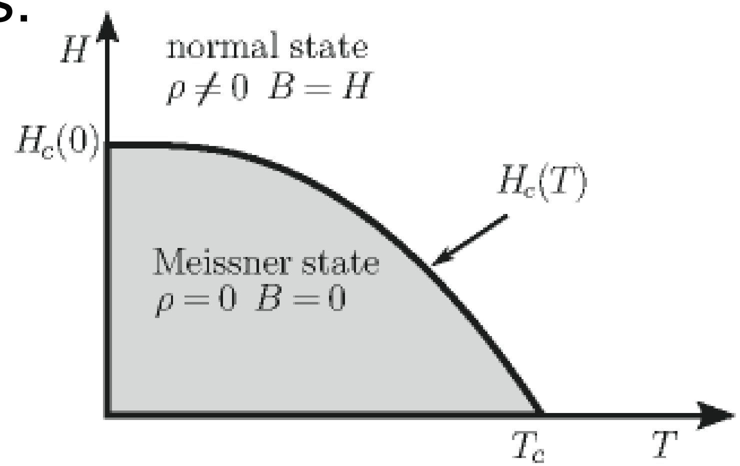


Figure 18.3 Schematic phase diagram illustrating normal and superconducting regions of a type-I superconductor.

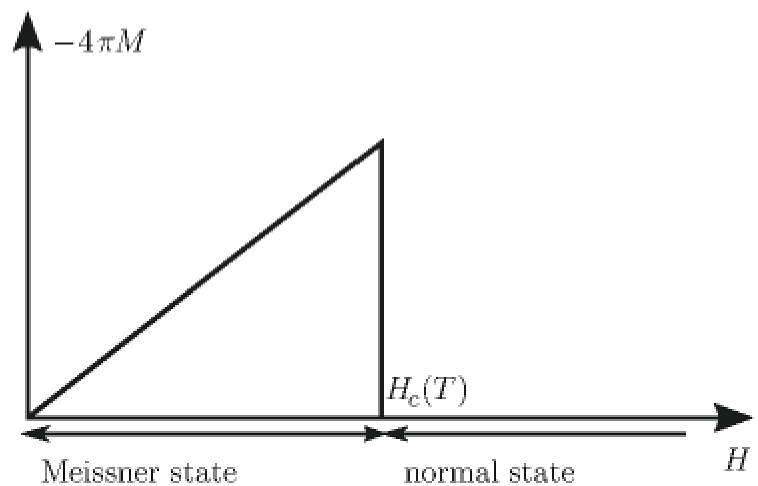


Figure 18.4 Magnetization versus applied field for type-I superconductors.

The following slides give a quick look of some of the intriguing aspects of superconducting materials and their properties --

Type II superconductors

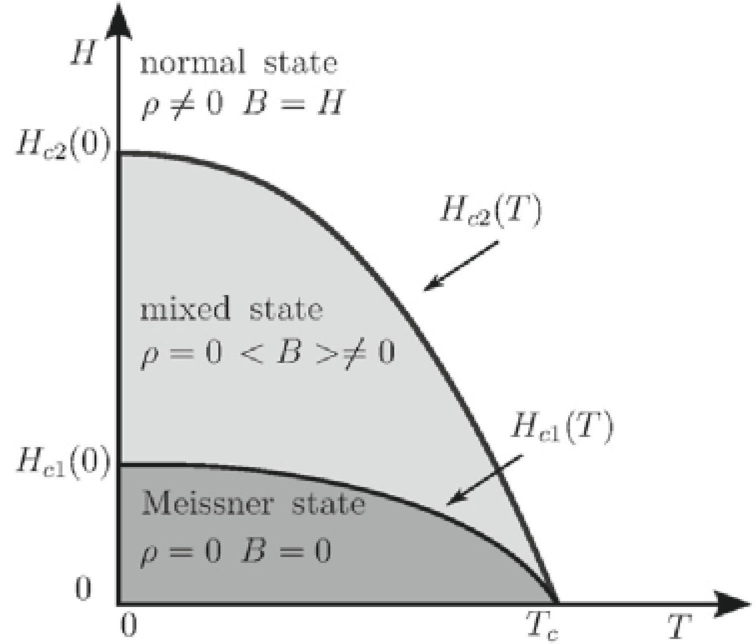


Figure 18.5 Schematic phase diagram illustrating normal, mixed and Meissner regions of a type-II superconductor (the vanishingly small resistivity of the mixed state occurs if flux lines are "pinned" by appropriate material defects); in the mixed state, $\langle B \rangle$ denotes the average magnetic field in the superconductor.

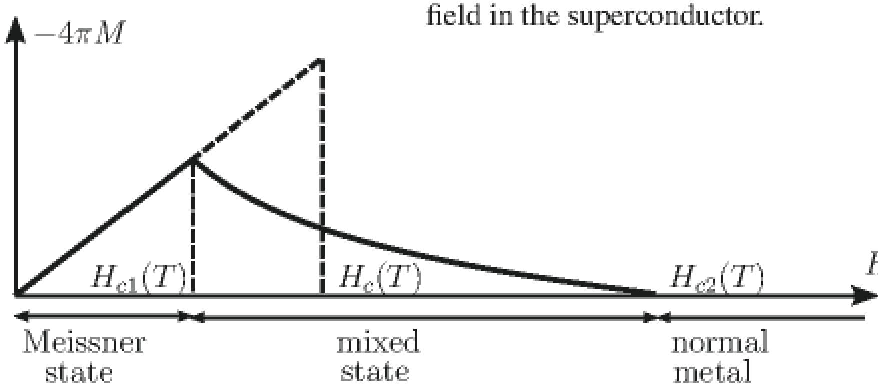


Figure 18.6 Magnetization versus applied field H for a type-II superconductor. The equivalent area construction of the thermodynamic field $H_c(T)$ is also illustrated.

Quantization of current flux associated with the superconducting state (Ref: Ashcroft and Mermin, ***Solid State Physics***)

From the London equations for the interior of the superconductor:

$$\left(m\mathbf{v} + \frac{e}{c}\mathbf{A} \right) = 0$$

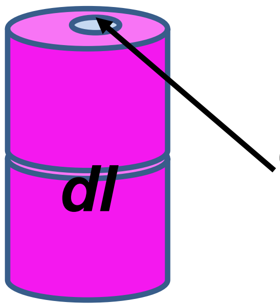
Now suppose that the current carrier is a pair of electrons characterized by a wavefunction of the form $\psi = |\psi| e^{i\phi}$

The quantum mechanical current associated with the electron pair is

$$\mathbf{j} = -\frac{e\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) - \frac{2e^2}{mc}\mathbf{A}|\psi|^2$$

$$= -\left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A} \right)|\psi|^2$$

Quantization of current flux associated with the superconducting state -- continued



Suppose a superconducting material has a cylindrical void. Evaluate the integral of the current in a closed path within the superconductor containing the void.

$$\oint \mathbf{j}\cdot d\mathbf{l} = 0 = -\oint \left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A} \right) |\psi|^2 \cdot d\mathbf{l}$$

$$\oint \mathbf{A}\cdot d\mathbf{l} = \int (\nabla\times\mathbf{A})\cdot d\mathbf{a} = \int \mathbf{B}\cdot d\mathbf{a} = \Phi \quad \text{magnetic flux}$$

$$\oint \nabla\phi\cdot d\mathbf{l} = 2\pi n \qquad \text{for some integer } n$$

$$\Rightarrow \text{Quantization of flux in the void: } |\Phi| = n\frac{hc}{2e} \equiv n\Phi_0$$

Such “vortex” fields can exist within type II superconductors.

Table 18.1 Critical temperature of some selected superconductors, and zero-temperature critical field. For elemental materials, the thermodynamic critical field $H_c(0)$ is given in gauss. For the compounds, which are type-II superconductors, the upper critical field $H_{c2}(0)$ is given in Tesla ($1\ T = 10^4\ G$). The data for metallic elements and binary compounds of V and Nb are taken from G. Burns (1992). The data for MgB_2 and iron pnictide are taken from the references cited in the text, and refer to the two principal crystallographic axes. The data for the other compounds are taken from D. R. Harshman and A. P. Mills, Phys. Rev. B *45*, 10684 (1992)]. A more extensive list of data can be found in the mentioned references.

Metallic elements	$T_c(K)$	$H_c(0)$ (gauss)
Al	1.17	105
Sn	3.72	305
Pb	7.19	803
Hg	4.15	411
Nb	9.25	2060
V	5.40	1410
Binary compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
V_3Ga	16.5	27
V_3Si	17.1	25
Nb_3Al	20.3	34
Nb_3Ge	23.3	38
MgB_2	40	≈ 5; ≈ 20
Other compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
UPt_3 (heavy fermion)	0.53	2.1
$PbMo_6S_8$ (Chevrel phase)	12	55
$\kappa-[BEDT-TTF]_2Cu[NCS]_2$ (organic phase)	10.5	≈ 10
Rb_2CsC_{60} (fullerene)	31.3	≈ 30
$NdFeAsO_{0.7}F_{0.3}$ (iron pnictide)	47	≈ 30; ≈ 50
Cuprate oxides	$T_c(K)$	$H_{c2}(0)$ (Tesla)
$La_{2-x}Sr_xCuO_4$ ($x \approx 0.15$)	38	≈ 45
$YBa_2Cu_3O_7$	92	≈ 140
$Bi_2Sr_2CaCu_2O_8$	89	≈ 107
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125	≈ 75

Crystal structure of one of the high temperature superconductors

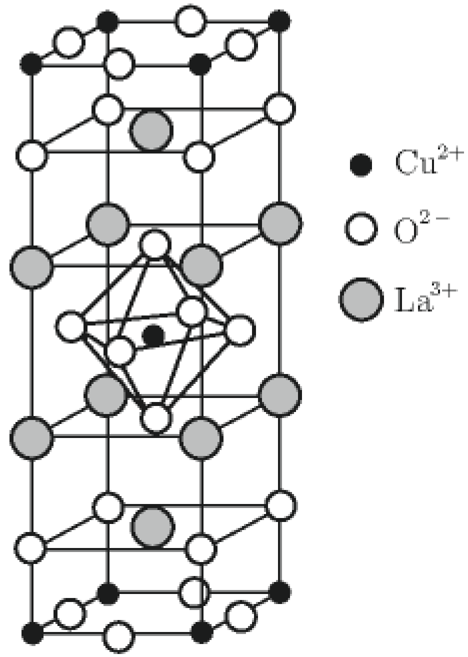
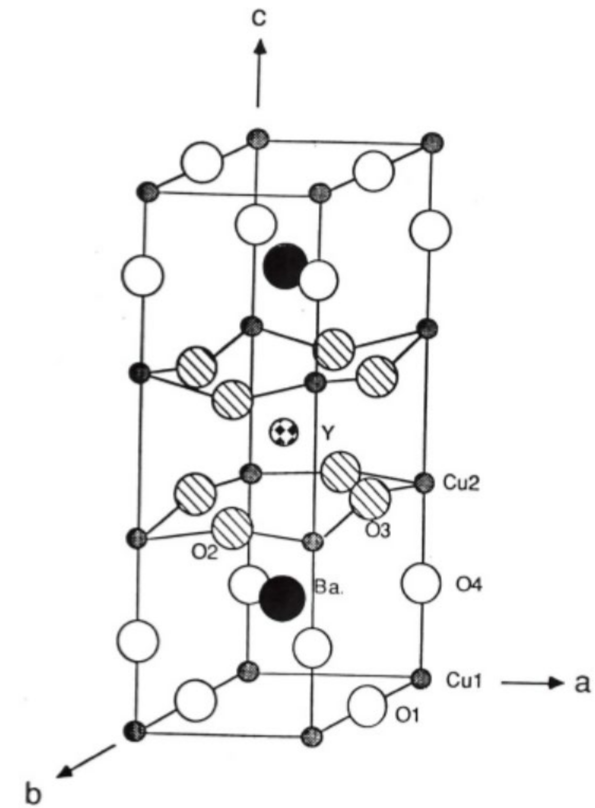


Figure 18.1 Crystal structure of the ceramic material La_2CuO_4. Appropriately doped, lanthanum-based cuprates opened the path to high-T_c superconductivity in 1986.



$YBa_2Cu_3O_7$

From MS thesis of Brent Howe (Minn State U, 2014)

Some details of single vortex in type II superconductor

London equation without vortices:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \text{where} \quad \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

Equation for field with single quantum of vortex along z - axis:

$$\nabla^2\mathbf{B} - \frac{1}{\lambda_L^2}\mathbf{B} = -\frac{\Phi_0}{\lambda_L^2}\hat{\mathbf{z}}\delta(\mathbf{r}) \qquad \Phi_0 = \frac{hc}{2e} \qquad \mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$$

Solution: $\quad \mathbf{B}(\mathbf{r})=\hat{\mathbf{z}}\frac{\Phi_0}{2\pi\lambda_L^2}K_0\left(\frac{r}{\lambda_L}\right)$

Check:

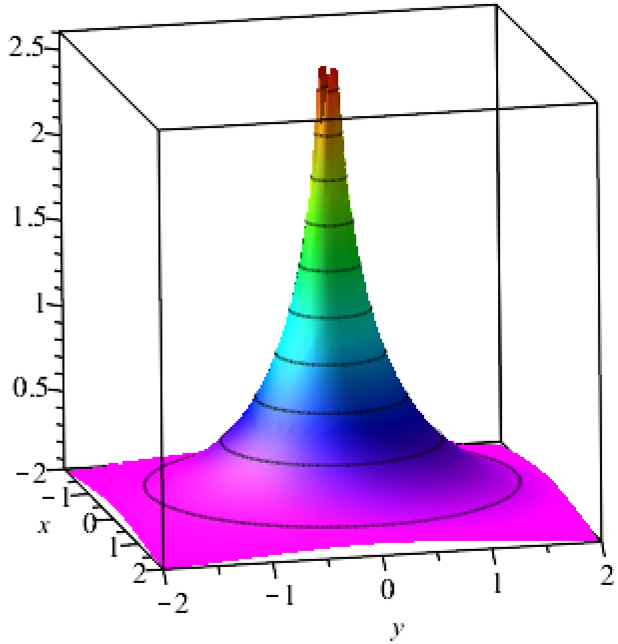
For $\quad r > 0 \quad \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = 0$

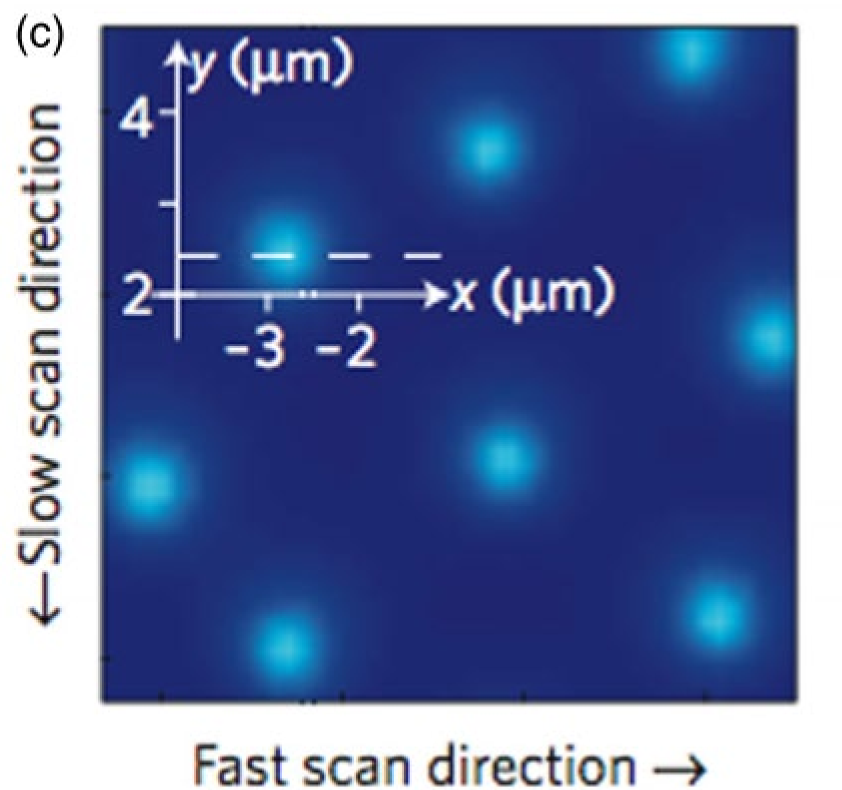
For $\quad r \to 0 \quad 2\pi\int_0^r dr'\, r'\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = -2\pi$

Since $\mathrm{K}_0(u) \underset{u\to 0}{\approx} -\ln u$

$$\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}} \frac{\Phi_0}{2\pi\lambda_L^2} K_0\left(\frac{r}{\lambda_L}\right)$$

Scanning probe images of vortices in YBCO at 22 K





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Fundamental studies of superconductors using scanning magnetic imaging

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Based on physics of the Josephson junction.

What we have not yet discussed is the microscopic mechanism for the phenomenon. --- to be continued on Monday 4/26/2021 (starting later due to QM exam)

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Theory of Superconductivity*

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(Received July 8, 1957)

A theory of superconductivity is presented, based on the fact that the interaction between electrons resulting from virtual exchange of phonons is attractive when the energy difference between the electrons states involved is less than the phonon energy, $\hbar\omega$. It is favorable to form a superconducting phase when this attractive interaction dominates the repulsive screened Coulomb interaction. The normal phase is described by the Bloch individual-particle model. The ground state of a superconductor, formed from a linear combination of normal state configurations in which electrons are virtually excited in pairs of opposite spin and momentum, is lower in energy than the normal state by amount proportional to an average $(\hbar\omega)^2$, consistent with the isotope effect. A mutually orthogonal set of excited states in one-to-one correspondence with those of the normal phase is obtained by specifying occupation of certain Bloch states and by using the rest to form a linear combination of virtual pair configurations. The theory yields a second-order phase transition and a Meissner effect in the form suggested by Pippard. Calculated values of specific heats and penetration depths and their temperature variation are in good agreement with experiment. There is an energy gap for individual-particle excitations which decreases from about $3.5kT_c$ at $T=0°K$ to zero at T_c. Tables of matrix elements of single-particle operators between the excited-state superconducting wave functions, useful for perturbation expansions and calculations of transition probabilities, are given.