

PHY 712 Electrodynamics

10-10:50 AM MWF Online

Discussion for Lecture 37:

Review of Electrodynamics --

Part 2

28	Fri: 04/09/2021	Chap. 14	Radiation from accelerating charged particles	#20	04/12/2021
29	Mon: 04/12/2021	Chap. 14	Synchrotron radiation	#21	04/14/2021
30	Wed: 04/14/2021	Chap. 14	Synchrotron radiation	#22	04/19/2021
31	Fri: 04/16/2021	Chap. 15	Radiation from collisions of charged particles	#23	04/21/2021
32	Mon: 04/19/2021	Chap. 15	Radiation from collisions of charged particles		
33	Wed: 04/21/2021	Chap. 13	Cherenkov radiation		
34	Fri: 04/23/2021		Special topic: E & M aspects of superconductivity		
35	Mon: 04/26/2021		Special topic: Overview of some optical properties of materials		
36	Wed: 04/28/2021		Review		
37	Fri: 04/30/2021		Review		
	Mon: 05/03/2021		Presentations I		
	Wed: 05/05/2021		Presentations II		

Review I Summary of concepts/equations

Review II Example problems

Timelines –

April 30 – sign up for presentations

May 3 – Presentations I

May 5 – Presentations II

May 6 – take home exam available

May 14 – all course materials due; outstanding homework, projects, and completed exams

Schedule for PHY 712 Presentations
Spring 2021

Monday, May 3, 2023

	Name	Topic
10:00-10:15		
10:15-10:30		
10:30-10:45		

Wednesday, May 5, 2021

	Name	Topic
10:00-10:15		
10:15-10:30		
10:30-10:45		

Review – Chapter 8 – waveguides and other boundary value situations for electromagnetic waves

Suppose for an isotropic medium: $\mathbf{D} = \varepsilon_b \mathbf{E}$ $\mathbf{J} = \sigma \mathbf{E}$

Maxwell's equations in terms of $\mathbf{H}$ and $\mathbf{E}$:

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon_b \frac{\partial \mathbf{E}}{\partial t}$$

$$\left(\nabla^2 - \mu \sigma \frac{\partial}{\partial t} - \mu \varepsilon_b \frac{\partial^2}{\partial t^2} \right) \mathbf{F} = 0 \quad \mathbf{F} = \mathbf{E}, \mathbf{H}$$

Plane wave form for $\mathbf{E}$:

$$\mathbf{E}(\mathbf{r}, t) = \Re(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}) \quad \text{where } \mathbf{k} = (n_R + in_I) \frac{\omega}{c} \hat{\mathbf{k}}$$

$$\Rightarrow \mathbf{E}(\mathbf{r}, t) = e^{-\hat{\mathbf{k}} \cdot \mathbf{r} / \delta} \Re(\mathbf{E}_0 e^{in_R(\omega/c) \hat{\mathbf{k}} \cdot \mathbf{r} - i\omega t})$$

Some details:

Plane wave form for $\mathbf{E}$:

$$\mathbf{E}(\mathbf{r}, t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}\right) \quad \text{where } \mathbf{k} = (n_R + in_I) \frac{\omega}{c} \hat{\mathbf{k}}$$

$$\left(\nabla^2 - \mu\sigma \frac{\partial}{\partial t} - \mu\epsilon_b \frac{\partial^2}{\partial t^2} \right) \mathbf{E} = 0$$

$$-(n_R + in_I)^2 + i \frac{\mu\sigma c^2}{\omega} + \mu\epsilon_b c^2 = 0$$

Fields near the surface on an ideal conductor -- continued

For our system:

$$\frac{\omega}{c} n_R = \omega \sqrt{\frac{\mu \epsilon_b}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon_b} \right)^2} + 1 \right)^{1/2}$$

$$\frac{\omega}{c} n_I = \omega \sqrt{\frac{\mu \epsilon_b}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon_b} \right)^2} - 1 \right)^{1/2}$$

$$\text{For } \frac{\sigma}{\omega} \gg 1 \quad \frac{\omega}{c} n_R \approx \frac{\omega}{c} n_I \approx \sqrt{\frac{\mu \sigma \omega}{2}} \equiv \frac{1}{\delta} \quad \leftarrow \text{“skin depth”}$$

$$\Rightarrow \mathbf{E}(\mathbf{r}, t) = e^{-\hat{\mathbf{k}} \cdot \mathbf{r} / \delta} \Re \left(\mathbf{E}_0 e^{i \hat{\mathbf{k}} \cdot \mathbf{r} / \delta - i \omega t} \right)$$

$$\Rightarrow \mathbf{H}(\mathbf{r}, t) = \frac{n}{c \mu} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t) = \frac{1+i}{\delta \mu \omega} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t)$$

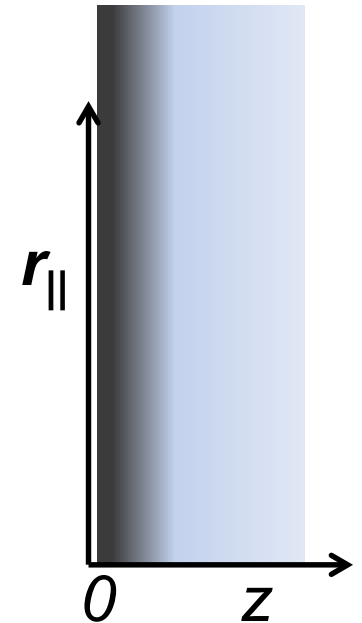
Fields near the surface on an ideal conductor

$$\text{For } \frac{\sigma}{\omega} \gg 1 \quad \frac{\omega}{c} n_R \approx \frac{\omega}{c} n_I \approx \sqrt{\frac{\mu \sigma \omega}{2}} \equiv \frac{1}{\delta}$$

$$\text{In this limit, } \sqrt{\frac{\mu \varepsilon}{\mu_0 \varepsilon_0}} = c \sqrt{\mu \varepsilon} = n_R + i n_I = \frac{c}{\omega} \frac{1}{\delta} (1 + i)$$

$$\mathbf{E}(\mathbf{r}, t) = e^{-\hat{\mathbf{k}} \cdot \mathbf{r} / \delta} \Re \left(\mathbf{E}_0 e^{i \hat{\mathbf{k}} \cdot \mathbf{r} / \delta - i \omega t} \right)$$

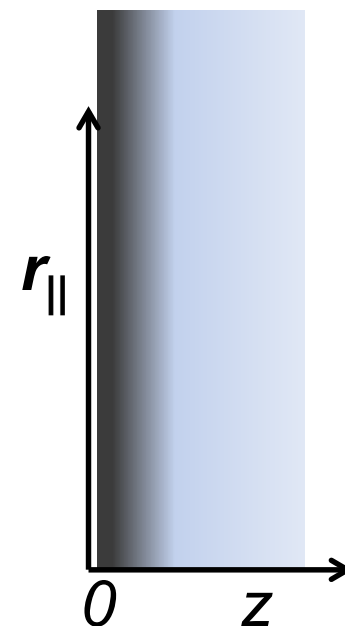
$$\mathbf{H}(\mathbf{r}, t) = \frac{n}{c \mu} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t) = \frac{1 + i}{\delta \mu \omega} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t)$$



Fields near the surface on an ideal conductor -- continued

$$\mathbf{E}(\mathbf{r}, t) = e^{-\hat{\mathbf{k}} \cdot \mathbf{r} / \delta} \Re \left(\mathbf{E}_0 e^{i \hat{\mathbf{k}} \cdot \mathbf{r} / \delta - i \omega t} \right)$$

$$\mathbf{H}(\mathbf{r}, t) = \frac{n}{c\mu} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t) = \frac{1+i}{\delta\mu\omega} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t)$$



Note that it is convenient to express the EM fields in terms of the $\mathbf{H}$ amplitude:

$$\mathbf{H}(\mathbf{r}, t) = e^{-\hat{\mathbf{k}} \cdot \mathbf{r} / \delta} \Re \left(\mathbf{H}_0 e^{i \hat{\mathbf{k}} \cdot \mathbf{r} / \delta - i \omega t} \right)$$

$$\mathbf{E}(\mathbf{r}, t) = \delta\mu\omega \frac{1-i}{2} \hat{\mathbf{k}} \times \mathbf{H}(\mathbf{r}, t)$$

Boundary values for ideal conductor

Inside the conductor :

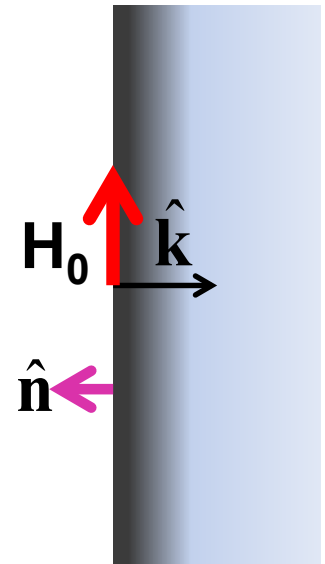
$$\mathbf{H}(\mathbf{r}, t) = e^{-\hat{\mathbf{k}} \cdot \mathbf{r} / \delta} \Re \left(\mathbf{H}_0 e^{i \hat{\mathbf{k}} \cdot \mathbf{r} / \delta - i \omega t} \right)$$

$$\mathbf{E}(\mathbf{r}, t) = \delta \mu \omega \frac{1-i}{2} \hat{\mathbf{k}} \times \mathbf{H}(\mathbf{r}, t)$$

At the boundary of an ideal conductor, the $\mathbf{E}$ and $\mathbf{H}$ fields decay in the direction normal to the interface.

Ideal conductor boundary conditions:

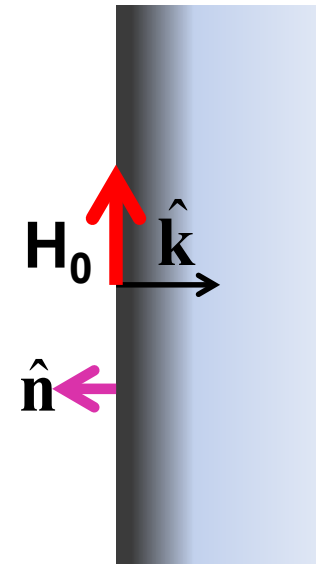
$$\hat{\mathbf{n}} \times \mathbf{E} \Big|_S = 0 \qquad \hat{\mathbf{n}} \cdot \mathbf{H} \Big|_S = 0$$



Wave guides – dielectric media with one or more metal boundary

Ideal conductor boundary conditions:

$$\hat{\mathbf{n}} \times \mathbf{E} \Big|_S = 0 \qquad \hat{\mathbf{n}} \cdot \mathbf{H} \Big|_S = 0$$



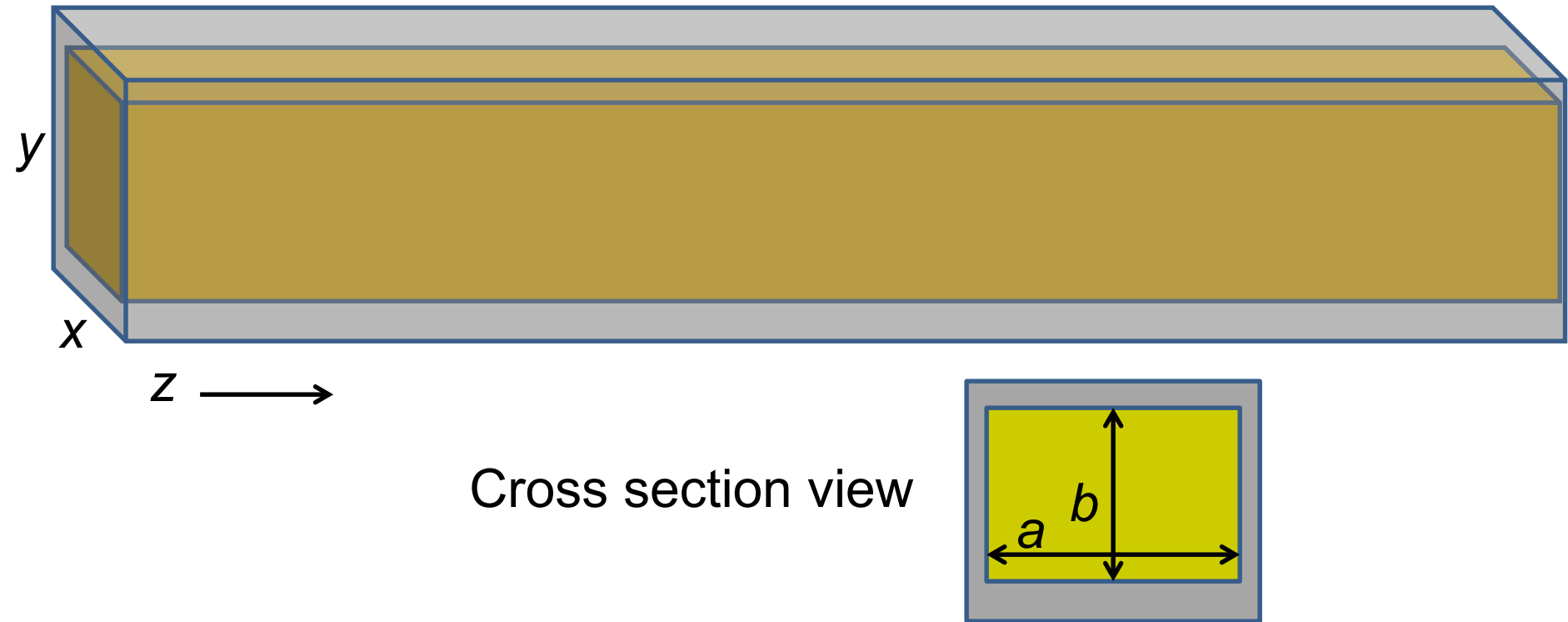
Waveguide terminology

- TEM: transverse electric and magnetic (both E and H fields are perpendicular to wave propagation direction)
- TM: transverse magnetic (H field is perpendicular to wave propagation direction)
- TE: transverse electric (E field is perpendicular to wave propagation direction)

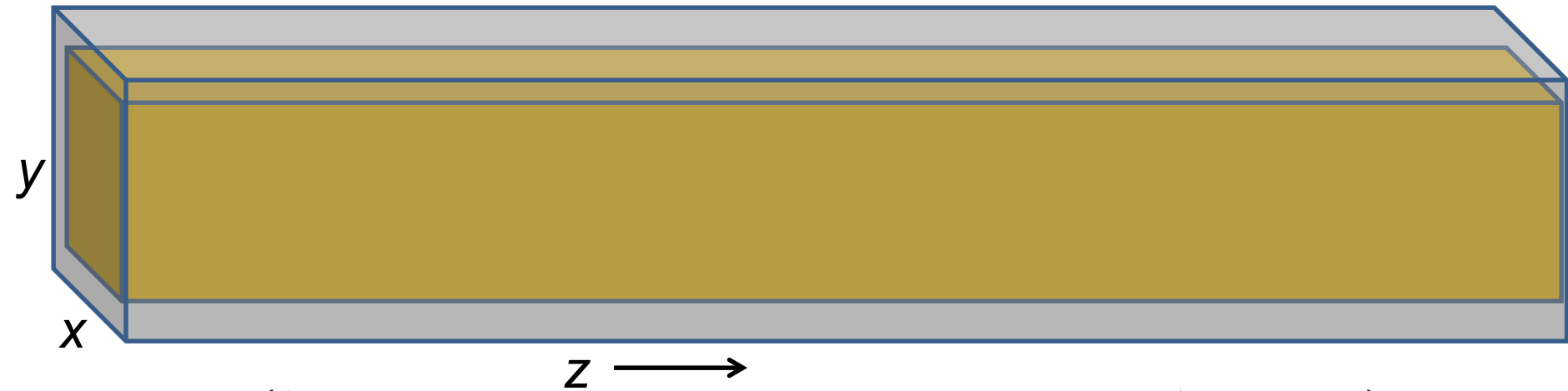
Analysis of rectangular waveguide

Boundary conditions at surface of waveguide:

$$\mathbf{E}_{\text{tangential}}=0, \quad \mathbf{B}_{\text{normal}}=0$$



Analysis of rectangular waveguide



$$\mathbf{B} = \Re \left\{ \left(B_x(x, y) \hat{\mathbf{x}} + B_y(x, y) \hat{\mathbf{y}} + B_z(x, y) \hat{\mathbf{z}} \right) e^{ikz - i\omega t} \right\}$$

$$\mathbf{E} = \Re \left\{ \left(E_x(x, y) \hat{\mathbf{x}} + E_y(x, y) \hat{\mathbf{y}} + E_z(x, y) \hat{\mathbf{z}} \right) e^{ikz - i\omega t} \right\}$$

Inside the dielectric medium: (assume ε to be real)

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \times \mathbf{B} - \varepsilon \mu \frac{\partial \mathbf{E}}{\partial t} = 0$$

Solution of Maxwell's equations within the pipe:

Combining Faraday's Law and Ampere's Law, we find that each field component must satisfy a two-dimensional Helmholtz equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - k^2 + \mu\epsilon\omega^2 \right) E_x(x, y) = 0.$$

For the rectangular wave guide discussed in Section 8.4 of your text a solution for a TE mode can have:

$$E_z(x, y) \equiv 0 \quad \text{and} \quad B_z(x, y) = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right),$$

$$\text{with } k^2 \equiv k_{mn}^2 = \mu\epsilon\omega^2 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right]$$

Maxwell's equations within the pipe in terms of all 6 components:

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + ikB_z = 0.$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + ikE_z = 0.$$

$$\frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x.$$

$$ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y.$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z.$$

For TE mode with $E_z \equiv 0$

$$B_x = -\frac{k}{\omega} E_y$$

$$B_y = \frac{k}{\omega} E_x$$

$$\frac{\partial B_z}{\partial y} - ikB_y = -i\mu\epsilon\omega E_x.$$

$$ikB_x - \frac{\partial B_z}{\partial x} = -i\mu\epsilon\omega E_y.$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -i\mu\epsilon\omega E_z.$$

TE modes for rectangular wave guide continued:

$$E_z(x, y) \equiv 0 \quad \text{and} \quad B_z(x, y) = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right),$$

$$E_x = \frac{\omega}{k} B_y = \frac{-i\omega}{k^2 - \mu\epsilon\omega^2} \frac{\partial B_z}{\partial y} = \frac{-i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \frac{n\pi}{b} B_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right),$$

$$E_y = -\frac{\omega}{k} B_x = \frac{i\omega}{k^2 - \mu\epsilon\omega^2} \frac{\partial B_z}{\partial x} = \frac{i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \frac{m\pi}{a} B_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right).$$

Check boundary conditions:

$$\mathbf{E}_{\text{tangential}} = 0 \quad \text{because:} \quad E_z(x, y) \equiv 0, \quad E_x(x, 0) = E_x(x, b) = 0$$

$$\text{and} \quad E_y(0, y) = E_y(a, y) = 0.$$

$$\mathbf{B}_{\text{normal}} = 0 \quad \text{because:} \quad B_y(x, 0) = B_y(x, b) = 0$$

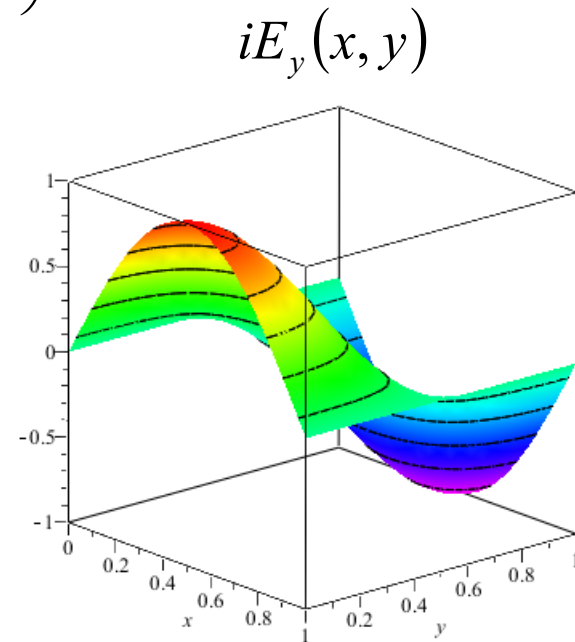
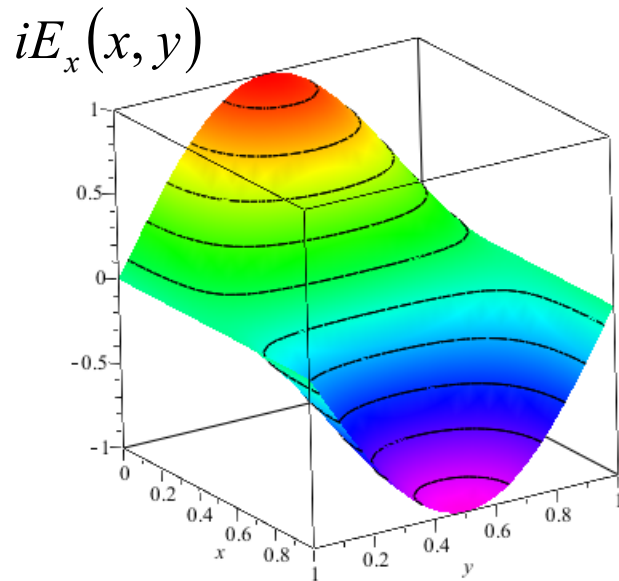
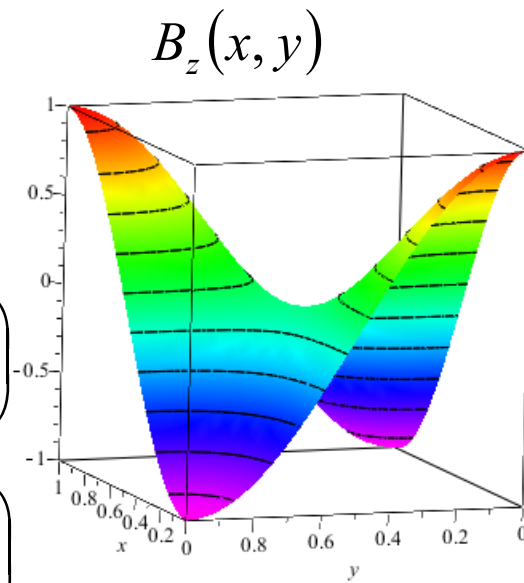
$$\text{and} \quad B_x(0, y) = B_x(a, y) = 0.$$

Solution for $m=n=1$

$$B_z(x, y) = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

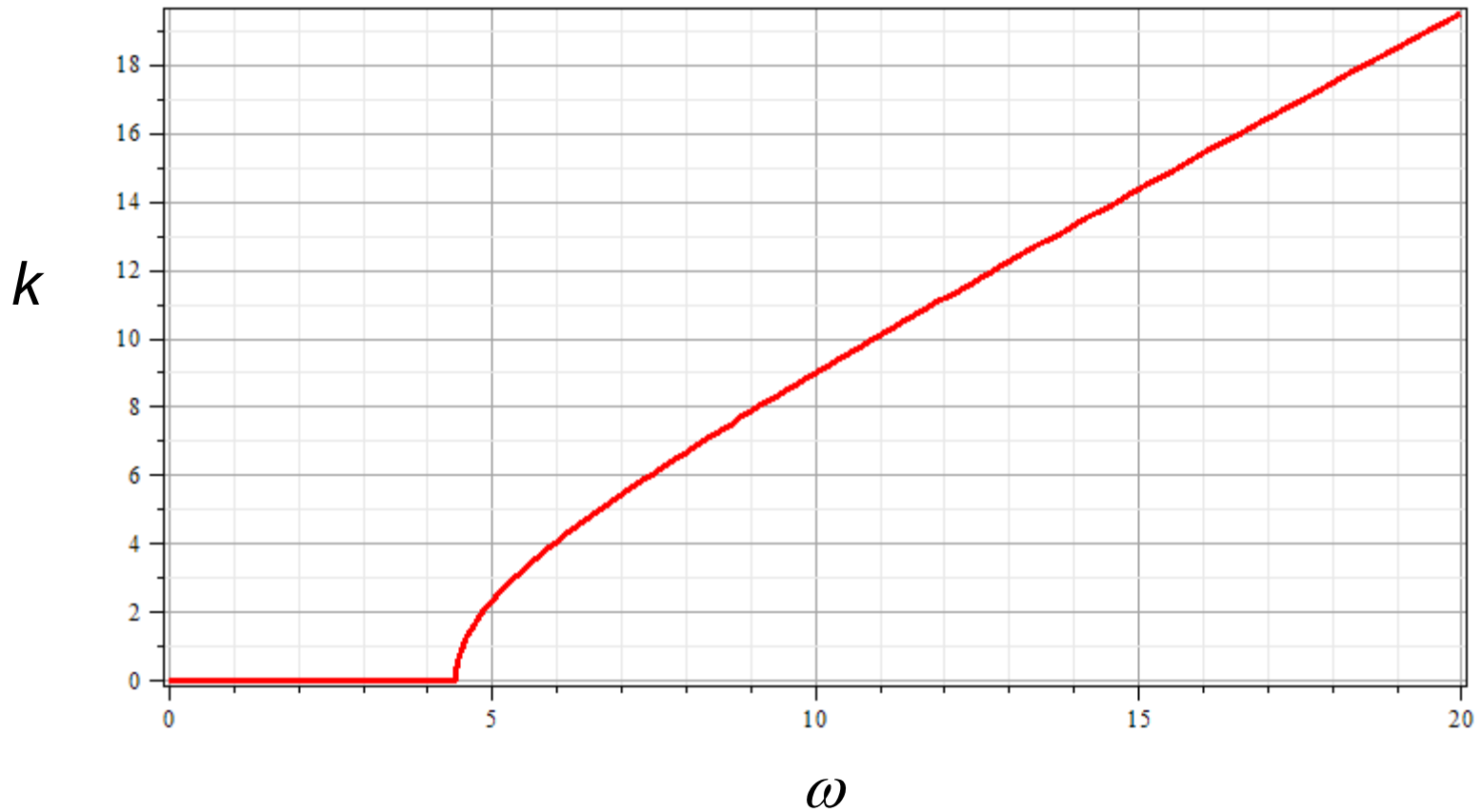
$$iE_x(x, y) = B_0 \left(\frac{\omega n \pi / b}{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$iE_y(x, y) = B_0 \left(\frac{-\omega m \pi / a}{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$



Solution for m=n=1

$$k^2 \equiv k_{mn}^2 = \mu\epsilon\omega^2 - \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]$$



TE modes for rectangular wave guide continued:

$$E_z(x, y) \equiv 0 \quad \text{and} \quad B_z(x, y) = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right),$$

$$E_x = \frac{\omega}{k} B_y = \frac{-i\omega}{k^2 - \mu\epsilon\omega^2} \frac{\partial B_z}{\partial y} = \frac{-i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \frac{n\pi}{b} B_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right),$$

$$E_y = -\frac{\omega}{k} B_x = \frac{i\omega}{k^2 - \mu\epsilon\omega^2} \frac{\partial B_z}{\partial x} = \frac{i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \frac{m\pi}{a} B_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right).$$

Note that it is assumed that all fields have the same harmonic time dependence $e^{-i\omega t}$

Time averaged Poynting vector inside wave guide:

$$\langle \mathbf{S} \rangle = \frac{1}{2} \Re(\mathbf{E} \times \mathbf{H}^*) = \frac{1}{2} \left(\hat{\mathbf{x}}(E_y H_z^* - E_z H_y^*) + \hat{\mathbf{y}}(E_z H_x^* - E_x H_z^*) + \hat{\mathbf{z}}(E_x H_y^* - E_y H_x^*) \right)$$

TE modes for rectangular wave guide continued:

$$E_z(x, y) \equiv 0 \quad \text{and} \quad B_z(x, y) = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right),$$

$$E_x = \frac{\omega}{k} B_y = \frac{-i\omega}{k^2 - \mu\epsilon\omega^2} \frac{\partial B_z}{\partial y} = \frac{-i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \frac{n\pi}{b} B_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right),$$

$$E_y = -\frac{\omega}{k} B_x = \frac{i\omega}{k^2 - \mu\epsilon\omega^2} \frac{\partial B_z}{\partial x} = \frac{i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} \frac{m\pi}{a} B_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right).$$

Time averaged Poynting vector inside wave guide:

$$\begin{aligned} \langle \mathbf{S} \rangle &= \frac{1}{2} \Re(\mathbf{E} \times \mathbf{H}^*) = \frac{1}{2} \Re\left(\hat{\mathbf{x}}(E_y H_z^* - E_z H_y^*) + \hat{\mathbf{y}}(E_z H_x^* - E_x H_z^*) + \hat{\mathbf{z}}(E_x H_y^* - E_y H_x^*)\right) \\ &= \frac{1}{2} \frac{k}{\mu\omega} \hat{\mathbf{z}} \left(|E_x|^2 + |E_y|^2 \right) \end{aligned}$$