PHY 712 Electrodynamics
11-11:50 AM in Olin 103

Discussion for Lecture 19:

Complete reading of Chapter 7

1. Comments on reflectivity of plane waves
2. Summary of complex response functions for electromagnetic fields
3. Comment on spectral properties of electromagnetic wavesg

Physics Colloquium – 4 PM 3/03/2022

Colloquium: "What can we learn from gravitational waves?" — Thursday, March 3, 2022 at 4 PM

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University of British Columbia

Thursday, March 3, 2022, 4 PM

George P. Williams, Jr. Lecture Hall, (Olin 101)

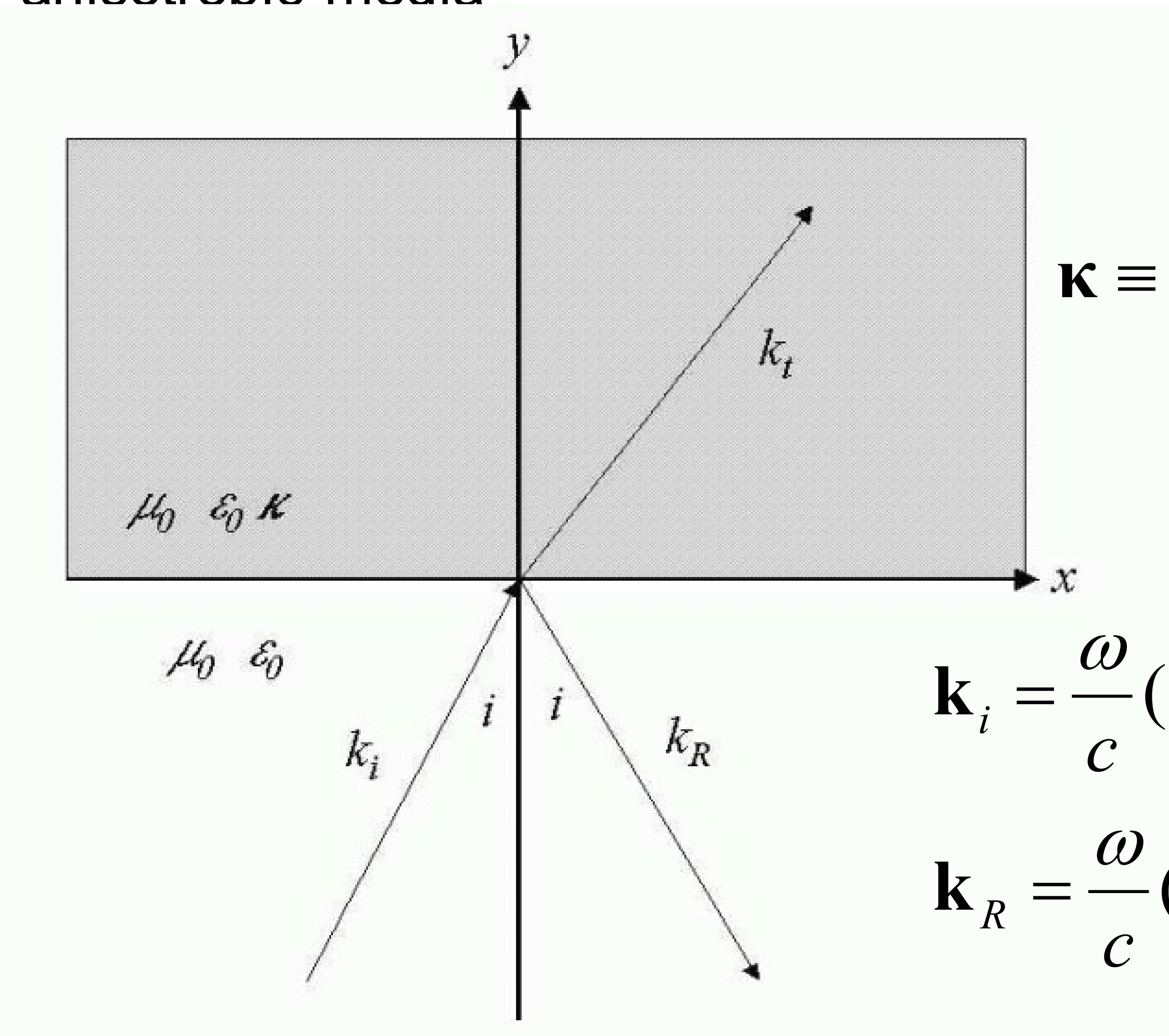
In just five years, the field of gravitational wave astronomy has grown from a groundbreaking first discovery to revealing new populations of dead stars through distant cosmic collisions. I'll summarize recent results from LIGO-Virgo-KAGRA and their wide-reaching implications for cosmology, general relativity, condensed matter physics, and what we know about how stars live and die. I'll also give an overview of the instrumentation of the current Advanced LIGO detectors, and discuss prospects for what we can learn with future gravitational wave detectors on Earth and in space.

A very useful paper to skim over before the talk: https://arxiv.org/abs/2111.03606
Or alternatively, this handy summary: https://www.ligo.org/science/Publication-O3bCatalog/index.php

16	Wed: 02/23/2022	Chap. 6	Electromagnetic energy and forces	#15	02/25/2022
17	Fri: 02/25/2022	Chap. 7	Electromagnetic plane waves	#16	02/28/2022
18	Mon: 02/28/2022	Chap. 7	Electromagnetic plane waves	#17	03/02/2022
19	Wed: 03/02/2022	Chap. 7	Optical effects of refractive indices		
20	Fri: 03/04/2022	Chap. 8	Brief introduction to wave guides		
	Mon: 03/07/2022	No class	*Spring Break*		
	Wed: 03/09/2022	No class	*Spring Break*		
	Fri: 03/11/2022	No class	*Spring Break*		
	Mon: 03/14/2022	No class	*APS March Meeting*	Prepare Project	
	Wed: 03/16/2022	No class	*APS March Meeting*	Prepare Project	
	Fri: 03/18/2022	No class	*APS March Meeting*	Prepare Project	
	Mon: 03/21/2022		Project presentations I		
	Wed: 03/23/2022		Project presentations II		
21	Fri: 03/25/2022	Chap. 9	Radiation from localized oscillating sources		

Please email your proposed projected topic(s) by Friday 3/4/2022

Comment on extension of reflection/refraction analysis to anisotropic media --



assume

$$\boldsymbol{\kappa} \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}$$

$$\mathbf{k}_i = \frac{\omega}{c}(\sin i\hat{\mathbf{x}} + \cos i\hat{\mathbf{y}}),$$

$$\mathbf{k}_R = \frac{\omega}{c}(\sin i\hat{\mathbf{x}} - \cos i\hat{\mathbf{y}}).$$

For s-polarization (E fields along z-axis)

$$\frac{E_0''}{E_0} = \frac{\cos i - n_y}{\cos i + n_y}.$$

$$n_y^2 = \kappa_{zz} - \sin^2 i$$

For p-polarization (E fields in x-y plane)

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx}\cos i - n_y}{\kappa_{xx}\cos i + n_y}.$$

$$n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}}(\kappa_{yy} - \sin^2 i).$$

Example of a birefringent crystal --

Some comments on the Fresnel Equations

1. Different behaviors of *s* and *p* polarization
2. Brewster's angle
3. Total internal reflection

Review: Electromagnetic plane waves in isotropic medium with linear and real permeability and permittivity: μ ε.

$$\mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right) \qquad n^2=c^2\mu\varepsilon$$

$$\mathbf{B}(\mathbf{r},t)=\frac{n}{c}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)=\sqrt{\mu\varepsilon}\,\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)$$

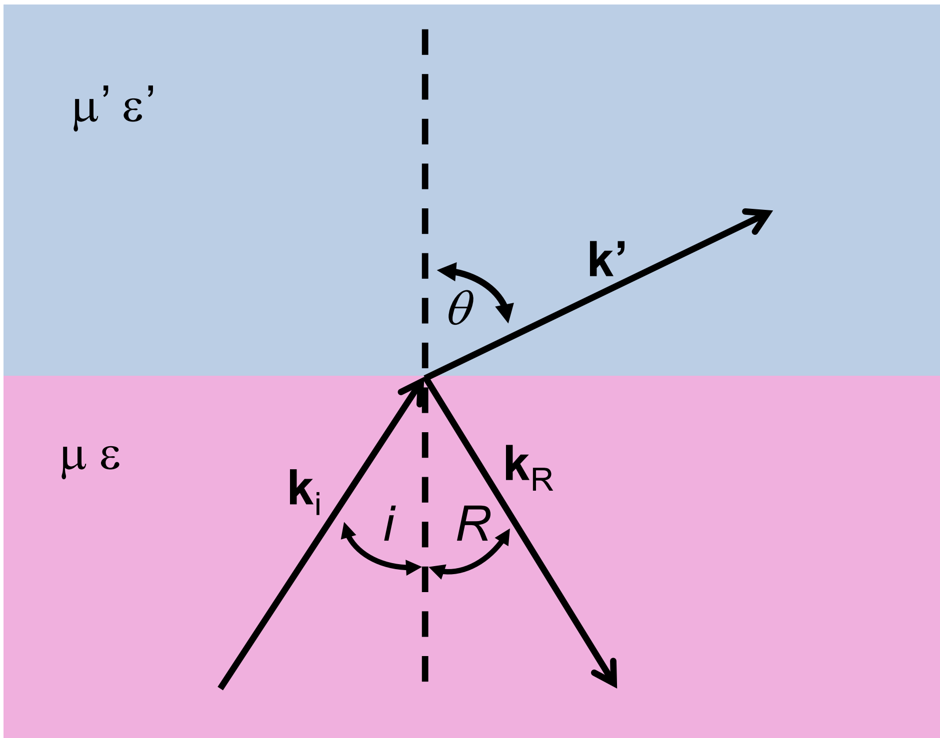
Poynting vector for plane electromagnetic waves:

$$\langle\mathbf{S}\rangle_{avg}=\frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}}=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\hat{\mathbf{k}}$$

Energy density for plane electromagnetic waves:

$$\langle u\rangle_{avg}=\frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$

Review:
Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



$$n' = \varepsilon' \mu'$$

$$n = \varepsilon \mu$$

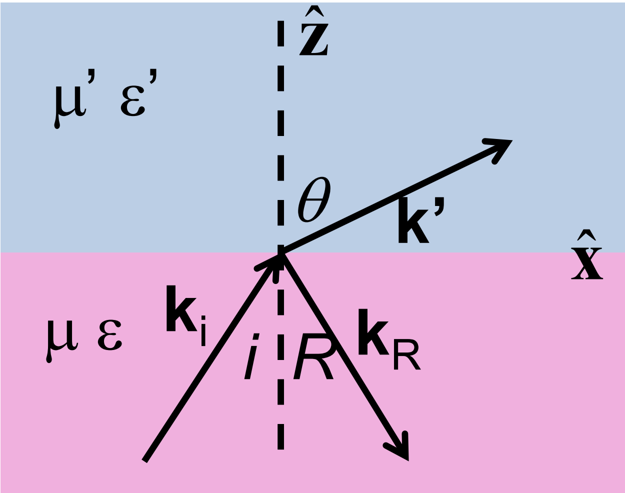
$$i = R$$

$$n \sin i = n' \sin \theta$$

$$|\mathbf{k}_i| = |\mathbf{k}_R| = n \frac{\omega}{c}$$

$$|\mathbf{k}'| = n' \frac{\omega}{c}$$

Review:
Reflection and refraction between two isotropic media



Reflectance, transmittance:

$$\mathrm{R}=\frac{\mathbf{S}_R\cdot\hat{\mathbf{z}}}{\mathbf{S}_i\cdot\hat{\mathbf{z}}}=\left|\frac{E_{0R}}{E_{0i}}\right|^2 \qquad \mathrm{T}=\frac{\mathbf{S}'\cdot\hat{\mathbf{z}}}{\mathbf{S}_i\cdot\hat{\mathbf{z}}}=\left|\frac{E'_0}{E_{0i}}\right|^2\frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i}$$

Note that $\quad \mathrm{R}+\mathrm{T}=1$

For s-polarization (E perpendicular to plane of incidence)

$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'}n'\cos\theta}{n\cos i + \frac{\mu}{\mu'}n'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'}n'\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

For p-polarization (E in plane of incidence)

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n'^2\cos i - nn'\cos\theta}{\frac{\mu}{\mu'}n'^2\cos i + nn'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2nn'\cos i}{\frac{\mu}{\mu'}n'^2\cos i + nn'\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

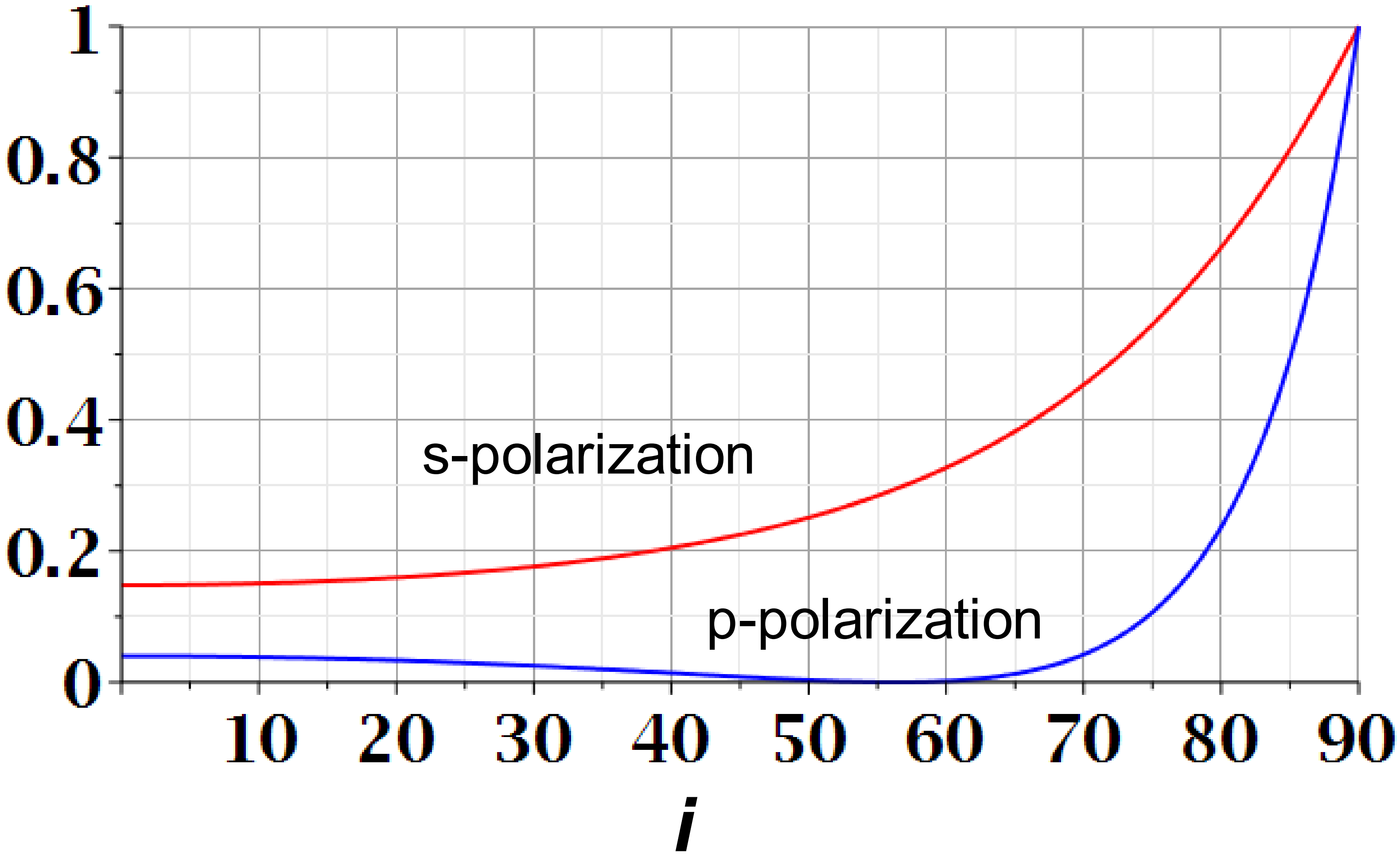
Reflectance for s-polarization

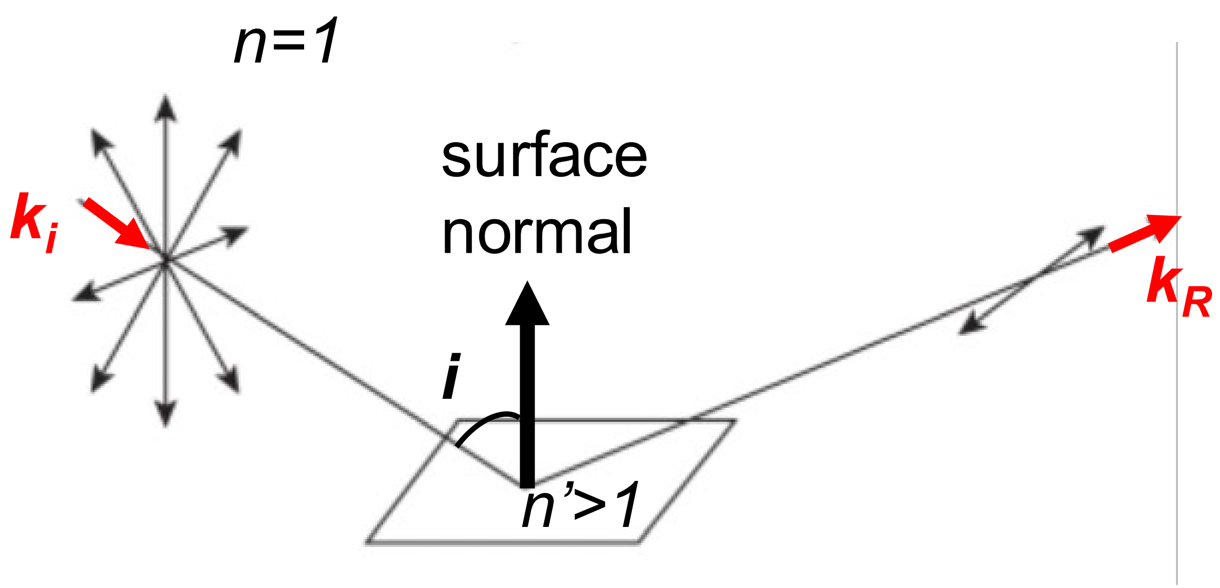
$$R_s = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Reflectance for p-polarization

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'\cos i - \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'\cos i + \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Example for $\mu = \mu'$; $n = 1$ and $n' = 1.5$





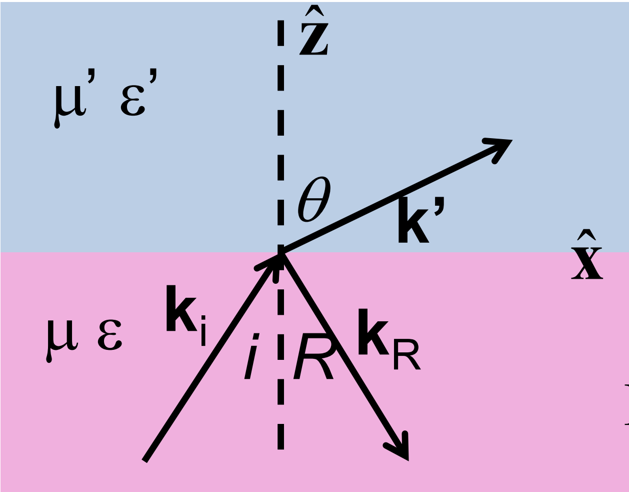
Polarization due to reflection from a refracting surface

Brewster's angle: for $i = i_B$, $\mathrm{R}_p(i_B) = 0$

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'\cos i - \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'\cos i + \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

For $\mu' = \mu$, $\quad i_B = \tan^{-1}\left(\frac{n'}{n}\right)$

Reflection and refraction between two isotropic media -- continued



For each wave:

$$\mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right) \qquad n^2=c^2\mu\varepsilon$$

$$\mathbf{B}(\mathbf{r},t)=\frac{n}{c}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)=\sqrt{\mu\varepsilon}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)$$

Matching condition at interface:

$$n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

Total internal reflection:

If $n>n'$, for $i>i_0\equiv\sin^{-1}\left(\frac{n'}{n}\right)$,

refracted field no longer propagates in medium $\mu'\ \varepsilon'$

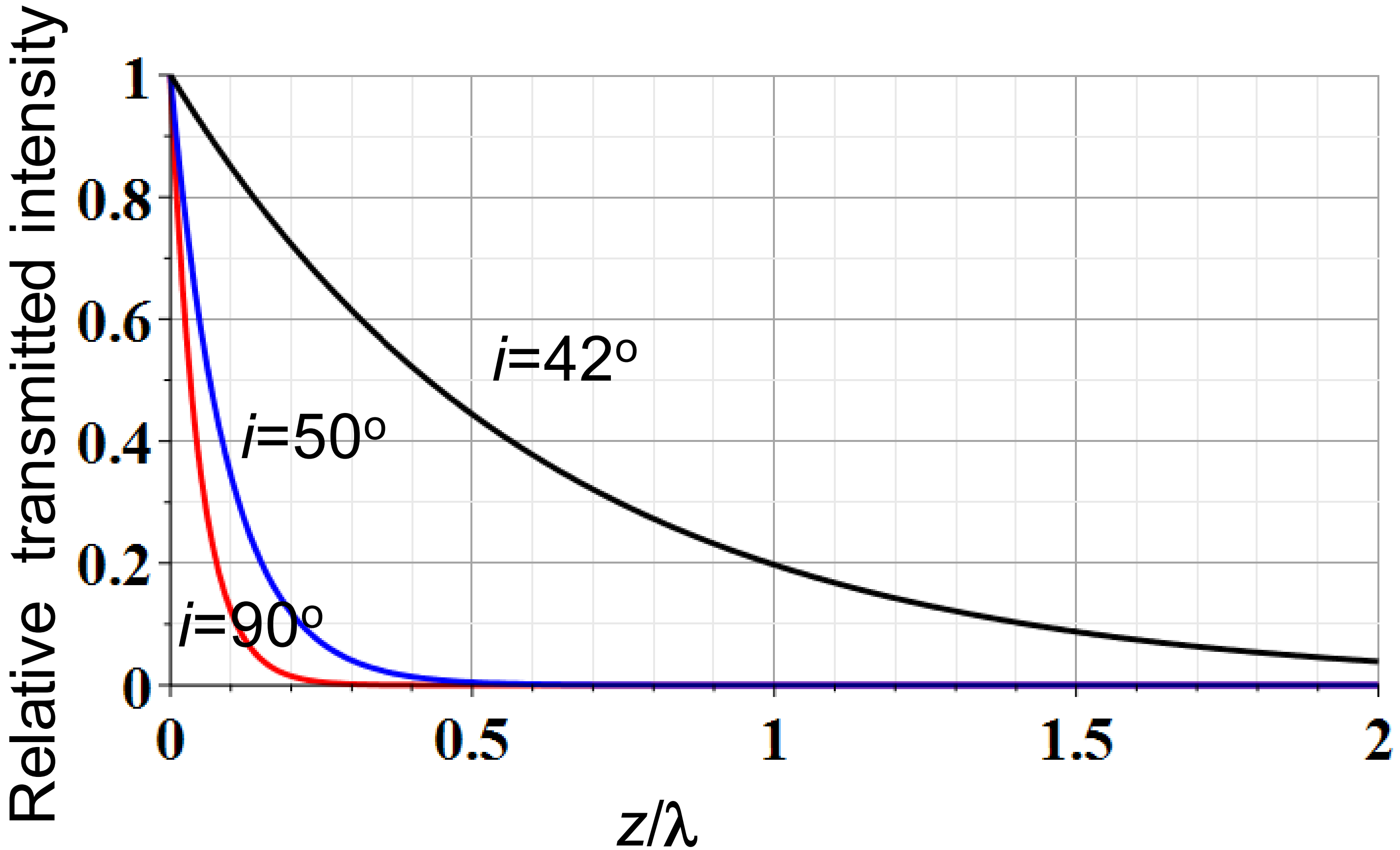
For $i > i_0$

$$n'\cos\theta=i\sqrt{n^2\sin^2 i-n'^2}\;=i\;n\sqrt{\frac{\sin^2 i}{\sin^2 i_0}-1}$$

$$\mathbf{E}'(\mathbf{r},t)=e^{-\left(\frac{n\omega}{c}\sqrt{\frac{\sin^2 i}{\sin^2 i_0}-1}\right)z}\Re\left(\mathbf{E}'_0\, e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_{\parallel}\cdot\mathbf{r}-ct\right)}\right)$$

Example of total internal reflection

n'=1 and n=1.5 ➔ $i_0 = \sin^{-1}(1/1.5)$=41.81°



Transmitted illumination confined within a few wavelengths of the surface.

TIRF (total internal reflection fluorescence)

www.nikon.com/products/microscope-solutions/bioscience.../nikon_note_10_lr.pdf

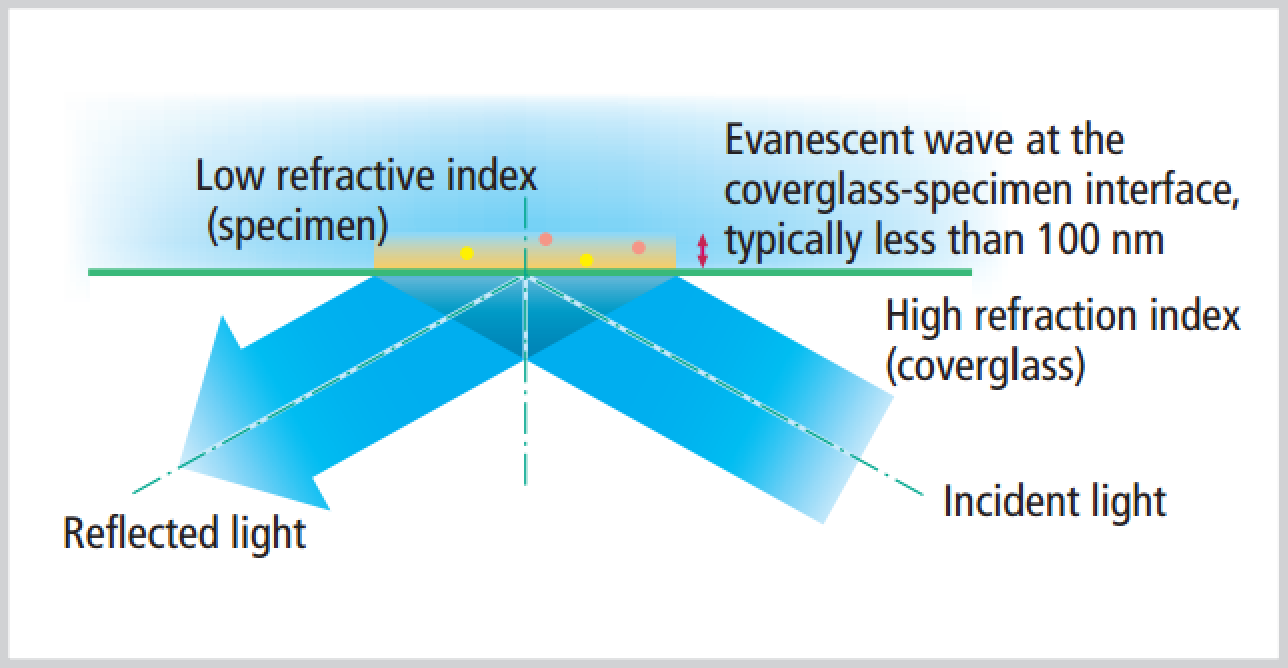


Figure 1: Creation of an evanescent wave at the coverglass-specimen interface

Design of TIRF device using laser and high power lens

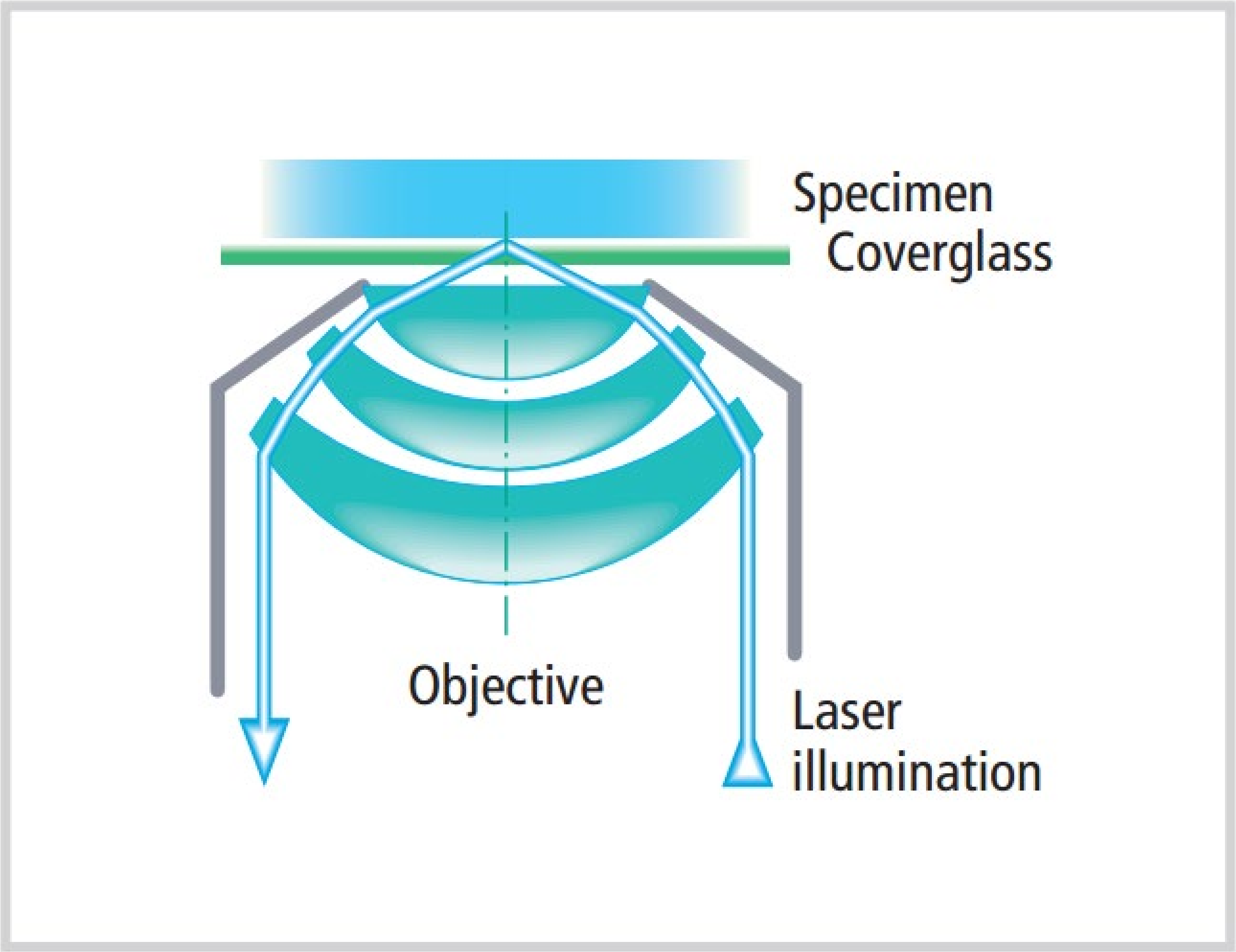


Figure 2: Through-the-lens laser TIRF.

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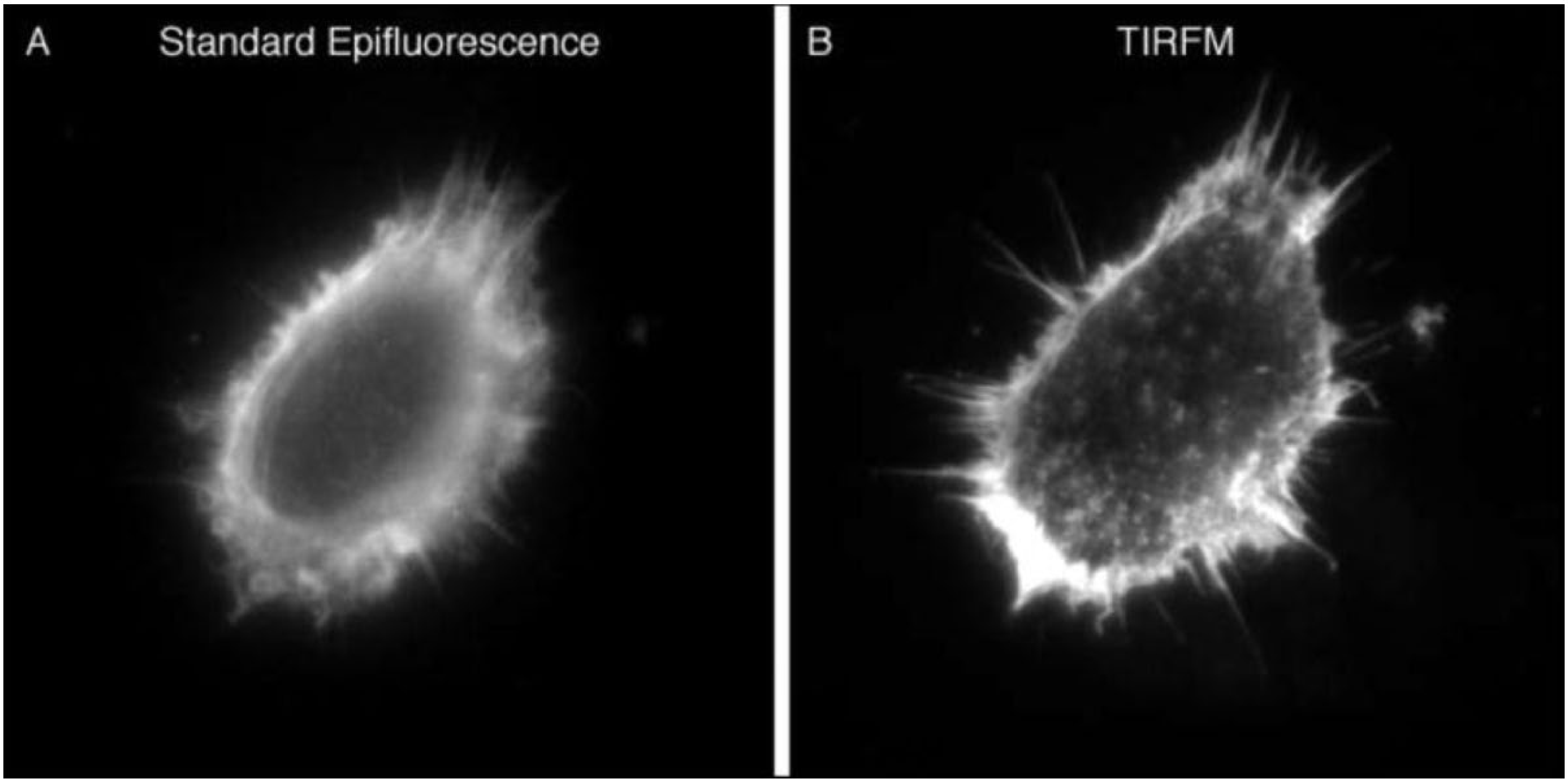
doi: 10.1002/0471142956.cy1218s50

Figure 1



Special case: normal incidence (i=0, θ=0)

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n} \qquad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'}n' + n}$$

Reflectance, transmittance:

$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n}\right|^2$$

$$\mathrm{T} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'} = \left|\frac{2n}{\frac{\mu}{\mu'}n' + n}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}$$

Extension to complex refractive index $n = n_R + i\, n_I$

Suppose $\mu = \mu'$, $n = \text{real}$, $n' = n'_R + i n'_I$

Reflectance at normal incidence:

$$\text{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n}\right|^2 = \frac{(n'_R - n)^2 + (n'_I)^2}{(n'_R + n)^2 + (n'_I)^2}$$

Note that for $n'_I >> |n'_R \pm n|$:

$$\text{R} \approx 1$$

Origin of imaginary contributions to permittivity --
Review: Drude model dielectric function:

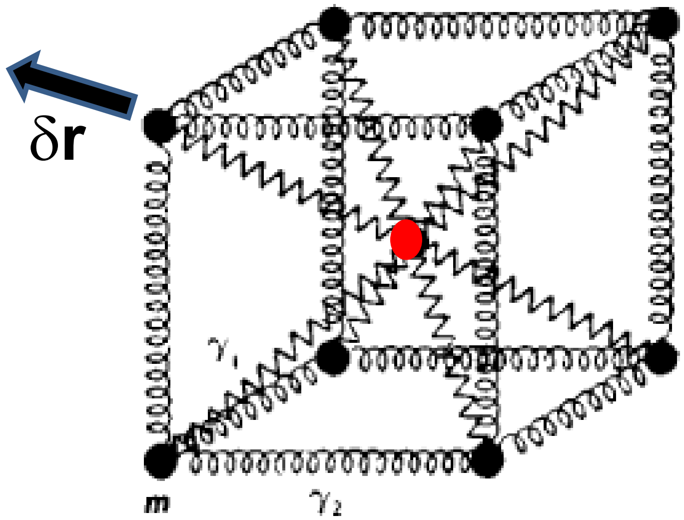
$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^{\,2}}{\varepsilon_0 m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$= \frac{\varepsilon_R(\omega)}{\varepsilon_0} + i\frac{\varepsilon_I(\omega)}{\varepsilon_0}$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^{\,2}}{\varepsilon_0 m_i} \frac{\omega_i^2 - \omega^2}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^{\,2}}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = N\sum_i f_i \frac{q_i^{\,2}}{\varepsilon_0 m_i} \frac{\omega\gamma_i}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^{\,2}}$$

Extensions of the Drude model for lattice vibrations



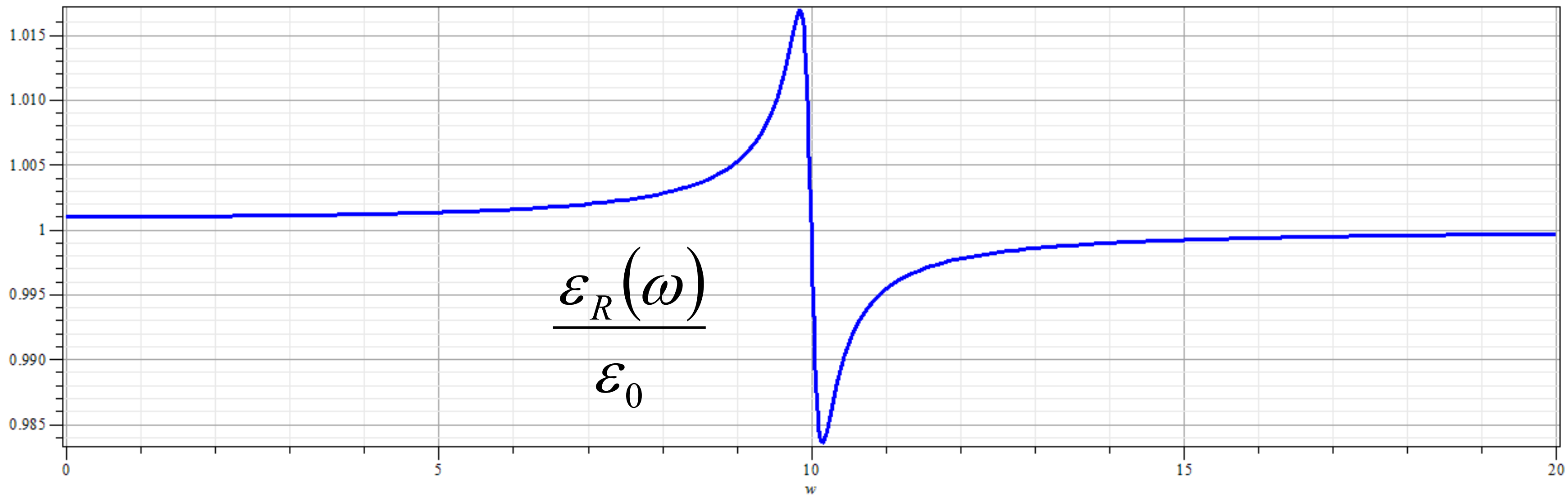
In principle, the ideas of the Drude model apply both to the ionic vibrations which occur at low frequency (~10^{12} Hz) contributing to the so called static permittivity function ε_s and to the electronic vibrations which occur at high frequency (~10^{15} Hz) contributing to the so called high frequency permittivity function ε_∞.

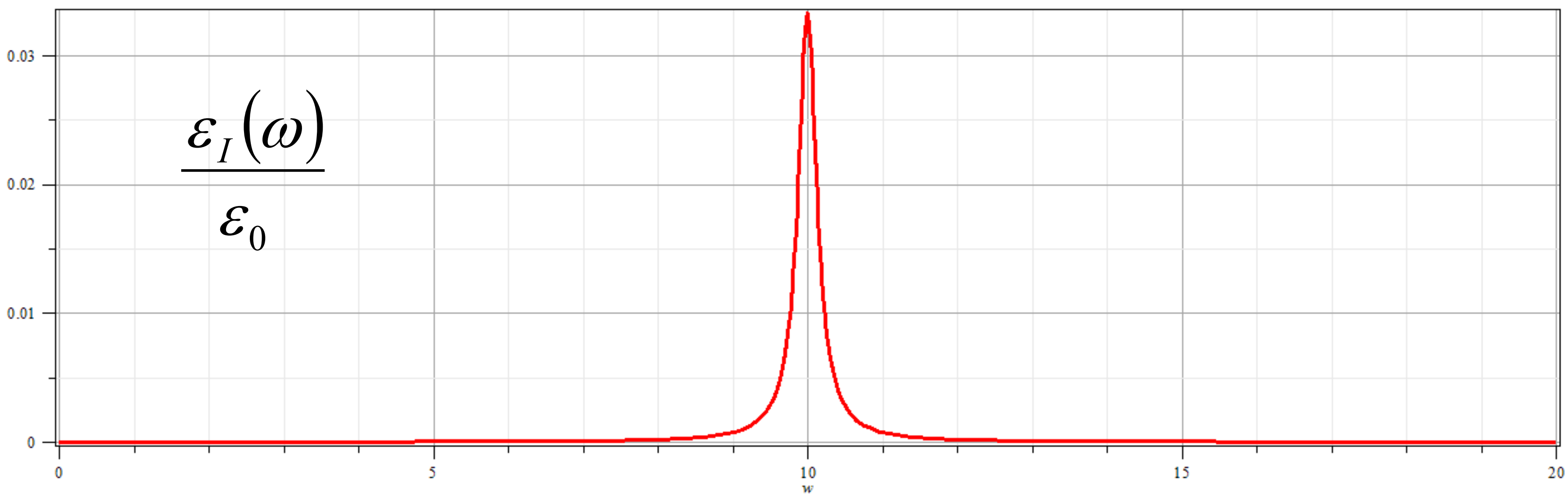
In this model at high frequencies, only the electrons contribute to the polarization: $\epsilon_\infty = \epsilon_0 + \frac{|\mathbf{P}_{electron}|}{|\mathbf{E}|}$

At low frequencies both electrons and ions contribute to the polarization: $\epsilon_s = \epsilon_0 + \frac{|\mathbf{P}_{electron}|}{|\mathbf{E}|} + \frac{|\mathbf{P}_{ion}|}{|\mathbf{E}|}$

$$\Rightarrow \frac{|\mathbf{P}_{ion}|}{|\mathbf{E}|} = \epsilon_s - \epsilon_\infty$$

Drude model dielectric function:





Summary –

Kramers-Kronig transform – for dielectric function:

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} - 1 = \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\varepsilon_I(\omega')}{\varepsilon_0} \frac{1}{\omega' - \omega}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = -\frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \left(\frac{\varepsilon_R(\omega')}{\varepsilon_0} - 1 \right) \frac{1}{\omega' - \omega}$$

$$\text{with} \quad \varepsilon_R(-\omega) = \varepsilon_R(\omega); \quad \varepsilon_I(-\omega) = -\varepsilon_I(\omega)$$

Practical applications -- It is often possible/more convenient to calculate the imaginary response and use KK to deduce the real response or visa versa.

Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

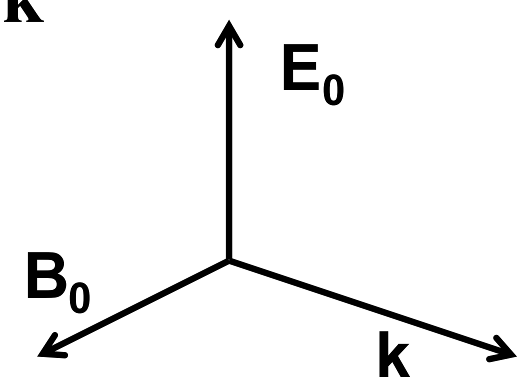
$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$$

Poynting vector and energy density:

$$\langle\mathbf{S}\rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\,\hat{\mathbf{k}}$$

$$\langle u\rangle_{avg} = \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$



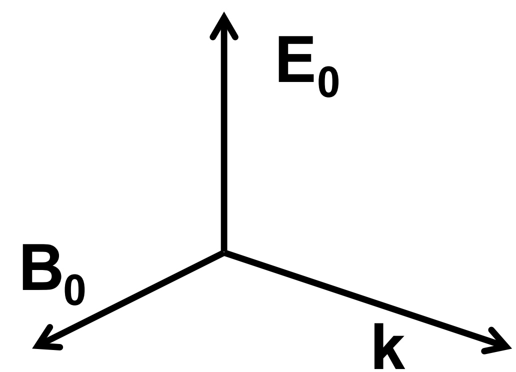
Transverse electric and magnetic waves (TEM)

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$$

TEM modes describe electromagnetic waves in lossless media and vacuum

For real
ε, μ, n, k



Effects of complex dielectric; fields near the surface on an ideal conductor

Suppose for an isotropic medium: $\mathbf{D} = \varepsilon_b \mathbf{E} \qquad \mathbf{J} = \sigma \mathbf{E}$

Maxwell's equations in terms of $\mathbf{H}$ and $\mathbf{E}$:

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon_b \frac{\partial \mathbf{E}}{\partial t}$$

$$\left(\nabla^2 - \mu\sigma \frac{\partial}{\partial t} - \mu\varepsilon_b \frac{\partial^2}{\partial t^2} \right) \mathbf{F} = 0 \qquad \mathbf{F} = \mathbf{E}, \mathbf{H}$$

Plane wave form for $\mathbf{E}$:

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t} \right) \qquad \text{where } \mathbf{k} = (n_R + i n_I)\frac{\omega}{c}\hat{\mathbf{k}}$$

$$\Rightarrow \mathbf{E}(\mathbf{r},t) = e^{-\hat{\mathbf{k}}\cdot\mathbf{r}/\delta} \Re\left(\mathbf{E}_0 e^{i n_R (\omega/c)\hat{\mathbf{k}}\cdot\mathbf{r} - i\omega t} \right)$$

Some details:

Plane wave form for $\mathbf{E}$:

$$\mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \text{where } \mathbf{k}=(n_R+in_I)\frac{\omega}{c}\hat{\mathbf{k}}$$

$$\left(\nabla^2-\mu\sigma\frac{\partial}{\partial t}-\mu\varepsilon_b\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=0$$

$$-(n_R+in_I)^2+i\frac{\mu\sigma c^2}{\omega}+\mu\varepsilon_b c^2=0$$

Fields near the surface on an ideal conductor -- continued

For our system:

$$\frac{\omega}{c}n_R = \omega\sqrt{\frac{\mu\varepsilon_b}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon_b}\right)^2}+1\right)^{1/2}$$

$$\frac{\omega}{c}n_I = \omega\sqrt{\frac{\mu\varepsilon_b}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon_b}\right)^2}-1\right)^{1/2}$$

For $\frac{\sigma}{\omega} >> 1 \qquad \frac{\omega}{c}n_R \approx \frac{\omega}{c}n_I \approx \sqrt{\frac{\mu\sigma\omega}{2}} \equiv \frac{1}{\delta}$

$$\Rightarrow \mathbf{E}(\mathbf{r},t) = e^{-\hat{\mathbf{k}}\cdot\mathbf{r}/\delta}\Re\left(\mathbf{E}_0 e^{i\hat{\mathbf{k}}\cdot\mathbf{r}/\delta - i\omega t}\right)$$

$$\Rightarrow \mathbf{H}(\mathbf{r},t) = \frac{n}{c\mu}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t) = \frac{1+i}{\delta\mu\omega}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)$$

Some representative values of skin depth
Ref: Lorrain and Corson

$$\frac{\omega}{c}n_R \approx \frac{\omega}{c}n_I \approx \sqrt{\frac{\mu\sigma\omega}{2}} \equiv \frac{1}{\delta}$$

	σ (10^7 S/m)	μ/μ_0	δ (0.001m) at 60 Hz	δ (0.001m) at 1 MHz
Al	3.54	1	10.9	84.6
Cu	5.80	1	8.5	66.1
Fe	1.00	100	1.0	10.0
Mumetal	0.16	2000	0.4	3.0
Zn	1.86	1	15.1	117

Relative energies associated with field

Electric energy density: $\varepsilon_b |\mathbf{E}|^2$

Magnetic energy density: $\mu |\mathbf{H}|^2$

Ratio inside conducting media: $$\frac{\varepsilon_b |\mathbf{E}|^2}{\mu |\mathbf{H}|^2} = \frac{\varepsilon_b}{\mu \left|\frac{1+i}{\delta\mu\omega}\right|^2} = \frac{\varepsilon_b \mu \omega^2 \delta^2}{2}$$

Here wavelength is defined:

$$\lambda = \frac{2\pi c}{\omega}$$

$$= 2\pi^2 \frac{\varepsilon_b}{\varepsilon_0} \frac{\mu}{\mu_0} \frac{\delta^2}{\lambda^2}$$

For $\frac{\varepsilon_b |\mathbf{E}|^2}{\mu |\mathbf{H}|^2} \ll 1 \quad \Rightarrow$ magnetic energy dominates

Note that in free space, $\frac{\varepsilon_0 |\mathbf{E}|^2}{\mu_0 |\mathbf{H}|^2} = 1$

Various wavelengths λ --