PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Notes for Lecture 1:

Reading: Appendix 1 and Chapters I&1

1. Course structure and expectations
2. Units – SI vs Gaussian
3. Electrostatics – Poisson equation

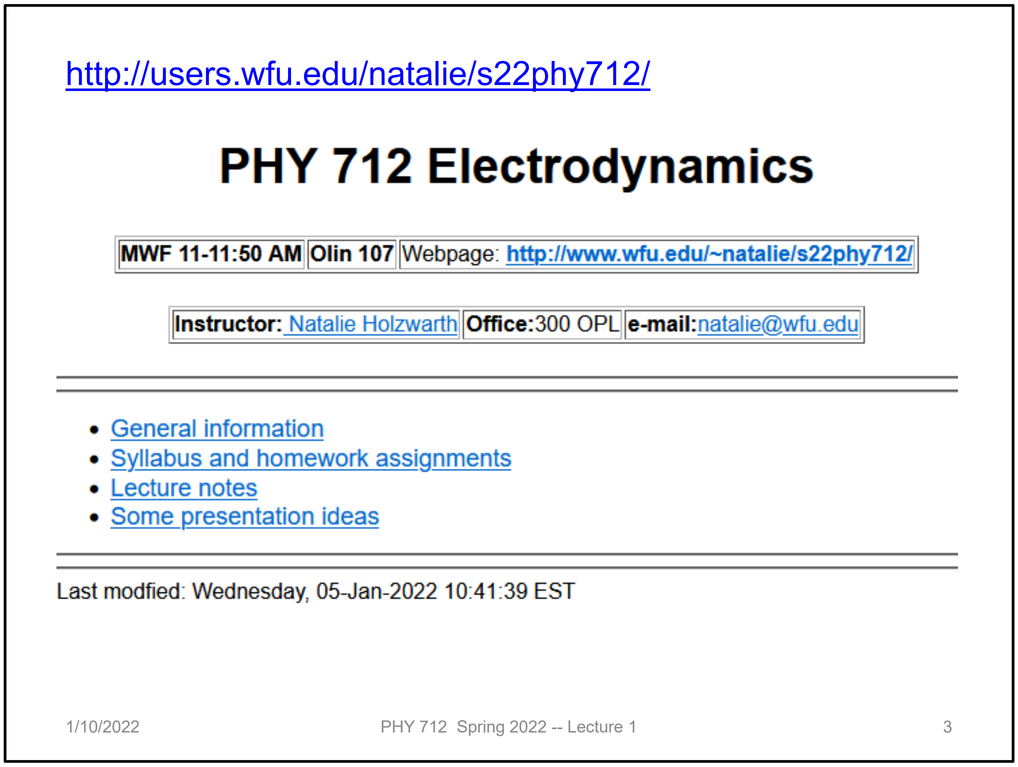
Welcome to Electrodynamics. In this lecture we will discuss the course structure and jump right in to Chapters I (introduction) and 1 (electrostatics).

Spring 2022 Schedule
for N. A. W. Holzwarth

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-11:00	Lecture Preparation & Office Hours	Physics Research	Lecture Preparation & Office Hours	Physics Research	Lecture Preparation & Office Hours
11:00-1:00	Electrodynamics PHY 712 & Quantum II PHY 742		Electrodynamics PHY 712 & Quantum II PHY 742		Electrodynamics PHY 712 & Quantum II PHY 742
1:00-4:00	Physics Research		Physics Research		Physics Research
4:00-5:00				Physics Colloquium	

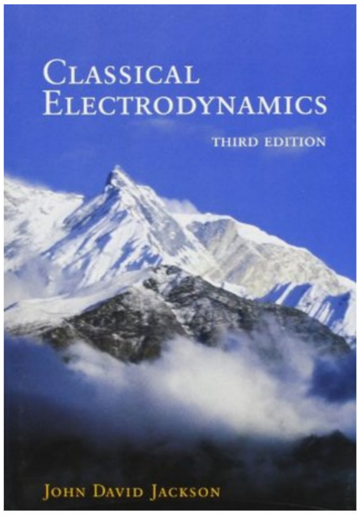
Note that Olin 107 is available at 10-11 AM, but there is a class at 1 PM.

Several of you are involved with PHY 663 which we still need to schedule….



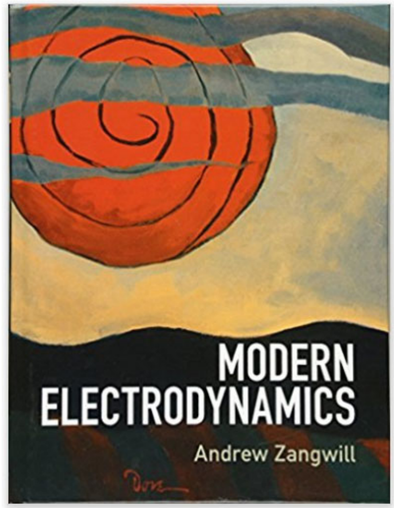
Back to our regularly scheduled program. Our webpage is set up and is the source of all information about the course.

Textbook



Third edition

Optional supplement



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The text book is the 3rd addition of the classic textbook by John David Jackson. (Please note that you will need the 3rd edition in order to follow the lectures.) Some of you may want another (perhaps less mathematically focused) and will find that the textbook by Andrew Zangwill to be helpful. Personally, I find that Jackson's text has served me well to understand the basics of electrodynamics.

http://users.wfu.edu/natalie/s22phy712/info

PHY 712 Electrodynamics

MWF 11-11:50 AM	Olin 107	Webpage: http://www.wfu.edu/~natalie/s22phy712/

Instructor: Natalie Holzwarth	Office:300 OPL	e-mail:natalie@wfu.edu

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General Information

This course is a one semester survey of Electrodynamics at the graduate level, using the textbook: **Classical Electrodynamics**, 3rd edition, by John David Jackson (John Wiley & Sons, Inc., 1999) -- "**JDJ**". (link to errata for early printings) Note that it is necessary to get the **third** edition in order to synchronize with the class lectures and homework. The more recent textbook: **Modern Electrodynamics**, by Andrew Zangwill (Cambridge University Press, 2013) will be used as a supplement.

The course will consist of the following components:

- In person meetings in Olin 107 MWF 11-11:50 AM. Zoom connections can be made available if requested, but not on a regular basis. The class sessions will focus on discussion of the material, particularly answering your prepared and spontaneous questions.
- Asynchronous review of annotated lecture notes and corresponding textbook sections. The reading assignment and annotated lecture notes will be available one day before the corresponding synchronous online discussion. For each class meeting, students will be expected to submit (by email) at least one question for class discussion at least 3 hours before the class meeting.
- Homework sets. Typically there will be one homework problem associated with each class meeting.
- There will be two take-home exams, one at mid-term and the other during finals week.
- There will be one project on a chosen topic related to electrodynamics.
- It is highly recommended that each student arrange for weekly one-on-one meetings with the instructor to discuss the course material, homework, and/or projects. These may be face-to-face or online as appropriate.

Information about the course structure is on our web page. The structure of the course is very much like the structure we used last semester for PHY 711. We want to focus class time on discussing your questions. Please prepare at least one question 3 hours before (preferably the night before) each class. We will use your questions as well as spontaneous questions to form the basis of the class discussions. Again, there will typically be one homework per class, nominally due the day of the next class meeting.

It is likely that your grade for the course will depend upon the following factors:

Class participation	15%
Problem sets*	35%
Project	15%
Exams	35%

*In general, there will a new assignment after each lecture, so that for optimal learning, it would be best to complete each assignment before the next scheduled lecture. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts.

➔ Schedule weekly one-on-one meetings and/or attend office hours

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This is the expected grade distribution. Last semester the weekly one-on-one meetings seemed to be mostly productive, particularly for improving the completely online format. We will try to continue this during the spring semester. I need information from you to set up the appropriate timing.

Some Ideas for Computational Project

The purpose of the "Computational Project" is to provide an opportunity for you to study a topic of your choice in greater depth. The general guideline for your choice of project is that it should have something to do with electrodynamics, and there should be some degree of computation or analysis with the project. The completed project will include a short write-up and a ~20min presentation to the class. You may design your own project or use one of the following list (which will be updated throughout the term).

- Evaluate the Ewald sum of various ionic crystals using Maple or a programing language. (Template available in Fortran code.)
- Work out the details of the finite difference or finite element methods.
- Work out the details of the hyperfine Hamiltonian as discussed in Chapter 5 of Jackson.
- Work out the details of Jackson problem 7.2 and related problems.
- Work out the details of reflection and refraction from birefringent materials.
- Analyze the Kramers-Kronig transform of some optical data or calculations.
- Determine the classical electrodynamics associated with an infrared or optical laser.
- Analyze the radiation intensity and spectrum from an interesting source such as an atomic or molecular transition, a free electron laser, etc.
- Work out the details of Jackson problem 14.15.

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This semester we will also have individual projects due at the end of the semester. Here is a list from last year. During the course, please keep your eye out for topics that interest you worthy of your extra attention.

http://users.wfu.edu/natalie/s22phy712/homework/

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Instructor: Natalie Holzwarth	Office:300 OPL	e-mail:natalie@wfu.edu

Course schedule for Spring 2022

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/10/2022	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/14/2022
2	Wed: 01/12/2022	Chap. 1	Electrostatic energy calculations	#2	01/19/2022
3	Fri: 01/14/2022	Chap. 1	Electrostatic energy calculations		
	Mon: 01/17/2022		MLK Holiday -- no class		

PHY 712 -- Assignment #1

January 10, 2022

Read Chapters I and 1 and Appendix 1 in **Jackson**.

1. Jackson Problem #1.5. Be careful to take into account the behavior of $\Phi(\mathbf{r})$ for r-->0.

The class schedule is available on the webpage and except for today's lecture topic, subject to change. (The schedule is based on the topics we covered last year.)

Comment about HW #1: (Jackson problem 1.5)

The time-averaged potential of a neutral hydrogen atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-2r/a_0}}{r}\left(1 + \frac{r}{a_0}\right)$$

where q denotes the magnitude of the elementary charge of an electron or proton and where a_0 denotes the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your results physically.

Be careful to take into account the behavior of the potential for r→0.

The homework for today's lecture is from the textbook and the content is listed here. It is an interesting problem relating the electrostatic potential of a neutral hydrogen atom. The result should be consistent with things you learned in Quantum Mechanics class.

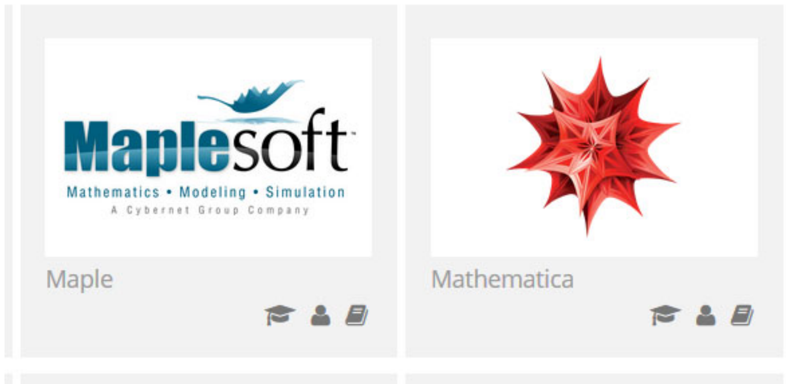
Tentative additional information –

- Mon Jan 17 – MLK Holiday
- Spring break March 5-13
- Mid term grades due March 7
- APS March meeting March 14-18 (no class)
- Fri April 15 – Good Friday Holiday
- Thurs Apr 21 – Student wellness day (no class)
- Wed Apr 27 – Last day of class
- Apr 29-May 6 – Final exams

This semester, we are starting the course late and there will be no spring break. The plan is to have a week where you will work on the mid term take home exam and there will be no lecture. (This happens to be the week when I plan to attend the March Meeting of the American Physical Society (virtually).

Remember to check your algebraic manipulation software --

https://software.wfu.edu/audience/students/



It is expected that some of your homework will need to use algebraic manipulation software such as Maple or Mathematica which are available at the website mentioned above.

Your questions –

From Wells -- Does epsilon_0 have a value of 1/4pi in CGS units on slide 12? And mu_0 = 4pi/c^2? In a few slides, it says that these values are dimensionless in CGS so I do not know if this is right, but it makes sense.

Comment – We will go through this systematically, and hopefully it will make sense in the end (or not).

From Owen -- For what sort of systems or problems is it useful to use Green's functions in electrodynamics?

Comment – Green's functions are particularly good for boundary value problems. In general differential and integral formulations of E&M should be interchangeable, but depending on the situation, one may be more convenient than the other.

Material discussed in Appendix of textbook --

Units - SI vs Gaussian

Coulomb's Law

$$F = K_C \frac{q_1 q_2}{r_{12}^2}. \tag{1}$$

Ampere's Law

$$F = K_A \frac{i_1 i_2}{r_{12}^2}\, d\mathbf{s_1} \times d\mathbf{s_2} \times \hat{\mathbf{r}}_{\mathbf{12}}, \tag{2}$$

In the equations above, the current and charge are related by $i_1 = dq_1/dt$ for all unit systems. The two constants K_C and K_A are related so that their ratio K_C/K_A has the units of $(m/s)^2$ and it is *experimentally* known that the ratio has the value $K_C/K_A = c^2$, where c is the speed of light.

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Now for some discussion of our favorite topic. First we need to be clear about units. In general, the standard unit system is “SI”. The appendix discusses alternative unit systems found in the literature and IN THE SECOND HALF OF THIS TEXTBOOK!!!!! While for mechanics this is not such a big deal, for E & M it causes great pain and suffering.

The NIST Reference on
Constants, Units, and Uncertainty

Fundamental Physical Constants

Constants Topics:
- **Values**
- Energy Equivalents
- Searchable Bibliography
- Background

speed of light in vacuum

c

Numerical value	299 792 458 m s^{-1}
Standard uncertainty	**(exact)**
Relative standard uncertainty	**(exact)**

Units - SI vs Gaussian – continued

The choices for these constants in the SI and Gaussian units are given below:

	CGS (Gaussian)	SI
K_C	1	$\frac{1}{4\pi\epsilon_0}$
K_A	$\frac{1}{c^2}$	$\frac{\mu_0}{4\pi}$

Here, $\frac{\mu_0}{4\pi} \equiv 10^{-7} N/A^2$ and $\frac{1}{4\pi\epsilon_0} = c^2 \cdot 10^{-7} N/A^2 = 8.98755 \times 10^9 N \cdot m^2/C^2$.

Here are some relationships between the two unit systems used in this text.

Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m^2/s$	$dyne$	$gm \cdot cm^2/s$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

More about units

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

More about units. Note that while SI units are the current “standard”, many papers in the literature and older textbooks use other unit systems.

Units - SI vs Gaussian – continued

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As we will see throughout the course, the E and B fields represent the basic electric and magnetic fields while the other fields include or represent electric and magnetic effects of matter.

More about units. Note that while SI units are the current "standard", many papers in the literature and older textbooks use other unit systems.

Basic equations of electrodynamics

CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$	$u = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

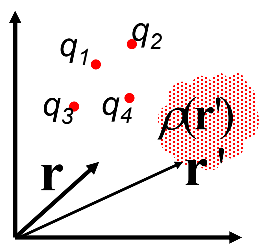
Maxwell’s equations and other relationships that we will see during this course in the two unit systems.

Units choice for this course:

SI units for Jackson in Chapters 1-10

Gaussian units for Jackson in Chapters 11-16

Electrostatics



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_i q_i \frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3}$$

$$= \frac{1}{4\pi\epsilon_0}\int d^3r' \rho(\mathbf{r}') \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

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Starting review of electrostatics both for a discrete collection of charges and for a continuous distribution of charge.

Electrostatics

Discrete versus continuous charge distributions

In terms of Dirac delta function:

$$\rho(\mathbf{r}) = \sum_i q_i \ \delta(\mathbf{r} - \mathbf{r}_i)$$

Digression: Note that in cartesian coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \delta(x - x_i)\delta(y - y_i)\delta(z - z_i)$$

in spherical polar coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \frac{1}{r^2}\delta(r - r_i)\delta(\cos\theta - \cos\theta_i)\delta(\phi - \phi_i)$$

Note that we can use a Dirac delta function to express discrete charges in terms of a distribution. The digression lists some useful relationships for various coordinate systems.

Differential equations --

Electrostatics

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{E} = 0$$

Electrostatic potential

$$\mathbf{E} = -\nabla\Phi(r).$$

$$\nabla^2\Phi(r) = -\rho(r)/\epsilon_0.$$

In addition to the integral relationships, electrostatics can also be expressed in terms of differential forms. Mathematically, the electrostatic potential is a convenient form in which to analyze electrostatic systems.

Relationship between integral and differential forms of electrostatics --

Differential form

$$\nabla^2\Phi(\mathbf{r}) = -\rho(\mathbf{r})/\epsilon_0$$

Integral form

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Comparison of differential and integral relationships.

Relationship between integral and differential forms of electrostatics --

Need to show: $$\nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -4\pi \delta^3(\mathbf{r} - \mathbf{r}').$$

Noting that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \delta^3(\mathbf{r} - \mathbf{r}') f(\mathbf{r}) = f(\mathbf{r}'),$$

we see that we must show that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) = -4\pi f(\mathbf{r}').$$

The identity of the differential and integral forms follows from the singular behavior of the Coulomb kernel expressed on the top line. We need to “prove” this identity.

We introduce a small radius a such that:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \lim_{a\to 0} \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}}.$$

For a fixed value of a,

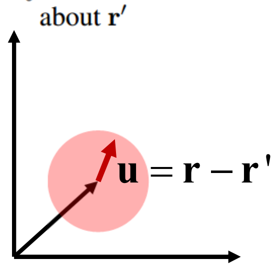
$$\nabla^2 \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}} = \frac{-3a^2}{(|\mathbf{r}-\mathbf{r}'|^2 + a^2)^{5/2}}.$$

Proof using the limit of an expression for a→0.

If the function $f(\mathbf{r})$ is continuous, we can make a Taylor expansion of it about the point $\mathbf{r} = \mathbf{r}'$, keeping only the first term. The integral over the small sphere about $\mathbf{r}'$ can be carried out analytically, by changing to a coordinate system centered at $\mathbf{r}'$;

so that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \, \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) \approx f(\mathbf{r}') \lim_{a \to 0} \int_{u<R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}}.$$



$$\mathbf{r} = \mathbf{r}' + \mathbf{u} \qquad f(\mathbf{r}) \approx f(\mathbf{r}')$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}} = 4\pi \int_0^R du \, \frac{-3a^2u^2}{(u^2 + a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2 + a^2)^{3/2}}.$$

Analytic results for finite a.

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du \, \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

$$\text{For } a \ll R, \quad 4\pi \frac{-R^3}{\left(R^2+a^2\right)^{3/2}} \approx -4\pi$$

$$\rightarrow \int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \, \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) \approx f(\mathbf{r}')(-4\pi),$$

$$\rightarrow \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$$

From analytic result, evaluate for fixed R and a→0. Find that the result is independent of R and the identity is found to be justified.

Example in HW1

The electrostatic potential of a neutral H atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right).$$

Find the charge density (both continuous and discrete) for this potential.

Hint #1: For continuous contribution you can use the identity: $\nabla^2\Phi(r) = \frac{1}{r}\frac{\partial^2(r\Phi(r))}{\partial r^2}$

Hint #2: Don't forget to consider possible discrete contributions.

Comment about homework problem.