

## PHY 712 Electrodynamics 11-11:50 AM MWF Olin 103

## Notes for Lecture 29:

**Continue reading Chap. 14 –** 

## **Radiation by moving charges**

- 1. Recap of results for synchrotron radiation from land based sources
- 2. Synchrotron radiation from astronomical sources
- 3. Compton scattering for next time

### **Physics Colloquium this week --**

### Annual Honors Presentations and Awards Ceremony — Part I Location: Olin 101 Time: Thursday, April 14, 2022 at 4 PM

**Annelise Brigham** — "Mechanical Properties of Electrospun PCL:Fibrinogen Fibers" — Mentor: Prof. Guthold

**Rich Pope** — "The Effects of LDL on the Mechanical Properties of Single Fibrin Fibers" — Mentor: Prof. Guthold

**Will Caulkins** — "Characterization of a Novel Diffractive Optical Element for 3-Dimensional Tracking of Fluorescent Particles" — Mentor: Prof. Bonin

**Jash Desai** — "Convolutional Neural Nets and Applications to 3D Imaging " — Mentor: Prof. Bonin

**Zach Scofield** — "Symmetric Two-Point Function for a Massive, Minimally Coupled Scalar Field in Two-Dimensional Schwarzschild de-Sitter Spacetime" — Mentor: Prof. Anderson

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21	Fri: 03/25/2022	Chap. 9	Radiation from localized oscillating sources	<u>#18</u>	03/30/2022
22	Mon: 03/28/2022	Chap. 9	Radiation from oscillating sources		
23	Wed: 03/30/2022	Chap. 9 & 10	Radiation and scattering	<u>#19</u>	04/01/2022
24	Fri: 04/01/2022	Chap. 11	Special Theory of Relativity	<u>#20</u>	04/04/2022
25	Mon: 04/04/2022	Chap. 11	Special Theory of Relativity	<u>#21</u>	04/06/2022
26	Wed: 04/06/2022	Chap. 11	Special Theory of Relativity		
27	Fri: 04/08/2022	Chap. 14	Radiation from moving charges	<u>#22</u>	04/11/2022
28	Mon: 04/11/2022	Chap. 14	Radiation from accelerating charged particles	<u>#23</u>	04/18/2022
29	Wed: 04/13/2022	Chap. 14	Synchrotron radiation		
	Fri: 04/15/2022	No class	Holiday		
30	Mon: 04/18/2022				

### Comment about units

Differential power (cgs Gaussian)

$$\frac{dP_r(t_r)}{d\Omega} = \frac{e^2}{4\pi c} \frac{\left|\hat{\mathbf{r}} \times \left((\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right)\right|^2}{\left(1 - \hat{\mathbf{r}} \cdot \boldsymbol{\beta}\right)^5}$$

e measured in Statcoulombs Length measured in cm Energy measured in ergs Differential power (SI)

$$\frac{dP_r(t_r)}{d\Omega} = \frac{e^2}{\left(4\pi\epsilon_0\right)4\pi c} \frac{\left|\hat{\mathbf{r}} \times \left(\left(\hat{\mathbf{r}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right)\right|^2}{\left(1 - \hat{\mathbf{r}} \cdot \boldsymbol{\beta}\right)^5}$$

*e* measured in Coulombs Length measured in m Energy measured in joules

Main results from synchrotron radiation spectra from man made sources --





$$\mathbf{R}_{q}(t_{r}) = \rho \hat{\mathbf{x}} \sin(\nu t_{r} / \rho) + \rho \hat{\mathbf{y}} (1 - \cos(\nu t_{r} / \rho)) \mathbf{\beta}(t_{r}) = \beta (\hat{\mathbf{x}} \cos(\nu t_{r} / \rho) + \hat{\mathbf{y}} \sin(\nu t_{r} / \rho)) For convenience, choose: 
$$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos\theta + \hat{\mathbf{z}} \sin\theta$$$$

Note that we have previous shown that in the radiation zone, the Poynting vector is in the  $\hat{\mathbf{r}}$  direction; we can then choose to analyze two orthogonal polarization directions:  $\mathbf{\epsilon}_{\parallel} = \hat{\mathbf{y}}$   $\mathbf{\epsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$  $\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{\beta}) = \beta \left(-\mathbf{\epsilon}_{\parallel} \sin(vt_r / \rho) + \mathbf{\epsilon}_{\perp} \sin\theta\cos(vt_r / \rho)\right)$ 

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We will analyze this expression for two different cases. The first case, is appropriate for man-made synchrotrons used as light sources. In this case, the light is produced by short bursts of electrons moving close to the speed of light ( $v \approx c(1-1/(2\gamma^2))$ ) passing a beam line port. In addition, because of the design of the radiation ports,  $\theta \approx 0$ , and the relevant integration times *t* are close to  $t \approx 0$ . This results in the form shown in Eq. 14.79 of your text. It is convenient to rewrite this form in terms of a critical

frequency 
$$\omega_c \equiv \frac{3c\gamma^3}{2\rho}$$
.  

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c} \left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{ \left[ K_{2/3} \left(\frac{\omega}{2\omega_c} (1+\gamma^2\theta^2)^{\frac{3}{2}}\right) \right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} \left[ K_{1/3} \left(\frac{\omega}{2\omega_c} (1+\gamma^2\theta^2)^{\frac{3}{2}}\right) \right]^2 \right\}$$

$$\frac{d^{2}I}{d\omega d\Omega} = \frac{3q^{2}\gamma^{2}}{4\pi^{2}c} \left(\frac{\omega}{\omega_{c}}\right)^{2} \left(1+\gamma^{2}\theta^{2}\right)^{2} \left\{ \left[K_{2/3}\left(\frac{\omega}{2\omega_{c}}\left(1+\gamma^{2}\theta^{2}\right)^{\frac{3}{2}}\right)\right]^{2} + \frac{\gamma^{2}\theta^{2}}{1+\gamma^{2}\theta^{2}} \left[K_{1/3}\left(\frac{\omega}{2\omega_{c}}\left(1+\gamma^{2}\theta^{2}\right)^{\frac{3}{2}}\right)\right]^{2} \right\}$$

By plotting the intensity as a function of  $\omega$ , we see that the intensity is largest near  $\omega \approx \omega_c$ . The plot below shows the intensity as a function of  $\omega/\omega_c$  for  $\gamma\theta=0$ , 0.5 and 1:



More details

$$\frac{d^{2}I}{d\omega d\Omega} = \frac{d^{2}I_{\parallel}}{d\omega d\Omega} + \frac{d^{2}I_{\perp}}{d\omega d\Omega}$$
$$\frac{d^{2}I_{\parallel}}{d\omega d\Omega} = \frac{3q^{2}\gamma^{2}}{4\pi^{2}c} \left(\frac{\omega}{\omega_{c}}\right)^{2} (1+\gamma^{2}\theta^{2})^{2} \left[K_{2/3}\left(\frac{\omega}{2\omega_{c}}(1+\gamma^{2}\theta^{2})^{\frac{3}{2}}\right)\right]^{2}$$
$$\frac{d^{2}I_{\perp}}{d\omega d\Omega} = \frac{3q^{2}\gamma^{2}}{4\pi^{2}c} \left(\frac{\omega}{\omega_{c}}\right)^{2} (1+\gamma^{2}\theta^{2})^{2} \frac{\gamma^{2}\theta^{2}}{1+\gamma^{2}\theta^{2}} \left[K_{1/3}\left(\frac{\omega}{2\omega_{c}}(1+\gamma^{2}\theta^{2})^{\frac{3}{2}}\right)\right]^{2}$$



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### https://lightsources.org/lightsources-of-the-world/



# Comment on light source facilities and the newer free electron laser technology

SCIENCE'S COMPASS





### **Free-Electron Lasers: Status and Applications**

Patrick G. O'Shea<sup>1</sup> and Henry P. Freund<sup>2</sup>

A free-electron laser consists of an electron beam propagating through a periodic magnetic field. Today such lasers are used for research in materials science, chemical technology, biophysical science, medical applications, surface studies, and solid-state physics. Free-electron lasers with higher average power and shorter wavelengths are under development. Future applications range from industrial processing of materials to light sources for soft and hard x-rays.

tions at wavelengths down to 1 Å, and this is illustrated by the peak brilliance of a wide range of the present-day synchrotrons and the projected performance of FELs (Fig. 3) (7). Consequently, applications in the x-ray region will undergo an upheaval similar to that which followed the invention of the laser at visible wavelengths.

Ultraviolet FEL oscillators using electron

### DOI: 10.1126/science.1055718

**Free-Electron Lasers: Status and Applications** 

SCIENCE'S COMPASS

**REVIEW:** LASER TECHNOLOGY

Patrick G. O'Shea<sup>1</sup> and Henry P. Freund<sup>2</sup>

• REVIEW



Fig. 3. Peak brilliance of x-ray FELs and undulators for spontaneous radiation at the TESLA Test Facility, in comparison with synchrotron radiation sources. Brilliance is expressed as photons s<sup>-1</sup> mrad<sup>-2</sup> mm<sup>-2</sup> per 0.1% bandwidth. For comparison, the spontaneous spectrum of x-ray FEL undulators is also shown. The label TTF-FEL indicates design values for the FEL at the TESLA Test Facility, with (M) for the planned seeded version (28).

Additional references on Free Electron Lasers

https://doi.org/10.1016/j.xinn.2021.100097

- J.M.J. Madey
   Stimulated emission of bremsstrahlung in a periodic magnetic field
   J. Appl. Phys., 42 (1971), pp. 1906-1913, 10.1063/1.1660466
   View PDF View Record in Scopus Google Scholar
- D.A.G. Deacon, L.R. Elias, J.M.J. Madey, G.J. Ramian, H.A. Schwettman, T.I. Smith
   First operation of a free-electron laser
   Phys. Rev. Lett., 38 (1977), pp. 892-894, 10.1103/PhysRevLett.38.892
   View PDF Google Scholar

Components of the FEL

- 1. Electrons moving in circular paths
- 2. Self-amplified spontaneous emission (SASE)
- 3. +++

Often designed for the X-ray range

Jefferson Free Electron Laser Lab in VA is designed in the microwave  $\leftarrow \rightarrow$  infrared range



The above analysis applies to a class of man-made facilities dedicated to producing intense radiation in the continuous spectrum. For more specific information on man-made synchrotron sources, the following web page is useful: <a href="http://www.als.lbl.gov/als/synchrotron\_sources.html">http://www.als.lbl.gov/als/synchrotron\_sources.html</a>.

Synchrotron radiation is also produced by astronomical sources as analyzed by Julian Schwinger –

#### On the Classical Radiation of Accelerated Electrons

JULIAN SCHWINGER Harvard University, Cambridge, Massachusetts (Received March 8, 1949)

This paper is concerned with the properties of the radiation from a high energy accelerated electron, as recently observed in the General Electric synchrotron. An elementary derivation of the total rate of radiation is first presented, based on Larmor's formula for a slowly moving electron, and arguments of relativistic invariance. We then construct an expression for the instantaneous power radiated by an electron moving along an arbitrary, prescribed path. By casting this result into various forms, one obtains the angular distribution, the spectral distribution, or the combined angular and spectral distributions of the radiation. The method is based on an examination of the rate at which the electron irreversibly transfers energy to the electromagnetic field, as determined by half the difference of retarded and advanced electric field intensities. Formulas are obtained for an arbitrary chargecurrent distribution and then specialized to a point charge. The total radiated power and its angular distribution are obtained for an arbitrary trajectory. It is found that the direction of motion is a strongly preferred direction of emission at high energies. The spectral distribution of the radiation depends upon the detailed motion over a time interval large compared to the period of the radiation. However, the narrow cone of radiation generated by an energetic electron indicates that only a small part of the trajectory is effective in producing radiation observed in a given direction, which also implies that very high frequencies are emitted. Accordingly, we evaluate the spectral and angular distributions of the high frequency radiation by an energetic electron, in their dependence upon the parameters characterizing the instantaneous orbit. The average spectral distribution, as observed in the synchrotron measurements, is obtained by averaging the electron energy over an acceleration cycle. The entire spectrum emitted by an electron moving with constant speed in a circular path is also discussed. Finally, it is observed that quantum effects will modify the classical results here obtained only at extraordinarily large energies.

### DOI:https://doi.org/10.1103/PhysRev.75.1912

Recalling general results of analysis --  

$$\mathbf{\epsilon}_{\parallel} = \hat{\mathbf{y}}$$
  $\mathbf{\epsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$   
 $\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{\beta}) =$   
 $\mathbf{\epsilon}_{\parallel} \mathbf{y}$   $\mathbf{\beta} (-\mathbf{\epsilon}_{\parallel}\sin(vt_r / \rho) + \mathbf{\epsilon}_{\perp}\sin\theta\cos(vt_r / \rho))$   
 $\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2c} \Big| \int_{-\infty}^{\infty} \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \beta) e^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} dt \Big|^2$   
 $\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2c} \Big\{ |C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2 \Big\}$   
 $C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt / \rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$   
 $C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt / \rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$ 

Useful identity involving Bessel functions

$$e^{-iA\sin\alpha} = \sum_{m=-\infty}^{\infty} J_m(A) e^{-im\alpha}$$
 Here  $J_m(A)$  is a

Bessel function of integer order *m*.

In our case, 
$$A = \frac{\omega \rho}{c} \cos \theta$$
 and  $\alpha = \frac{vt}{\rho}$ .  
 $C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt / \rho) e^{i\omega(t - \frac{\rho}{c} \cos \theta \sin(vt / \rho))}$   
 $= \frac{c}{-i\omega\rho} \frac{\partial}{\partial \cos \theta} \int_{-\infty}^{\infty} dt e^{i\omega(t - \frac{\rho}{c} \cos \theta \sin(vt / \rho))}$ 

$$=\frac{c}{-i\omega\rho}\frac{\partial}{\partial\cos\theta}\sum_{m=-\infty}^{\infty}J_{m}\left(\frac{\omega\rho}{c}\cos\theta\right)2\pi\delta(\omega-m\frac{v}{\rho}).$$

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Some details for last step --

$$e^{-iA\sin\alpha} = \sum_{m=-\infty}^{\infty} J_m(A) e^{-im\alpha}$$
 Here  $J_m(A)$  is a

Bessel function of integer order *m*.

In our case, 
$$A = \frac{\omega \rho}{c} \cos \theta$$
 and  $\alpha = \frac{vt}{\rho}$ .

$$\int_{-\infty}^{\infty} dt e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))} = \sum_{m=-\infty}^{\infty} J_m \left(\frac{\omega\rho}{c}\cos\theta\right) \int_{-\infty}^{\infty} dt e^{i\omega(t-m\frac{v}{c}t))}$$
$$= \sum_{m=-\infty}^{\infty} J_m \left(\frac{\omega\rho}{c}\cos\theta\right) 2\pi\delta(\omega-m\frac{v}{\rho}).$$

### Astronomical synchrotron radiation -- continued: Note that:

$$\int_{-\infty}^{\infty} dt e^{i(\omega - m\frac{v}{\rho})t} = 2\pi\delta(\omega - m\frac{v}{\rho}).$$
  

$$\Rightarrow C_{\parallel}(\omega) = 2\pi i \sum_{m=-\infty}^{\infty} J'_{m} \left(\frac{\omega\rho}{c}\cos\theta\right) \delta(\omega - m\frac{v}{\rho}),$$
  
where  $J'_{m}(A) \equiv \frac{dJ_{m}(A)}{dA}$ 

Similarly:

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin \theta \cos(vt / \rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta \sin(vt / \rho))}$$
$$= 2\pi \frac{\tan \theta}{v / c} \sum_{m = -\infty}^{\infty} J_m \left(\frac{\omega \rho}{c}\cos\theta\right) \delta(\omega - m\frac{v}{\rho}).$$

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Astronomical synchrotron radiation -- continued:

In both of the expressions, the sum over *m* includes both negative and positive values. However, only the positive values of  $\omega$  and therefore positive values of *m* are of interest. Using the identity:  $J_{-m}(A) = (-1)^m J_m(A)$ , the result becomes:

$$\frac{d^2I}{d\omega d\Omega} =$$

$$\frac{q^2\omega^2\beta^2}{c}\sum_{m=0}^{\infty}\delta(\omega-m\frac{\nu}{\rho})\left\{\left[J'_m\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2+\frac{\tan^2\theta}{\nu^2/c^2}\left[J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2\right\}$$

These results were derived by Julian Schwinger (Phys. Rev. **75**, 1912-1925 (1949)). The discrete case is similar to the result quoted in Problem 14.15 in Jackson's text.

Back to fundamental processes – Thompson and Compton scattering (see section 14.8 in Jackson)

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass  $m_q$ 

