PHY 712 Electrodynamics
11-11:50 AM MWF Olin 103

Notes for Lecture 30:

Finish reading Chap. 14 and start Chap. 15 –

Radiation from scattering charged particles

1. **Thompson and Compton scattering**
2. **Radiation from particle collisions**

21	Fri: 03/25/2022	Chap. 9	Radiation from localized oscillating sources	#18	03/30/2022
22	Mon: 03/28/2022	Chap. 9	Radiation from oscillating sources		
23	Wed: 03/30/2022	Chap. 9 & 10	Radiation and scattering	#19	04/01/2022
24	Fri: 04/01/2022	Chap. 11	Special Theory of Relativity	#20	04/04/2022
25	Mon: 04/04/2022	Chap. 11	Special Theory of Relativity	#21	04/06/2022
26	Wed: 04/06/2022	Chap. 11	Special Theory of Relativity		
27	Fri: 04/08/2022	Chap. 14	Radiation from moving charges	#22	04/11/2022
28	Mon: 04/11/2022	Chap. 14	Radiation from accelerating charged particles	#23	04/18/2022
29	Wed: 04/13/2022	Chap. 14	Synchrotron radiation		
	Fri: 04/15/2022	No class	*Holiday*		
30	Mon: 04/18/2022	Chap. 14 & 15	Thompson and Compton scattering	#24	04/20/2022
31	Wed: 04/20/2022	Chap. 15	Radiation from collisions of charged particles		

PHY 712 -- Assignment #24

April 18, 2022

Finish reading Chap. 14 and start Chap. 15 in ***Jackson*** *.*

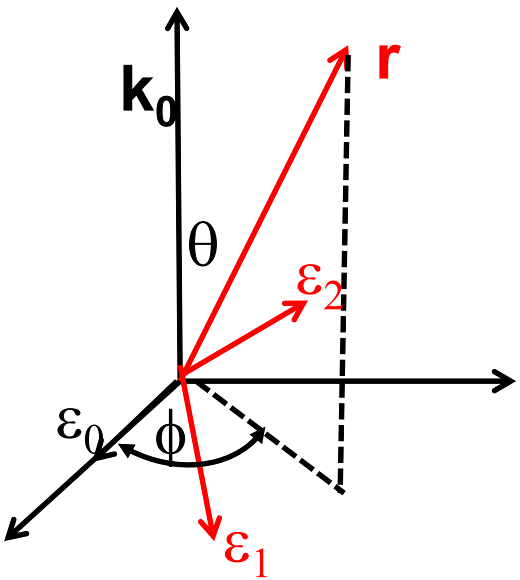
1. *This problem concerns the Compton scattering of a photon having an initial momentum magnitude of p and a final momentum magnitude p' at an angle θ due to an electron of mass m, initially at rest, as discussed in lecture and on page 696 of* ***Jackson****. The wavelength of the photon before the collision is $\lambda=h/p$ and after is $\lambda'=h/p'$, where h is Planck's constant. Show that*

   $$\lambda'-\lambda=(h/(mc^2))(1-\cos\theta).$$

Note – this week is more or less “normal” except for no classes will be held on Thursday 4/21/2022 (and therefore no physics colloquium this week).

Thompson scattering -- classical picture

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



Incident electomagnetic wave:

$\mathbf{k}_0$ propagation direction

$\boldsymbol{\varepsilon}_0$ polarization direction

$$\mathbf{E}(\mathbf{r}',t') = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}'-i\omega t'}\right)$$

Scattered radiation:

$\mathbf{r}$ observed position

$\boldsymbol{\varepsilon}_1,\boldsymbol{\varepsilon}_2$ polarization directions

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \left| \hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \dot{\mathbf{v}} \right) \right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q)

is caused by an electric field: $\mathbf{E}(\mathbf{r}, t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r} - i\omega t} \right)$

$$\dot{\mathbf{v}} = \frac{q}{m_q} \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r} - i\omega t} \right)$$

Time averaged power: $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 \left| E_0 \right|^2 \left| \hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0 \right) \right|^2$

What assumptions are made to conclude that

$$\dot{\mathbf{v}} = \frac{q}{m_q} \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r} - i\omega t}\right) \quad ?$$

Is it always true?

Comment on acceleration

Lorentz force: $$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$$

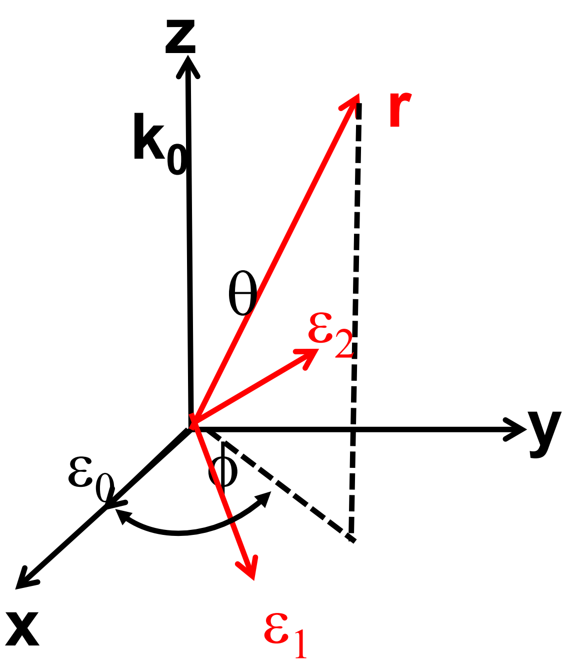
For $v << c$, the dominate force on a charged particle is from the electric field. According to Newton:

$$m_q \frac{d\mathbf{v}}{dt} \equiv m_q \dot{\mathbf{v}} = q\mathbf{E}(\mathbf{r},t) = q\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0\right)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\boldsymbol{\varepsilon}_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

Are these polarizations unique?

Note that we are associating the vector $\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \dot{\mathbf{v}} \right)$ with the polarization of the light. Why?

Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3} \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \left(1 - \frac{v^2}{c^2} \right) + \left(\mathbf{R} \times \left\{ \left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right\} \right) \right]. \tag{19}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}} \cdot \mathbf{R}}{c^2} \right) - \frac{\mathbf{R} \times \dot{\mathbf{v}}/c}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^2} \right]. \tag{20}$$

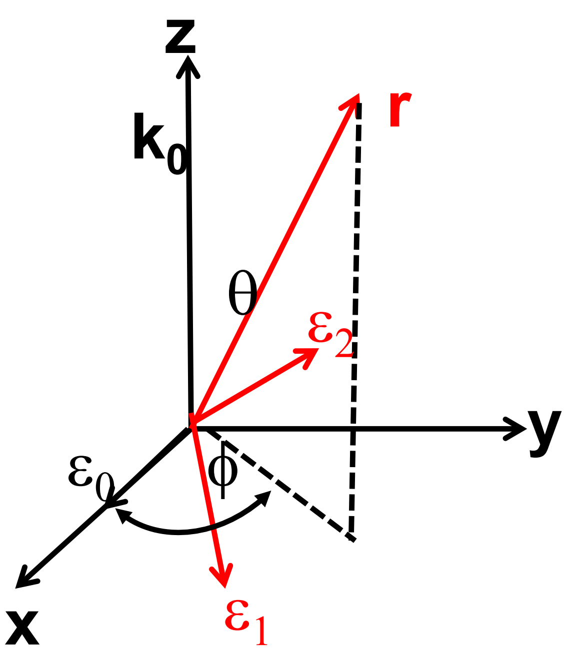
In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r},t)}{R}. \tag{21}$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right) = \hat{\mathbf{r}}\left(\hat{\mathbf{r}}\cdot\boldsymbol{\varepsilon}_0\right) - \boldsymbol{\varepsilon}_0 \quad \text{(perpendicular to } \hat{\mathbf{r}}\text{)}$$

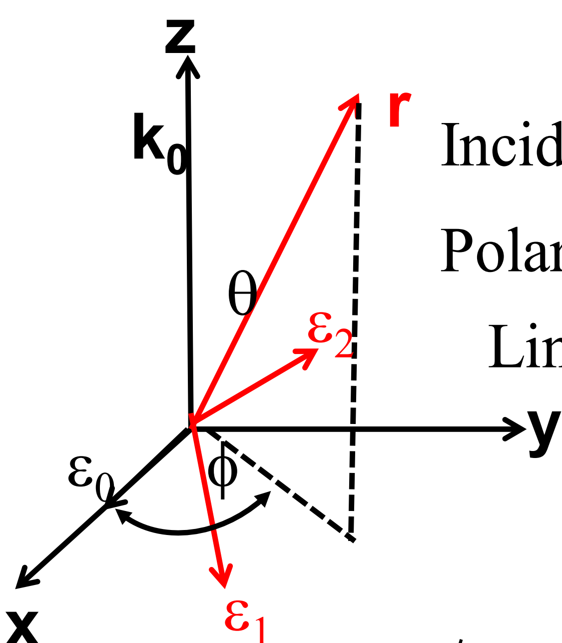
denote scattered light polarization by $\boldsymbol{\varepsilon}^*$

$$\boldsymbol{\varepsilon}^*\cdot\left(\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right) = -\boldsymbol{\varepsilon}^*\cdot\boldsymbol{\varepsilon}_0$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light: $\boldsymbol{\varepsilon}^*$

Linear combination of

$$\boldsymbol{\varepsilon}_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

$$\left\langle \left|\boldsymbol{\varepsilon}^*\cdot\boldsymbol{\varepsilon}_0\right|^2 \right\rangle = \left\langle \left|\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\varepsilon}_0\right|^2 \right\rangle + \left\langle \left|\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\varepsilon}_0\right|^2 \right\rangle = \left\langle \cos^2\theta\cos^2\phi \right\rangle + \left\langle \sin^2\phi \right\rangle$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization $\boldsymbol{\varepsilon}^*$:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2$$

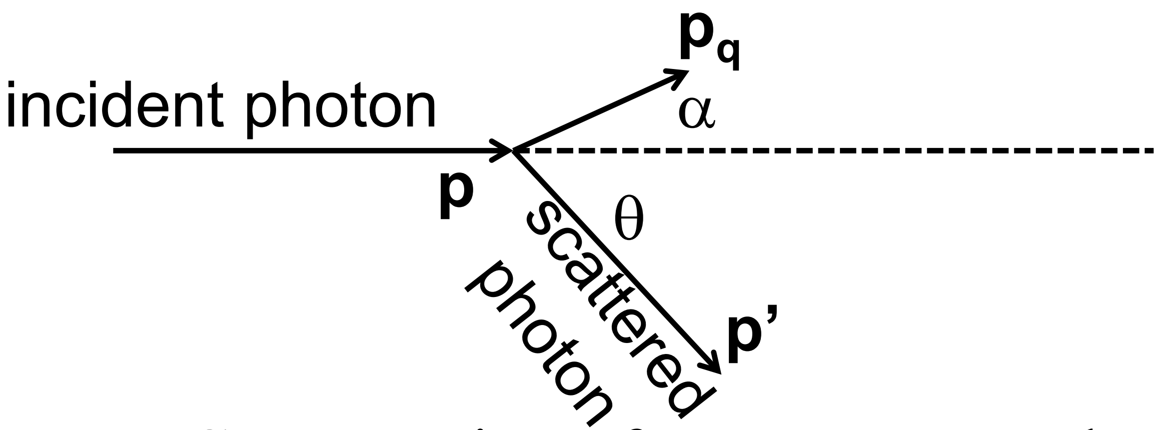
Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

$$\left\langle |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \left\langle |\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi + \left\langle |\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \frac{1}{2}\cos^2\theta + \frac{1}{2}$$

Averaged cross section: $$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy:

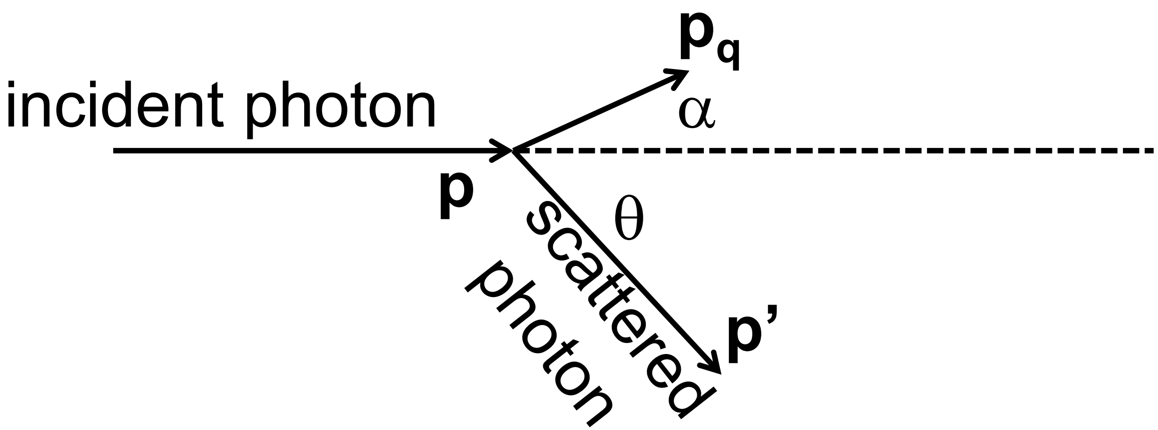
$$p = p'\cos\theta + p_q \cos\alpha \qquad\qquad pc = \hbar\omega$$

$$0 = p'\sin\theta - p_q \sin\alpha \qquad\qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p'^2_q c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$

Thompson scattering – relativistic and quantum modifications

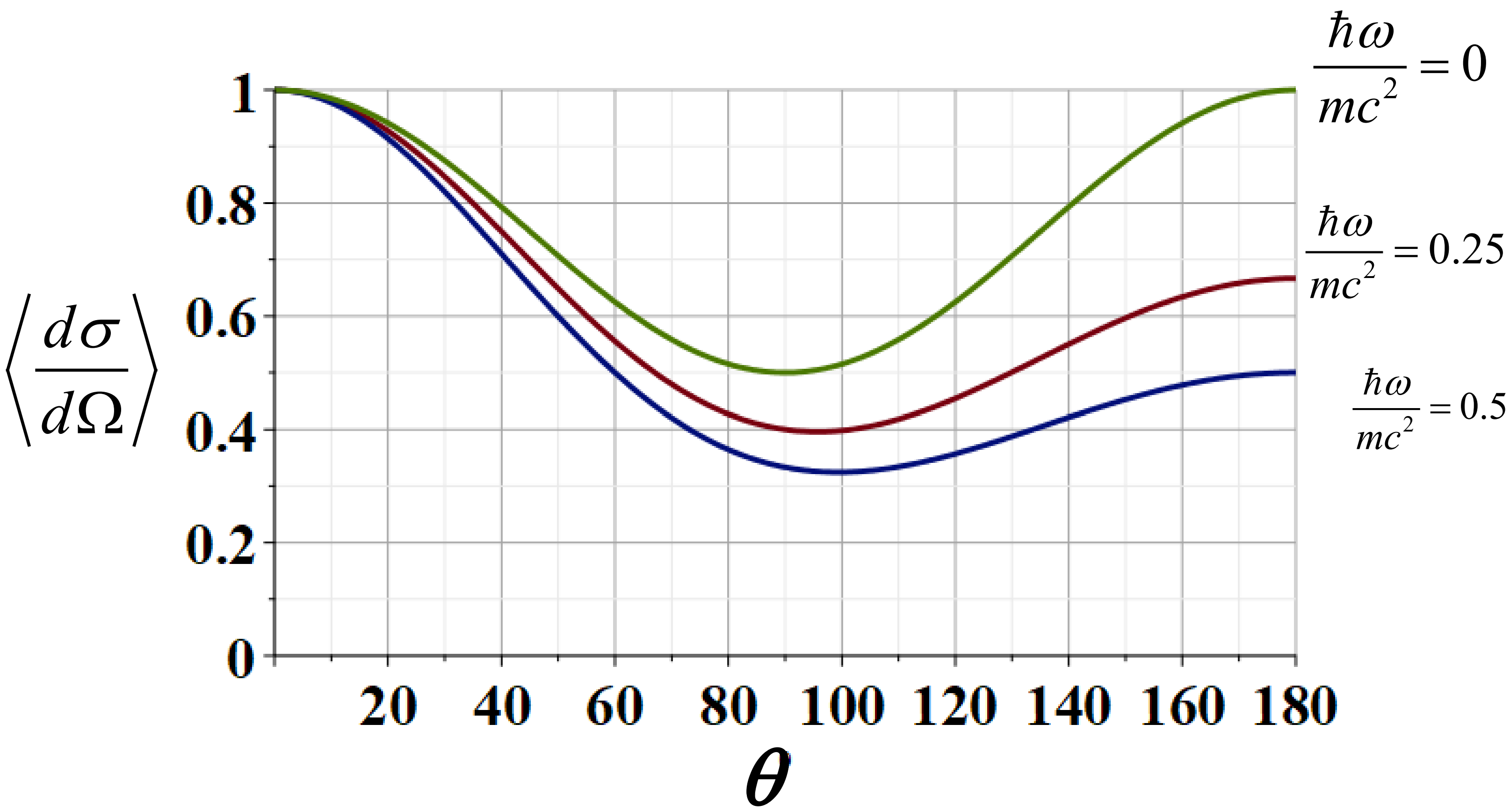


Relativistic and quantum modifications to averaged cross section:

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

$$\frac{p'}{p} = \frac{1}{1+\dfrac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$

Modified Thompson scattering cross section



In fact, the more accurate treatment by Klein and Nishina gives

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$

Klein-Nishina formula

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2} \left(\frac{p'}{p} + \frac{p}{p'} - \sin^2\theta \right)$$

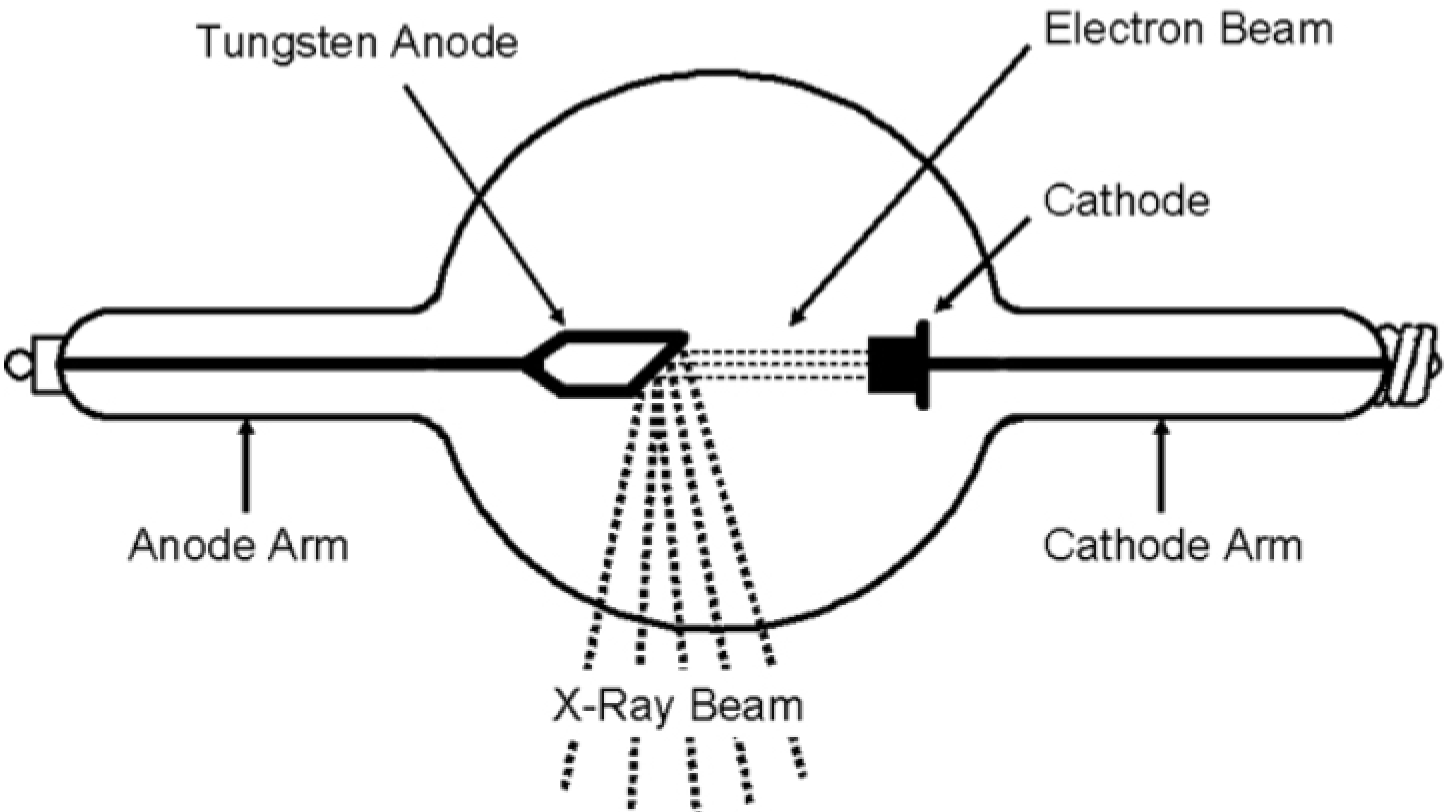
Note that for $\frac{\hbar\omega}{m_q c^2} << 1$ all results are consistent

Up to now, we have been considering (re)radiation due to a charged particle interacting with an electromagnetic field. Next time we will consider radiation due to interactions (collisions) of charged particles themselves.

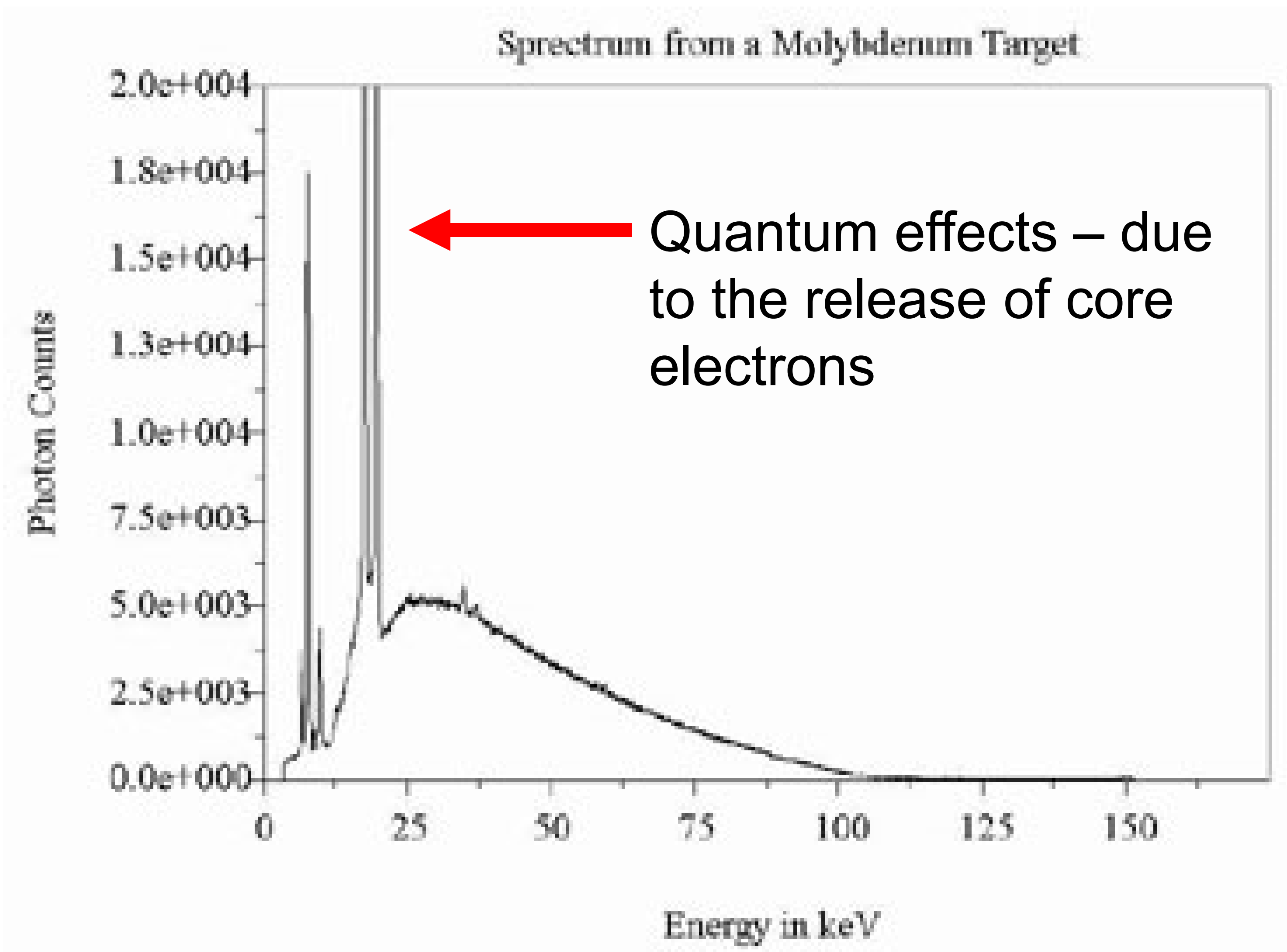
Radiation produced by collisions of charged particles

Generation of X-rays in a Coolidge tube

https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm



http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Quantum effects – due to the release of core electrons