PHY 742 Quantum Mechanics II
12-12:50 PM MWF in Olin 103:

Plan for Lecture 23

Quantization of the Electromagnetic fields

Complete the reading of Chap. 17 in your textbook, Quantizing Electromagnetic Fields.

1. Quantum Hamiltonian for the electromagnetic fields
2. Eigenstates of the electromagnetic Hamiltonian
3. Quantum expressions for the electromagnetic fields
4. Variance of measurable properties of the electromagnetic fields
5. Properties of a single mode coherent state

	Mon: 03/21/2022		Project presentations I		
	Wed: 03/23/2022		Project presentations II		
21	Fri: 03/25/2022	Chap. 5 & 17	Quantization of the Electromagnetic Field	#17	03/28/2022
22	Mon: 03/28/2022	Chap. 17	Quantization of the Electromagnetic Field	#18	03/30/2022
23	Wed: 03/30/2022	Chap. 17	Quantization of the Electromagnetic Field	#19	04/01/2022
24	Fri: 04/01/2022	Chap. 18	Absorption and emission of photons		

PHY 742 -- Assignment #19

March 30, 2022

Finish reading Chapter 17 in **Professor Carlson's QM textbook.**.

1. Consider a Glauber coherent state for a single photon mode of wavevector **k** and polarization index σ with amplitude $\alpha = E_0 \exp(i\psi)$. Calculate the average and variance of the **E** and **B** fields for this coherent state.

Digression – list of possible topics for remainder of the course

- **Multiparticle systems;   reviewing Chap. 10 and introducing "second quantization" formalism**
- **Hubbard model**
- **Superconductivity**
- **Hartree-Fock formalism**
- **Density functional formalism**

Summary of previous results for the electromagnetic Hamiltonian

In terms of the operators $a_{\mathbf{k}\sigma}$ and $a^\dagger_{\mathbf{k}\sigma}$ operators for wavevector $\mathbf{k}$ and polarization σ.

With commutation relations: $\left[a_{\mathbf{k}\sigma}, a^\dagger_{\mathbf{k}'\sigma'}\right] = \delta_{\mathbf{k}\mathbf{k}'}\delta_{\sigma\sigma'}$ $\left[a_{\mathbf{k}\sigma}, a_{\mathbf{k}'\sigma'}\right] = 0$ $\left[a^\dagger_{\mathbf{k}\sigma}, a^\dagger_{\mathbf{k}'\sigma'}\right] = 0$

The eigenstates of the EM Field Hamiltonian (omitting diverging term) are integers $n_{\mathbf{k}\sigma}$:

$$H^{\text{fixed}}_{\text{field}} \left| n_{\mathbf{k}\sigma} \right\rangle = \sum_{\mathbf{k}'\sigma'} \left(\hbar\omega_{\mathbf{k}'} a^\dagger_{\mathbf{k}'\sigma'} a_{\mathbf{k}'\sigma'} \right) \left| n_{\mathbf{k}\sigma} \right\rangle = \hbar\omega_{\mathbf{k}} n_{\mathbf{k}\sigma} \left| n_{\mathbf{k}\sigma} \right\rangle$$

It is convenient to define the photon number operator

$$\mathbf{N}_{\mathbf{k}'\sigma'} \equiv a^\dagger_{\mathbf{k}'\sigma'} a_{\mathbf{k}'\sigma'} \quad \text{such that } \mathbf{N}_{\mathbf{k}\sigma} \left| n_{\mathbf{k}\sigma} \right\rangle = n_{\mathbf{k}\sigma} \left| n_{\mathbf{k}\sigma} \right\rangle$$

Properties of the creation and annihilation operators:

$$a_{\mathbf{k}\sigma}\left|n_{\mathbf{k}\sigma}\right\rangle = \sqrt{n_{\mathbf{k}\sigma}}\left|n_{\mathbf{k}\sigma}-1\right\rangle$$

$$a^{\dagger}_{\mathbf{k}\sigma}\left|n_{\mathbf{k}\sigma}\right\rangle = \sqrt{n_{\mathbf{k}\sigma}+1}\left|n_{\mathbf{k}\sigma}+1\right\rangle$$

Quantum mechanical form of vector potential --

$$\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} + a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

Note: We are assuming that the polarization vector is real.

Quantum mechanical form of vector potential and corresponding fields --

$$\mathbf{A}(\mathbf{r},t)=\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}+a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

Electric field:

$$\mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}\quad\Rightarrow\mathbf{E}(\mathbf{r},t)=i\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}-a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

Magnetic field:

$$\mathbf{B}=\nabla\times\mathbf{A}\quad\Rightarrow\mathbf{B}(\mathbf{r},t)=i\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}-a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

Embarassing/puzzling expectation values --

$$\left\langle n_{\mathbf{k}'\sigma'}\right|\mathbf{A}(\mathbf{r},t)\left|n_{\mathbf{k}'\sigma'}\right\rangle=\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left\langle n_{\mathbf{k}'\sigma'}\right|\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}+a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)\left|n_{\mathbf{k}'\sigma'}\right\rangle=0$$

Electric field:

$$\mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}\quad\Rightarrow\left\langle n_{\mathbf{k}'\sigma'}\right|\mathbf{E}(\mathbf{r},t)\left|n_{\mathbf{k}'\sigma'}\right\rangle=i\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left\langle n_{\mathbf{k}'\sigma'}\right|\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}-a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)\left|n_{\mathbf{k}'\sigma'}\right\rangle=0$$

Magnetic field:

$$\mathbf{B}=\nabla\times\mathbf{A}\quad\Rightarrow\left\langle n_{\mathbf{k}'\sigma'}\right|\mathbf{B}(\mathbf{r},t)\left|n_{\mathbf{k}'\sigma'}\right\rangle=i\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left\langle n_{\mathbf{k}'\sigma'}\right|\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t}-a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)\left|n_{\mathbf{k}'\sigma'}\right\rangle=0$$

In order to compare the classical treatment to the quantum approach we need to calculate expectation values of the observables. In addition to mean value of an observable, its statistical properties are also of interest, particularly the variance and the standard deviation (its square root) which is defined in terms of the average of the squared value of the observable and the average value itself:

$$\text{Standard deviation:} \quad \Delta V \equiv \sqrt{\left\langle V^2 \right\rangle - \left|\left\langle V \right\rangle\right|^2}$$

The next few slides review the relationship of this variance to observables in quantum mechanics which have non trivial commutation relationships and thus have built in variance values.

Digression -- Commutator formalism in quantum mechanics

Definition:

Given two Hermitian operators A and B, their commutator is

$$[A,B] \equiv AB - BA$$

Theorem:

Given Hermitian operators A, B, C such that

$$[A,B] = iC,$$

it follows that $\Delta A \Delta B \geq \frac{1}{2}\left|\langle C \rangle\right|$

Proof --

Note that:

$$[A,B]^\dagger = (iC)^\dagger$$

$$(AB-BA)^\dagger = B^\dagger A^\dagger - A^\dagger B^\dagger = -iC^\dagger$$

$$= BA - AB = -iC$$

Calculation of the variance:

$$(\Delta A)^2 \equiv \langle\psi|(A-\langle A\rangle)^2|\psi\rangle$$

$$= \langle (A-\langle A\rangle)\psi | (A-\langle A\rangle)\psi\rangle$$

Similarly,

$$(\Delta B)^2 \equiv \langle\psi|(B-\langle B\rangle)^2|\psi\rangle$$

$$= \langle (B-\langle B\rangle)\psi | (B-\langle B\rangle)\psi\rangle$$

Define $|\psi_A\rangle \equiv |(A-\langle A\rangle)\psi\rangle$

$$|\psi_B\rangle \equiv |(B-\langle B\rangle)\psi\rangle$$

Schwarz inequality:

$$\langle\psi_A|\psi_A\rangle\langle\psi_B|\psi_B\rangle \geq |\langle\psi_A|\psi_B\rangle|^2$$

Define $\left|\psi_A\right\rangle \equiv \left|\left(A-\langle A\rangle\right)\psi\right\rangle$ and $\left|\psi_B\right\rangle \equiv \left|\left(B-\langle B\rangle\right)\psi\right\rangle$

Schwarz inequality:

$$\langle\psi_A|\psi_A\rangle\langle\psi_B|\psi_B\rangle \geq \left|\langle\psi_A|\psi_B\rangle\right|^2$$

$$\langle\psi_A|\psi_B\rangle = \langle\psi|\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right)|\psi\rangle$$

$$\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right) = \frac{1}{2}\left(\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right)+\left(B-\langle B\rangle\right)\left(A-\langle A\rangle\right)\right)$$

$$+\frac{1}{2}\left(\left(A-\langle A\rangle\right)\left(B-\langle B\rangle\right)-\left(B-\langle B\rangle\right)\left(A-\langle A\rangle\right)\right)$$

$$= F + \frac{i}{2}C$$

$$\langle \psi_A | \psi_B \rangle = \langle \psi | (A - \langle A \rangle)(B - \langle B \rangle) | \psi \rangle \qquad = \langle \psi | F | \psi \rangle + \frac{i}{2} \langle \psi | C | \psi \rangle$$

$$\left| \langle \psi_A | \psi_B \rangle \right|^2 = \left| \langle \psi | F | \psi \rangle \right|^2 + \frac{1}{4} \left| \langle \psi | C | \psi \rangle \right|^2 \geq \frac{1}{4} \left| \langle \psi | C | \psi \rangle \right|^2$$

Putting it all together:

$$\langle \psi_A | \psi_A \rangle \langle \psi_B | \psi_B \rangle \geq \left| \langle \psi_A | \psi_B \rangle \right|^2 \geq \frac{1}{4} \left| \langle \psi | C | \psi \rangle \right|^2$$

$$\Rightarrow (\Delta A)^2 (\Delta B)^2 \geq \frac{1}{4} \left| \langle C \rangle \right|^2$$

Therefore: $[A, B] = iC$ implies $\Delta A \Delta B \geq \frac{1}{2} \left| \langle C \rangle \right|$

Example: $A = X, \quad B = P$

$[X, P] = i\hbar$ implies $\Delta X \Delta P \geq \frac{\hbar}{2}$

What does this have to do with quantum EM fields?

In fact, your textbook shows that although

$$\left\langle n_{\mathbf{k}'\sigma'}\middle|\mathbf{E}(\mathbf{r},t)\middle|n_{\mathbf{k}'\sigma'}\right\rangle = 0 \quad \text{and} \quad \left\langle n_{\mathbf{k}'\sigma'}\middle|\mathbf{B}(\mathbf{r},t)\middle|n_{\mathbf{k}'\sigma'}\right\rangle = 0,$$

the variances of the fields are both infinite for a pure eigenstate --

$$\langle 0|\mathbf{E}^2(\mathbf{r})|0\rangle = \left|\mathbf{E}(\mathbf{r})|0\rangle\right|^2 = \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k}\sigma}\sum_{\mathbf{k}'\sigma'}\sqrt{\omega_k\omega_{k'}}\left(\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\cdot\boldsymbol{\varepsilon}^*_{\mathbf{k}'\sigma'}\right)e^{i\mathbf{k}\cdot\mathbf{r}-i\mathbf{k}'\cdot\mathbf{r}}\langle 1,\mathbf{k},\sigma|1,\mathbf{k}',\sigma'\rangle$$

$$= \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k}\sigma}\omega_k = \frac{\hbar c}{\varepsilon_0 V}\sum_{\mathbf{k}}k = \frac{\hbar c}{\varepsilon_0}\int\frac{d^3\mathbf{k}}{(2\pi)^3}k, \qquad (17.19\text{a})$$

infinite

$$\langle 0|\mathbf{B}^2(\mathbf{r})|0\rangle = \left|\mathbf{B}(\mathbf{r})|0\rangle\right|^2 = \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k},\sigma}\sum_{\mathbf{k}',\sigma'}\frac{e^{i\mathbf{k}\cdot\mathbf{r}-i\mathbf{k}'\cdot\mathbf{r}}}{\sqrt{\omega_k\omega_{k'}}}\left(\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\right)\cdot\left(\mathbf{k}'\times\boldsymbol{\varepsilon}^*_{\mathbf{k}'\sigma'}\right)\langle 1,\mathbf{k},\sigma|1,\mathbf{k}',\sigma'\rangle$$

$$= \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k},\sigma}\frac{\left|\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\right|^2}{\omega_k} = \frac{\hbar}{2\varepsilon_0 V}\sum_{\mathbf{k},\sigma}\frac{k^2}{\omega_k} = \frac{\hbar}{\varepsilon_0 V c}\sum_{\mathbf{k}}k = \frac{\hbar}{\varepsilon_0 c}\int\frac{d^3\mathbf{k}}{(2\pi)^3}k, \qquad (17.19\text{b})$$

infinite

It is also possible to show that components of the E and B field have nontrivial commutation relations, indicating that in general it is not possible to simultaneously determine E and B at the same point in space to arbitrary accuracy.

Effects of the phase of each mode.

In deriving these equations, we neglected the phase of each mode. A more careful treatment of photon number and phase show that these also have nontrivial commutation relations.

How is this quantum treatment of the electromagnetic fields consistent with the classical picture?

1. **There is no need for consistency.?**
2. **There should be consistency in certain ranges of the parameters.?**

Glauber's coherent state: $\left|c_\alpha\right\rangle \equiv \sum_{n=0}^{\infty} \frac{\alpha^n e^{-|\alpha|^2/2}}{\sqrt{n!}}\left|n\right\rangle$ based on a single mode $n \rightarrow n_{\mathbf{k}\sigma}$

Electric field: $$\left\langle c_\alpha\right|\mathbf{E}(\mathbf{r},t)\left|c_\alpha\right\rangle = i\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(\alpha_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - \alpha^*_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

Magnetic field: $$\left\langle c_\alpha\right|\mathbf{B}(\mathbf{r},t)\left|c_\alpha\right\rangle = i\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(\alpha_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - \alpha^*_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

Note that α is a complex number which can be written in terms of a real amplitude and phase: E_0 and ψ :

$$\left\langle c_\alpha\right|\mathbf{E}(\mathbf{r},t)\left|c_\alpha\right\rangle = -2\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}E_0\sin\left(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t+\psi\right)$$

$$\left\langle c_\alpha\right|\mathbf{B}(\mathbf{r},t)\left|c_\alpha\right\rangle = -2\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}E_0\sin\left(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t+\psi\right)$$

Let $\alpha = E_0 e^{i\Psi}$

Single mode coherent state continued

It can also be shown that

$$\langle c_\alpha | |\mathbf{E}(\mathbf{r},t)|^2 | c_\alpha \rangle = \frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}\left(4E_0^2 \sin^2(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t+\psi)+1\right)$$

Therefore

$$\langle c_\alpha | |\mathbf{E}(\mathbf{r},t)|^2 | c_\alpha \rangle - |\langle c_\alpha | \mathbf{E}(\mathbf{r},t) | c_\alpha \rangle|^2 = \frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}$$

This means that variance of the E field for the coherent state is independent of the amplitude E_0. Therefore, for large E_0 the variance is small in comparison.

Visualization of coherent state electric fields for various amplitudes

Source: Rodney Loudon, "The Quantum Theory of Light"

The quantized radiation field **151**

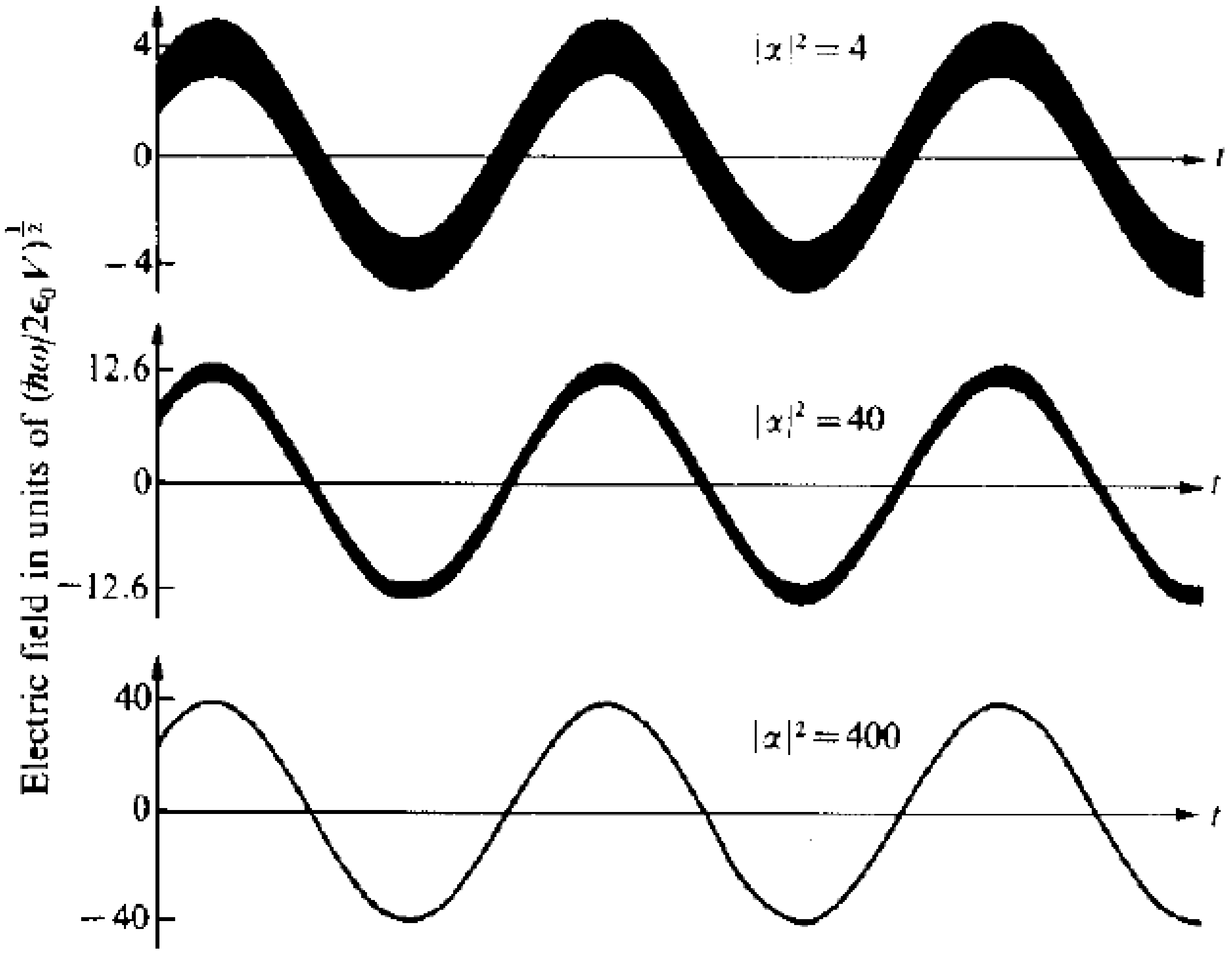

FIG. 4.3. Pictorial representation of the electric-field variation in a cavity mode excited to state $|\alpha\rangle$. Three different values of the mean photon number $|\alpha|^2$ are shown, the vertical scales being different for the three cases. The uncertainties in field values are indicated by the vertical widths $2\Delta E$ of the sine waves. These widths can also be regarded as combinations of the amplitude uncertainty associated with Δn and the phase uncertainty associated with $\Delta \cos\phi$.

Single mode coherent state continued

Now consider the expectation values of the number operator and its square:

$$\mathbf{N}_{\mathbf{k}\sigma} \equiv a^{\dagger}_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}$$

$$\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle = |\alpha|^2 \qquad \langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma}\mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle = |\alpha|^4 + |\alpha|^2$$

Square of the variance: $\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma}\mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle - \left|\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle\right|^2 = |\alpha|^2$

Fractional uncertainty in the number of photons for the coherent state:

$$\frac{\sqrt{\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma}\mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle - \left|\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle\right|^2}}{\langle c_\alpha | \mathbf{N}_{\mathbf{k}\sigma} | c_\alpha \rangle} = \frac{1}{|\alpha|}$$

Interpretation of a single mode coherent state

$$\left|c_\alpha\right\rangle \equiv \sum_{n=0}^{\infty} \frac{\alpha^n e^{-|\alpha|^2/2}}{\sqrt{n!}} \left|n\right\rangle$$ based on a single mode $n \rightarrow n_{\mathbf{k}\sigma}$

The probability of finding n photons in this state is given by:

$$\left|\left\langle n \middle| c_\alpha \right\rangle\right|^2 = \frac{|\alpha|^{2n} e^{-|\alpha|^2}}{n!}$$

This is the form of a Poisson distribution

for a mean value of $|\alpha|^2$.

More reading -- **page 231**

REVIEWS OF

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Coherence Properties of Optical Fields*

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This article presents a review of coherence properties of electromagnetic fields and their measurements, with special emphasis on the optical region of the spectrum. Analyses based on both the classical and quantum theories are described. After a brief historical introduction, the elementary concepts which are frequently employed in the discussion of interference phenomena are summarized. The measure of second-order coherence is then introduced in connection with the analysis of a simple interference experiment and some of the more important second-order coherence effects are studied. Their uses in stellar interferometry and interference spectroscopy are described. Analysis of partial polarization from the standpoint of correlation theory is also outlined. The general statistical description of the field is discussed in some detail. The recently discovered universal "diagonal" representation of the density operator for free fields is also considered and it is shown how, with the help of the associated generalized phase-space distribution function, the quantum-mechanical correlation functions may be expressed in the same form as the classical ones. The sections which follow deal with the statistical properties of thermal and nonthermal light, and with the temporal and spatial coherence of black-body radiation. Later sections, dealing with fourth- and higher-order coherence effects include a discussion of the photoelectric detection process. Among the fourth-order effects described in detail are bunching phenomena, the Hanbury Brown–Twiss effect and its application to astronomy. The article concludes with a discussion of various transient superposition effects, such as light beats and interference fringes produced by independent light beams.