PHY 712 Electrodynamics
10-10:50 AM MWF in Olin 103

Class notes for Lecture 13:

Continue reading Chapter 5

- **A. Examples of magnetostatic fields**
- **B. Magnetic dipoles**
- **C. Hyperfine interaction**

PHYSICS COLLOQUIUM

THURSDAY

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FEBRUARY 9, 2023

What spectral line-shapes can reveal

Spectroscopy provides us a window into the quantum dynamics of system. Traditionally, we focus upon the transition frequencies since this reveals the energy eigenspectrum and details of the symmetry and structure of the system we are trying to study. And for most purposes, this is sufficient. However, the spectral line shape provides us an additional window into the environment surrounding the system. In my colloquium, I'll discuss a general theory of spectral line shapes and in particular discuss our recent efforts into understanding how non-stationary/non-equilibrium many-body effects are manifest in multi-dimensional/ultrafast spectroscopy--especially when applied to semiconducting systems. I will highlight our theoretical efforts against recent

Eric R. Bittner, Ph.D.

Moores Professor of Chemical Physics,

University of Houston

4:00 pm - Olin 101*

Note: For additional information on the seminar, contact wfuphys@wfu.edu


Ellen Ochoa
Chien-Shung Wu
Mae Jemison
Annie Easley
marking the
International
Day
of
Women
and
Girls
in Science
the Physics department will
host a panel discussion with
faculty and students
Friday, 2/10
11:30 am– 1:00 pm, Olin 105
Free
refreshments
served by
Panera
BREAD

Course schedule for Spring 2023

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/9/2023	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/13/2023
2	Wed: 01/11/2023	Chap. 1	Electrostatic energy calculations	#2	01/18/2023
3	Fri: 01/13/2023	Chap. 1	Electrostatic energy calculations thanks to Ewald	#3	01/18/2023
	Mon: 01/16/2023		MLK Holiday -- no class		
4	Wed: 01/18/2023	Chap. 1 & 2	Electrostatic potentials and fields	#4	01/20/2023
5	Fri: 01/20/2023	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions	#5	01/23/2023
6	Mon: 01/23/2023	Chap. 1 - 3	Brief introduction to numerical methods	#6	01/25/2023
7	Wed: 01/25/2023	Chap. 2 & 3	Image charge constructions	#7	01/30/2023
8	Fri: 01/27/2023	Chap. 2 & 3	Cylindrical and spherical geometries		
9	Mon: 01/30/2023	Chap. 3 & 4	Spherical geometry and multipole moments	#8	02/01/2023
10	Wed: 02/01/2023	Chap. 4	Dipoles and Dielectrics	#9	02/03/2023
11	Fri: 02/03/2023	Chap. 4	Dipoles and Dielectrics	#10	02/06/2023
12	Mon: 02/06/2023	Chap. 5	Magnetostatics	#11	02/08/2023
13	Wed: 02/08/2023	Chap. 5	Magnetic dipoles and hyperfine interaction	#12	02/10/2023
14	Fri: 02/10/2023	Chap. 5	Magnetic dipoles and dipolar fields		
15	Mon: 02/13/2023	Chap. 6	Maxwell's Equations		

PHY 712 -- Assignment #12

February 08, 2023

Continue reading Chapter 5 in **Jackson** .

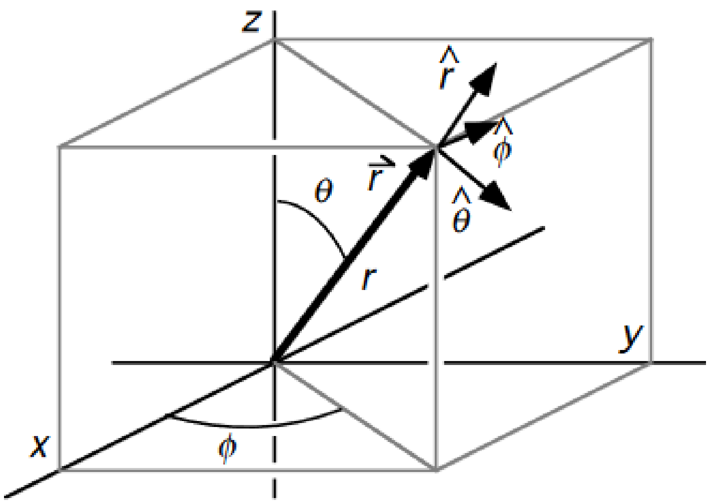
1. Consider the following equation and verify its validity (or otherwise).

$$\int d\Omega' \sum_{lm} Y_{lm}(\hat{\mathbf{r}}) Y^*_{lm}(\hat{\mathbf{r}'})\, \mathbf{r}' = \frac{r'}{r}\mathbf{r} \equiv r'\, \hat{\mathbf{r}}\ .$$

Comment about spherical polar coordinates

Ref: https://www.cpp.edu/~ajm/materials/delsph.pdf

Spherical Coordinates



Transforms

The forward and reverse coordinate transformations are

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arctan\left(\sqrt{x^2 + y^2}, z\right)$$

$$\phi = \arctan(y, x)$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

where we *formally* take advantage of the *two argument* arctan function to eliminate quadrant confusion.

Unit Vectors

The unit vectors in the spherical coordinate system are functions of position. It is convenient to express them in terms of the *spherical* coordinates and the unit vectors of the *rectangular* coordinate system which are *not* themselves functions of position.

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\hat{\phi} = \frac{\hat{z}\times\hat{r}}{\sin\theta} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$$\hat{\theta} = \hat{\phi}\times\hat{r} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$$

Lecture Notes

- Lecture 1 -- Introduction and electrostatics PP slides PDF Extra PP Extra PDF
- Lecture 2 -- Evaluation of electrostatic energy PP slides PDF Ewald notes PDF Extra PP Extra PDF
- Lecture 3 -- Ewald summation methods PP slides PDF Ewald notes PDF CsCl Maple input CsCl PDF Extra PP Extra PDF
- Lecture 4 -- Electrostatic potentials and fields PP slides PDF Extra PP Extra PDF
- Lecture 5 -- Poisson equation in 2 and 3 dimensions PP slides PDF Extra PP Extra PDF
- Lecture 6 -- Short introduction to numerical methods PP slides PDF Detailed notes (PDF)
- Lecture 7 -- Image charge tricks PP slides PDF Extra PP Extra PDF
- Lecture 8 -- Laplace/Poisson equation in cylindrical coordinates PP slides PDF Extra PP Extra PDF
- Lecture 9 -- Laplace/Poisson equation in spherical coordinates PP slides PDF
- Lecture 10 -- Dipole moments and dielectrics PP slides PDF Extra PP Extra PDF
- Lecture 11 -- Microscopic and macroscopic dipolar effects PP slides PDF Extra PP Extra PDF
- Lecture 12 -- Magnetostatics PP slides PDF Extra PP Extra PDF
- Lecture 13 -- Magnetic dipoles PP slides PDF Detailed PDF

For the model current density --

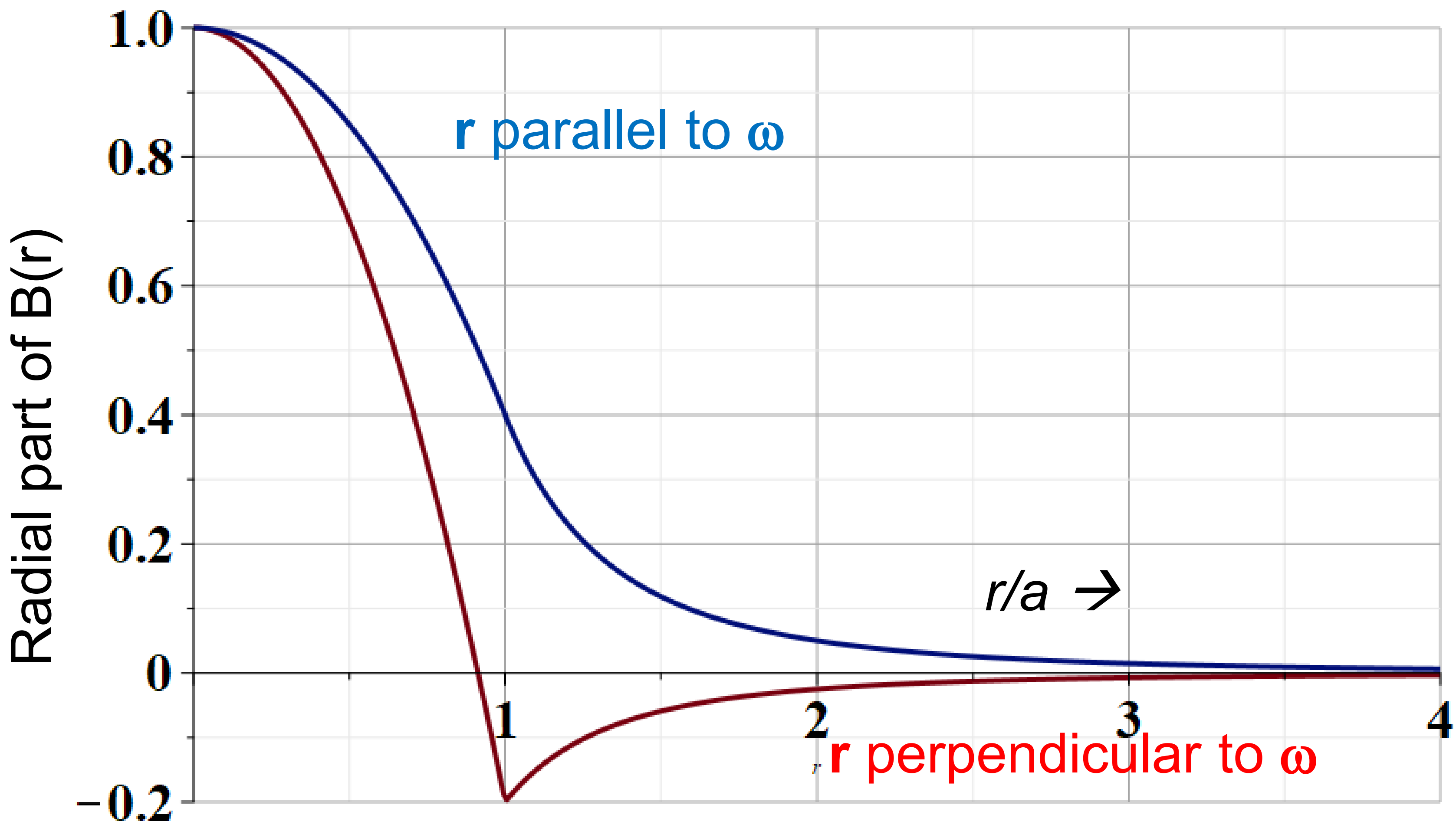
$$\mathbf{J}(\mathbf{r}) = \begin{cases} \rho_0 \boldsymbol{\omega} \times \mathbf{r} & \text{for r } \leq a \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{A}(\mathbf{r}) = \begin{cases} \frac{\mu_0 \rho_0}{3} \boldsymbol{\omega} \times \mathbf{r} \left(\frac{a^2}{2} - \frac{3r^2}{10} \right) & \text{for } r \leq a \\ \\ \frac{\mu_0 \rho_0}{3} \boldsymbol{\omega} \times \mathbf{r} \frac{a^5}{5r^3} & \text{for } r \geq a \end{cases} .$$

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) = \begin{cases} \frac{\mu_0 \rho_0}{3} \left[\boldsymbol{\omega} \left(a^2 - \frac{6}{5} r^2 \right) + \frac{3}{5} \mathbf{r} (\boldsymbol{\omega} \cdot \mathbf{r}) \right] & \text{for } r \leq a \\ \frac{\mu_0 \rho_0}{3} \left[-\boldsymbol{\omega} \frac{a^5}{5r^3} + \frac{3a^5}{5r^5} \mathbf{r} (\boldsymbol{\omega} \cdot \mathbf{r}) \right] & \text{for } r \geq a \end{cases}$$

$$\mathbf{A}(\mathbf{r}) = \begin{cases} \dfrac{\mu_0 \rho_0}{3} \boldsymbol{\omega} \times \mathbf{r} \left(\dfrac{a^2}{2} - \dfrac{3r^2}{10} \right) & \text{for } r \le a \\ \\ \dfrac{\mu_0 \rho_0}{3} \boldsymbol{\omega} \times \mathbf{r} \dfrac{a^5}{5r^3} & \text{for } r \ge a \end{cases} .$$

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) = \begin{cases} \frac{\mu_0 \rho_0}{3} \left[\boldsymbol{\omega} \left(a^2 - \frac{6}{5} r^2 \right) + \frac{3}{5} \mathbf{r}(\boldsymbol{\omega} \cdot \mathbf{r}) \right] & \text{for } r \leq a \\ \frac{\mu_0 \rho_0}{3} \left[-\boldsymbol{\omega} \frac{a^5}{5r^3} + \frac{3a^5}{5r^5} \mathbf{r}(\boldsymbol{\omega} \cdot \mathbf{r}) \right] & \text{for } r \geq a \end{cases}$$



Various forms of Ampere's law :

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

$$\text{Vector potential:} \quad \mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

$$\text{For Coulomb gauge:} \quad \nabla \cdot \mathbf{A}(\mathbf{r}) = 0$$

$$\Rightarrow \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r})$$

$$\text{For confined current density:}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3 r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Other examples of current density sources:

Quantum mechanical expression for current density

for a particle of mass M and charge e and of probability amplitude $\Psi(\mathbf{r})$:

$$\mathbf{J}(\mathbf{r}) = -\frac{e\hbar}{2Mi}\left(\Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r}) - \Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r})\right)$$

For an electron in a spherical potential (such as in an atom):

$$\Psi(\mathbf{r}) \equiv \Psi_{nlm_l}(\mathbf{r}) = R_{nl}(r)Y_{lm_l}(\hat{\mathbf{r}})$$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{2Mi}\left|R_{nl}(r)\right|^2 \frac{1}{r\sin\theta}\left(Y_{lm_l}{}^*(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}(\hat{\mathbf{r}})}{\partial\varphi} - Y_{lm_l}(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}{}^*(\hat{\mathbf{r}})}{\partial\varphi}\right)\hat{\boldsymbol{\varphi}}$$

$$= \frac{e\hbar}{M}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})^2\right|\hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}} = \dfrac{\hat{\mathbf{z}}\times\mathbf{r}}{r\sin\theta}$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{M}\frac{m_l}{r^2\sin^2\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2(\hat{\mathbf{z}}\times\mathbf{r})$$

Details of the electron orbital magnetic dipole moment

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{m_e}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})^2\right|\hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}}$

Magnetic dipole moment:

$$\mathbf{m} = \frac{1}{2}\int d^3r'\mathbf{r}'\times\mathbf{J}(\mathbf{r}') = -\frac{e\hbar m_l}{2m_e}\int d^3r'\frac{\mathbf{r}'\times\hat{\boldsymbol{\varphi}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})^2\right|$$

$$= -\frac{e\hbar m_l}{2m_e}\int d^3r'\frac{-r'\hat{\boldsymbol{\theta}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})^2\right|$$

Note that: $\hat{\boldsymbol{\theta}} = \cos\theta\cos\varphi\,\hat{\mathbf{x}} + \cos\theta\sin\varphi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$

$$\mathbf{m} = -\frac{e\hbar m_l\hat{\mathbf{z}}}{2m_e}\int d^3r'\left|\Psi_{nlm_l}(\mathbf{r})^2\right|$$

$$= -\frac{e\hbar m_l}{2m_e}\hat{\mathbf{z}}$$

Significance of magnetic dipole – multipole approximation to vector potential and magnetic field

Magnetic dipolar field

The magnetic dipole moment is defined by

$$\mathbf{m} = \frac{1}{2}\int d^3r'\,\mathbf{r}' \times \mathbf{J}(\mathbf{r}'),$$

with the corresponding potential

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2},$$

and magnetostatic field

$$\mathbf{B}_{\mathbf{m}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left\{\frac{3\hat{\mathbf{r}}(\mathbf{m}\cdot\hat{\mathbf{r}}) - \mathbf{m}}{r^3} + \frac{8\pi}{3}\mathbf{m}\delta^3(\mathbf{r})\right\}.$$

Summary of magnetic field generated by point magnetic dipole moment discussed in the detailed notes:

$$\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\hat{\mathbf{r}}\left(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}\right) - \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_e \delta(\mathbf{r})\right)$$

Magnetic field near nucleus due to orbiting electron:

$$\mathbf{B}_O(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{e}{m_e} L_z \hat{\mathbf{z}} \left\langle \frac{1}{r^3} \right\rangle$$

"Hyperfine" interaction energy:

$$\mathcal{H}_{HF} = -\boldsymbol{\mu}_N \cdot \left(\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r}) + \mathbf{B}_O(\mathbf{r})\right)$$

$$= \frac{\mu_0}{4\pi}\left(\frac{3\left(\boldsymbol{\mu}_N \cdot \hat{\mathbf{r}}\right)\left(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}\right) - \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e \delta(\mathbf{r}) + \frac{e}{m_e}\left\langle \frac{\mathbf{L} \cdot \boldsymbol{\mu}_N}{r^3} \right\rangle\right)$$

$$\mathcal{H}_{HF} = \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{\mu}_N \cdot \hat{\mathbf{r}})(\boldsymbol{\mu}_e \cdot \hat{\mathbf{r}}) - \boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_N \cdot \boldsymbol{\mu}_e \delta(\mathbf{r}) + \frac{e}{m_e}\left\langle \frac{\mathbf{L} \cdot \boldsymbol{\mu}_N}{r^3} \right\rangle \right)$$