PHY 712 Electrodynamics
10-10:50 AM in Olin 103

Discussion about Lecture 17:

Read Chapter 7

1. **Plane polarized electromagnetic waves**
2. **Reflectance and transmittance of electromagnetic waves – extension to anisotropy and complexity**

11	Fri: 02/03/2023	Chap. 4	Dipoles and Dielectrics	#10	02/06/2023
12	Mon: 02/06/2023	Chap. 5	Magnetostatics	#11	02/08/2023
13	Wed: 02/08/2023	Chap. 5	Magnetic dipoles and hyperfine interaction	#12	02/10/2023
14	Fri: 02/10/2023	Chap. 5	Magnetic dipoles and dipolar fields	#13	02/13/2023
15	Mon: 02/13/2023	Chap. 6	Maxwell's Equations	#14	02/17/2023
16	Wed: 02/15/2023	Chap. 6	Electromagnetic energy and forces		
17	Fri: 02/17/2023	Chap. 7	Electromagnetic plane waves	#15	02/20/2023
18	Mon: 02/20/2023	Chap. 7	Electromagnetic plane waves		
19	Wed: 02/22/2023	Chap. 7	Optical effects of refractive indices		
20	Fri: 02/24/2023	Chap. 1-7	Review		

PHY 712 -- Assignment #15

February 17, 2023

Start reading Chapter 7 in **Jackson** .

1. Consider the reflectivity of a plane polarized electromagnetic wave incident from air (*n*=1) on a material with refractive index *n'*=1.5 at an angle of incidence *i*. Assume μ=μ'. Plot the reflectance

   $R(i)=|E''_0/E_0|^2$

   as a function of *i* for $0 \leq i \leq 90$ deg for both cases of polarization of ($\mathbf{E}_0$ in the plane of incidence or perpendicular to the plane of incidence). What is the qualitative difference between the two cases?

Comment on schedule --

14	Fri: 02/10/2023	Chap. 5	Magnetic dipoles and dipolar fields	#13	02/13/2023
15	Mon: 02/13/2023	Chap. 6	Maxwell's Equations	#14	02/17/2023
16	Wed: 02/15/2023	Chap. 6	Electromagnetic energy and forces		
17	Fri: 02/17/2023	Chap. 7	Electromagnetic plane waves	#15	02/20/2023
18	Mon: 02/20/2023	Chap. 7	Electromagnetic plane waves		
19	Wed: 02/22/2023	Chap. 7	Optical effects of refractive indices		
20	Fri: 02/24/2023	Chap. 1-7	Review		

It will be great if all outstanding HW's are turned in by this date.

Maxwell's equations

For linear isotropic media and no sources: $\mathbf{D} = \varepsilon \mathbf{E}; \quad \mathbf{B} = \mu \mathbf{H}$

Coulomb's law: $\nabla \cdot \mathbf{E} = 0$

Ampere-Maxwell's law: $\nabla \times \mathbf{B} - \mu\varepsilon \frac{\partial \mathbf{E}}{\partial t} = 0$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

Analysis of Maxwell's equations without sources -- continued:

$$\text{Coulomb's law:} \qquad \nabla \cdot \mathbf{E} = 0$$

$$\text{Ampere-Maxwell's law:} \qquad \nabla \times \mathbf{B} - \mu\varepsilon \frac{\partial \mathbf{E}}{\partial t} = 0$$

$$\text{Faraday's law:} \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\text{No magnetic monopoles:} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \left(\nabla \times \mathbf{B} - \mu\varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) = -\nabla^2 \mathbf{B} - \mu\varepsilon \frac{\partial (\nabla \times \mathbf{E})}{\partial t}$$

$$= -\nabla^2 \mathbf{B} + \mu\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$\nabla \times \left(\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} \right) = -\nabla^2 \mathbf{E} + \frac{\partial (\nabla \times \mathbf{B})}{\partial t}$$

$$= -\nabla^2 \mathbf{E} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Analysis of Maxwell's equations without sources -- continued:

Both E and B fields are solutions to a wave equation:

$$\nabla^2\mathbf{B} - \frac{1}{v^2}\frac{\partial^2\mathbf{B}}{\partial t^2} = 0$$

$$\nabla^2\mathbf{E} - \frac{1}{v^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$

$$\text{where} \quad v^2 \equiv c^2\frac{\mu_0\varepsilon_0}{\mu\varepsilon} \equiv \frac{c^2}{n^2}$$

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

Analysis of Maxwell's equations without sources -- continued:

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r},t)=\Re\left(\mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2=\left(\frac{\omega}{v}\right)^2=\left(\frac{n\omega}{c}\right)^2 \qquad \text{where } n\equiv\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

Note: ε, μ, n, k can all be complex; for the moment we will assume that they are all real (no dissipation).

Note that $\mathbf{E}_0$ and $\mathbf{B}_0$ are not independent;

$$\mathbf{k}=n\frac{\omega}{c}\hat{\mathbf{k}}$$

from Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

$$\Rightarrow \mathbf{B}_0=\frac{\mathbf{k}\times\mathbf{E}_0}{\omega}=\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}$$

also note: $\hat{\mathbf{k}}\cdot\mathbf{E}_0=0$ and $\hat{\mathbf{k}}\cdot\mathbf{B}_0=0$

Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

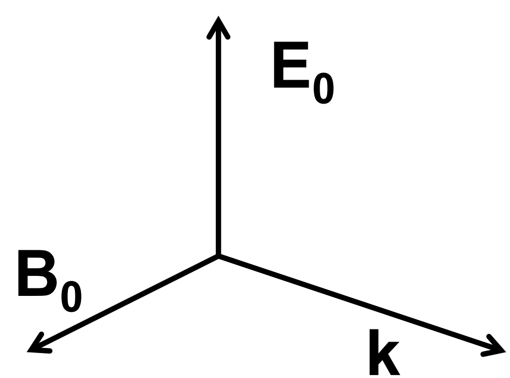
$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$$

$$\mathbf{B}_0 = \frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c} \quad \text{and} \quad \hat{\mathbf{k}}\cdot\mathbf{B}_0 = 0$$

Poynting vector and energy density:

$$\langle\mathbf{S}\rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\hat{\mathbf{k}}$$

$$\langle u\rangle_{avg} = \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$



Detailed analysis of reflection and refraction of plane polarized electromagnetic waves

Note that apparently, Fresnel deduced properties of reflection and refraction before Maxwell's equations were published.

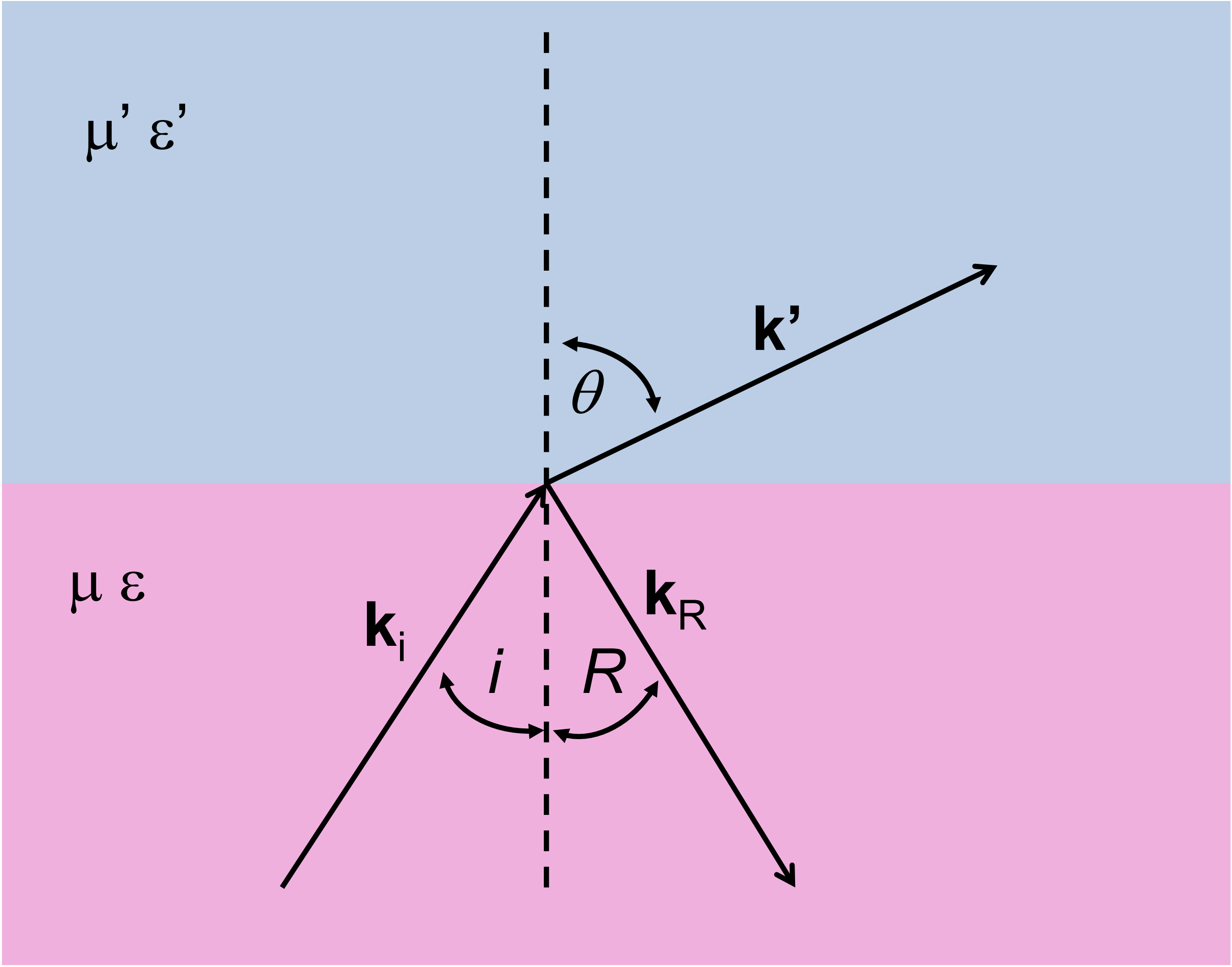
However, we can see how these results follow directly from Maxwell's equations.

Augustin-Jean Fresnel

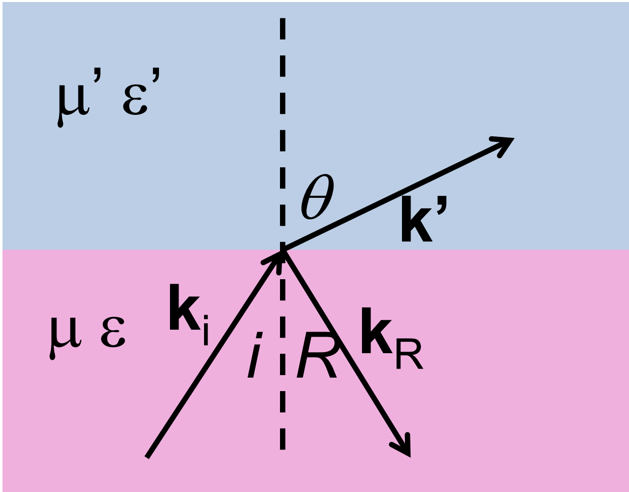
Portrait of "Augustin Fresnel" from the frontispiece of his collected works (1866)

1788-1827

Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



Reflection and refraction -- continued



In medium $\mu'\varepsilon'$:

$$\mathbf{E}'(\mathbf{r},t) = \Re\left(\mathbf{E}'_0\, e^{i\frac{\omega}{c}\left(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}'(\mathbf{r},t) = \frac{n'}{c}\hat{\mathbf{k}}'\times\mathbf{E}'(\mathbf{r},t) = \sqrt{\mu'\varepsilon'}\hat{\mathbf{k}}'\times\mathbf{E}(\mathbf{r},t)$$

In medium $\mu\varepsilon$:

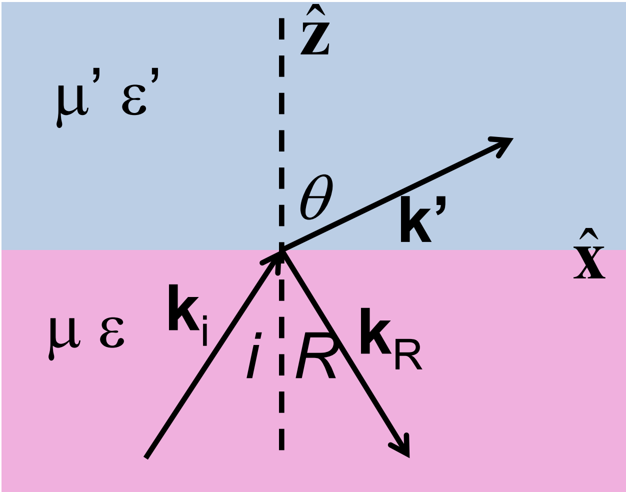
$$\mathbf{E}_i(\mathbf{r},t) = \Re\left(\mathbf{E}_{0i} e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_i(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t) = \sqrt{\mu\varepsilon}\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t)$$

$$\mathbf{E}_R(\mathbf{r},t) = \Re\left(\mathbf{E}_{0R} e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_R(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t) = \sqrt{\mu\varepsilon}\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t)$$

Reflection and refraction -- continued



Snell's law – matching phase factors at boundary plane *z=0*.

$$\left. e^{i\frac{\omega}{c}\left(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct\right)}\right|_{z=0} = \left. e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct\right)}\right|_{z=0}$$

$$= \left. e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct\right)}\right|_{z=0}$$

$$\text{matching plane:}\quad \mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$$

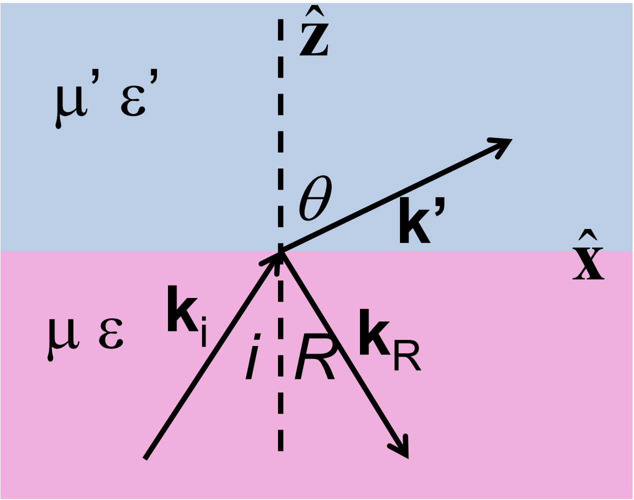
$$\hat{\mathbf{k}}'\cdot\mathbf{r} = x\sin\theta$$

$$\hat{\mathbf{k}}_i\cdot\mathbf{r} = x\sin i = \hat{\mathbf{k}}_R\cdot\mathbf{r} = x\sin R \quad \Rightarrow i = R$$

$$n'\hat{\mathbf{k}}'\cdot\mathbf{r} = n\hat{\mathbf{k}}_i\cdot\mathbf{r} \quad \Rightarrow n'x\sin\theta = nx\sin i$$

$$\text{Snell's law}:\quad n'\sin\theta = n\sin i$$

Reflection and refraction -- continued



Continuity equations at boundary with no sources:

$$\nabla \cdot \mathbf{D} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

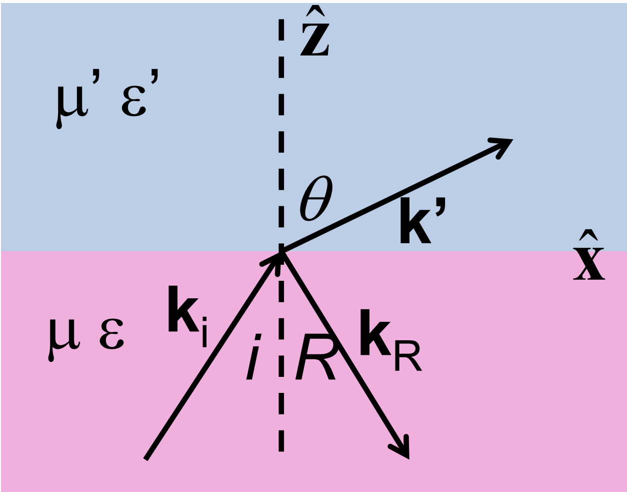
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0 \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

Matching field amplitudes at boundary plane:

$\mathbf{D} \cdot \hat{\mathbf{z}}, \ \mathbf{B} \cdot \hat{\mathbf{z}}$ continuous

$\mathbf{H} \times \hat{\mathbf{z}}, \mathbf{E} \times \hat{\mathbf{z}}$ continuous

Reflection and refraction -- continued



Matching field amplitudes at boundary plane:

$\mathbf{E}\times\hat{\mathbf{z}}$ continuous:

$$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}$$

$$=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$\mathbf{D}\cdot\hat{\mathbf{z}}$ continuous:

$$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

$\mathbf{B}\cdot\hat{\mathbf{z}}$ continuous:

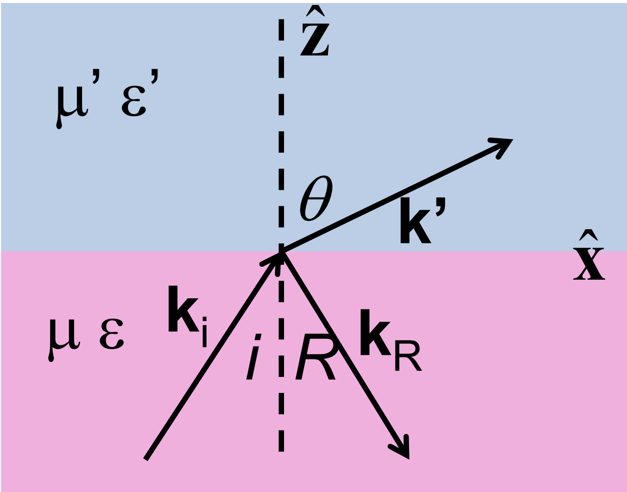
$$n\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=$$

$$n'\hat{\mathbf{k}}'\times\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

Known: $\mathbf{E}_{0i},\hat{\mathbf{k}}_i$

Unknown: $\mathbf{E}'_0,\mathbf{E}_{0R},\hat{\mathbf{k}}'$

Reflection and refraction -- continued



s-polarization – **E** field "polarized" perpendicular to plane of incidence

$$\mathbf{E}\times\hat{\mathbf{z}} \quad \text{continuous:}$$

$$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$$

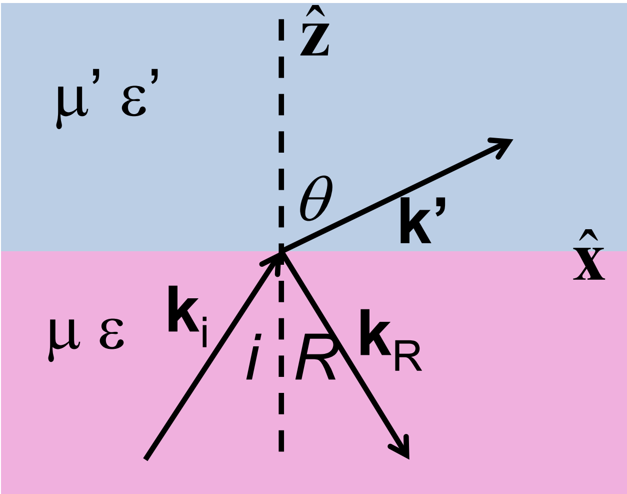
$$\mathbf{H}\times\hat{\mathbf{z}} \quad \text{continuous:}$$

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{n\cos i-\frac{\mu}{\mu'}n'\cos\theta}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta} \qquad \frac{E'_0}{E_{0i}}=\frac{2n\cos i}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta}$$

$$\text{Note that:}\quad n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

Reflection and refraction -- continued



p-polarization – **E** field "polarized" parallel to plane of incidence

$\mathbf{D}\cdot\hat{\mathbf{z}}$ continuous:

$$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

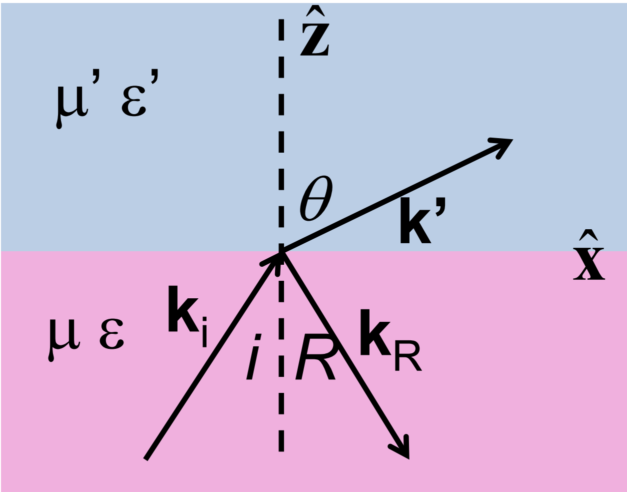
$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{\frac{\mu}{\mu'}n'\cos i-n\cos\theta}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta}\qquad\frac{E'_0}{E_{0i}}=\frac{2n\cos i}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta}$$

$$\text{Note that}:\quad n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

Reflection and refraction -- continued



Intensity in terms of Poynting vector:

$$\langle \mathbf{S} \rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\,\hat{\mathbf{k}}$$

Reflectance, transmittance:

$$R = \left|\frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}}\right| = \left|\frac{E_{0R}}{E_{0i}}\right|^2 \qquad T = \frac{\mathbf{S}'\cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i}$$

Note that $R + T = 1$

For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'} n'\cos\theta}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta}$$

$$\text{Note that: } \quad n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$$

$$R = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{n\cos i - \frac{\mu}{\mu'} n'\cos\theta}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta}\right|^2$$

$$T = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i} = \frac{4nn'\cos i\cos\theta}{\left|n\cos i + \frac{\mu}{\mu'} n'\cos\theta\right|^2}\frac{\mu}{\mu'}$$

For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos\theta}{\frac{\mu}{\mu'} n' \cos i + n \cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos\theta}$$

Note that : $n' \cos\theta = \sqrt{n'^2 - n^2 \sin^2 i}$

$$R = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{\frac{\mu}{\mu'} n' \cos i - n \cos\theta}{\frac{\mu}{\mu'} n' \cos i + n \cos\theta} \right|^2$$

$$T = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n'}{n} \frac{\mu}{\mu'} \frac{\cos\theta}{\cos i} = \frac{4nn' \cos i \cos\theta}{\left| \frac{\mu}{\mu'} n' \cos i + n \cos\theta \right|^2} \frac{\mu}{\mu'}$$

Special case: normal incidence (i=0, θ=0)

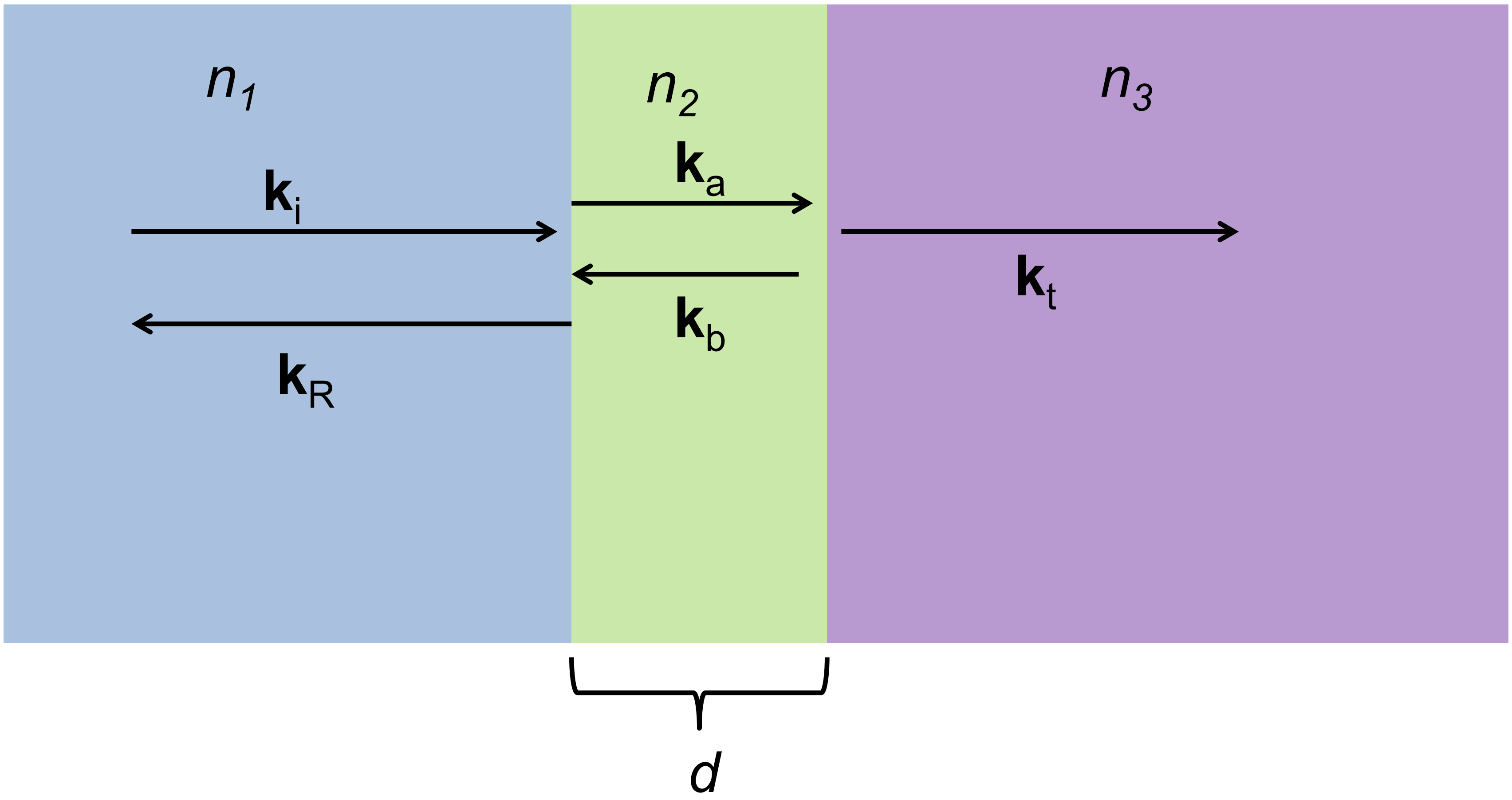
$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n'-n}{\frac{\mu}{\mu'}n'+n} \qquad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'}n'+n}$$

Reflectance, transmittance:

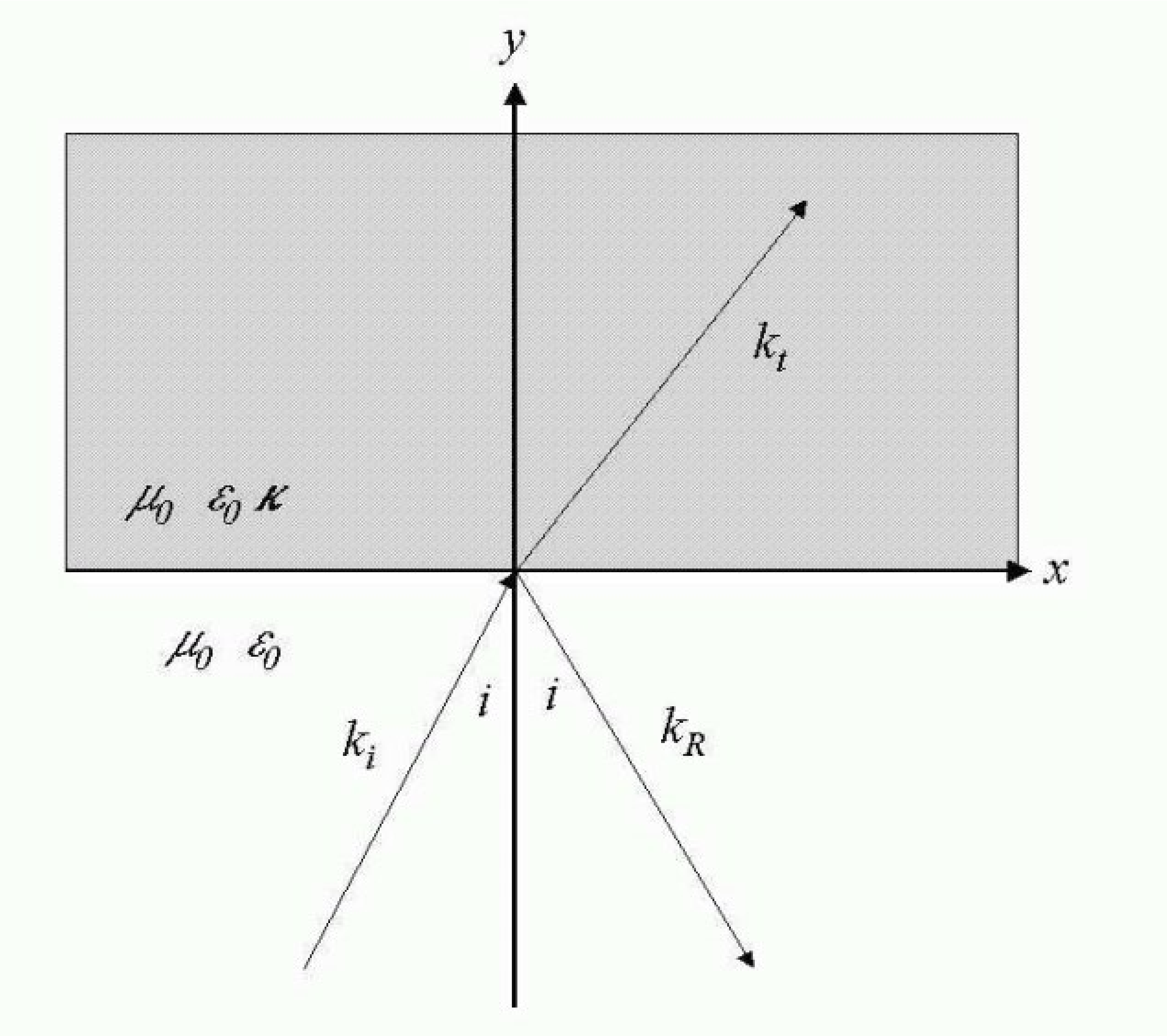
$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'-n}{\frac{\mu}{\mu'}n'+n}\right|^2$$

$$\mathrm{T} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'} = \left|\frac{2n}{\frac{\mu}{\mu'}n'+n}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}$$

Multilayer dielectrics (Problem #7.2)



Extension of analysis to anisotropic media --



Consider the problem of determining the reflectance from an anisotropic medium with isotropic permeability μ_0 and anisotropic permittivity $\varepsilon_0 \boldsymbol{\kappa}$ where:

$$\boldsymbol{\kappa} \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}$$

By assumption, the wave vector in the medium is confined to the *x*-*y* plane and will be denoted by

$$\mathbf{k}_t \equiv \frac{\omega}{c}(n_x \hat{\mathbf{x}} + n_y \hat{\mathbf{y}}), \text{where } n_x \text{ and } n_y \text{ are to be determined.}$$

The electric field inside the medium is given by:

$$\mathbf{E} = (E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{z}}) \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

Inside the anisotropic medium, Maxwell's equations are:

$$\nabla \cdot \mathbf{H} = 0 \qquad \nabla \cdot \boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} - i\omega\mu_0 \mathbf{H} = 0 \qquad \nabla \times \mathbf{H} + i\omega\epsilon_0 \boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

After some algebra, the equation for **E** is:

$$\begin{pmatrix} \kappa_{xx} - n_y^2 & n_x n_y & 0 \\ n_x n_y & \kappa_{yy} - n_x^2 & 0 \\ 0 & 0 & \kappa_{zz} - (n_x^2 + n_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0.$$

From **E, H** can be determined from

$$\mathbf{H} = \frac{1}{\mu_0 c} \left\{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) + (E_y n_x - E_x n_y) \hat{\mathbf{z}} \right\} \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

The fields for the incident and reflected waves are the same as for the isotropic case.

$$\mathbf{k}_i = \frac{\omega}{c}(\sin i\hat{\mathbf{x}} + \cos i\hat{\mathbf{y}}),$$

$$\mathbf{k}_R = \frac{\omega}{c}(\sin i\hat{\mathbf{x}} - \cos i\hat{\mathbf{y}}).$$

Note that, consistent with Snell's law: $n_x = \sin i$

Continuity conditions at the *y=0* plane must be applied for the following fields:

$$\mathbf{H}(x,0,z,t),\ \ E_x(x,0,z,t),\ \ E_z(x,0,z,t),\ \text{and}\ \ D_y(x,0,z,t).$$

There will be two different solutions, depending of the polarization of the incident field.

Solution for s-polarization

$$E_x = E_y = 0 \quad \Rightarrow \; n_y^2 = \kappa_{zz} - n_x^2 \qquad\qquad n_x = \sin i$$

$$\mathbf{E} = E_z \hat{\mathbf{z}} \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t} \qquad \mathbf{H} = \frac{1}{\mu_0 c}\left\{E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}})\right\} \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}$$

E_z must be determined from the continuity conditions:

$$E_0 + E_0'' = E_z \qquad (E_0 - E_0'')\cos i = E_z n_y \qquad (E_0 + E_0'')\sin i = E_z n_x$$

$$\frac{E_0''}{E_0} = \frac{\cos i - n_y}{\cos i + n_y}.$$

Solution for p-polarization

$$E_z = 0 \qquad \Rightarrow \; n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}}(\kappa_{yy} - n_x^2).$$

$$\mathbf{E} = E_x \left(\hat{\mathbf{x}} - \frac{\kappa_{xx} n_x}{\kappa_{yy} n_y} \hat{\mathbf{y}} \right) e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

$$\mathbf{H} = -\frac{E_x}{\mu_0 c} \frac{\kappa_{xx}}{n_y} \hat{\mathbf{z}} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

E_x must be determined from the continuity conditions:

$$(E_0 - E_0'')\cos i = E_x \qquad (E_0 + E_0'') = \frac{\kappa_{xx}}{n_y} E_x \qquad (E_0 + E_0'')\sin i = \frac{\kappa_{xx} n_x}{n_y} E_x.$$

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx}\cos i - n_y}{\kappa_{xx}\cos i + n_y}.$$

Extension of analysis to complex dielectric functions

For simplicity assume that $\mu = \mu_0$

Suppose the dielectric function is complex:

$$\varepsilon = \varepsilon_R + i\varepsilon_I \qquad \frac{\varepsilon}{\varepsilon_0} = \left(n_R + in_I\right)^2 \equiv \alpha + i\beta$$

$$n_R = \left(\frac{\sqrt{\alpha^2+\beta^2}+\alpha}{2}\right)^{1/2} \qquad n_I = \left(\frac{\sqrt{\alpha^2+\beta^2}-\alpha}{2}\right)^{1/2}$$

$$\mathbf{E}\left(\mathbf{r},t\right) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n_R\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right)e^{-\frac{\omega}{c}n_I\hat{\mathbf{k}}\cdot\mathbf{r}}$$