PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Notes for Lecture 1:

Reading: Appendix 1 and Chapters I&1

1. **Course structure and expectations**
2. **Units – SI vs Gaussian**
3. **Electrostatics – Poisson equation**

Spring 2023 Schedule
for N. A. W. Holzwarth

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Lecture Preparation	Physics Research	Lecture Preparation	Physics Research	Lecture Preparation
10:00-11:00	Electrodynamics: PHY 712		Electrodynamics: PHY 712		Electrodynamics: PHY 712
11:00-1:00	Office Hours		Office Hours		Office Hours
1:00-2:00	Physics Research		Physics Research	Condensed Matter Seminar	Physics Research
2:00-4:00				Physics Research	
4:00-5:00				Physics Department Colloquium	

http://users.wfu.edu/natalie/s23phy712/

PHY 712 Electrodynamics

MWF 10-10:50 AM	Olin 103	Webpage: http://www.wfu.edu/~natalie/s23phy712/

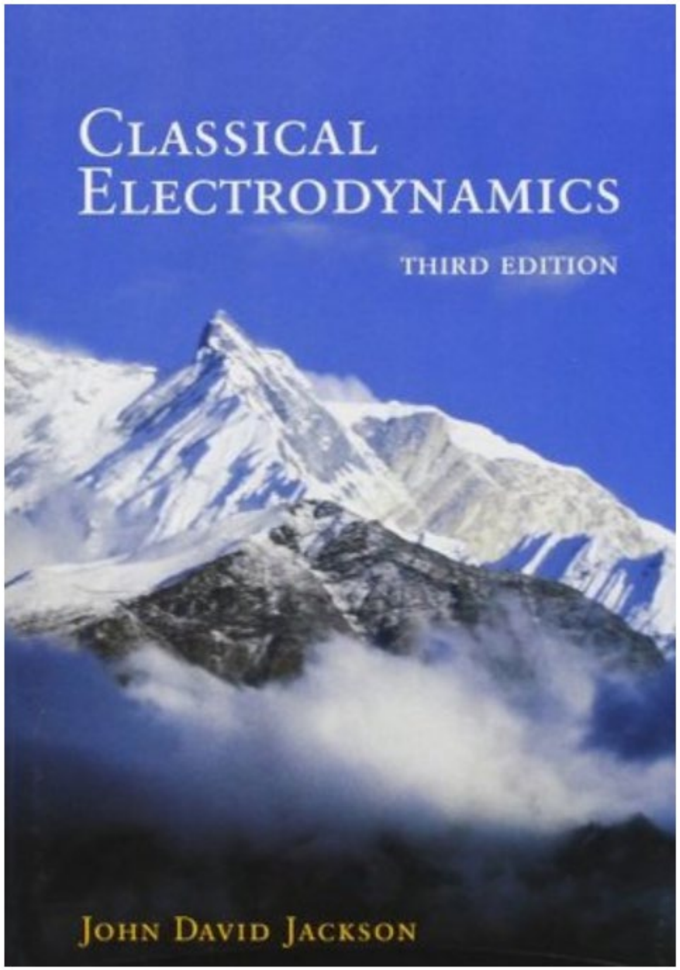
Instructor: Natalie Holzwarth	Office:300 OPL	e-mail:natalie@wfu.edu

- General information
- Syllabus and homework assignments
- Lecture notes
- Some presentation ideas

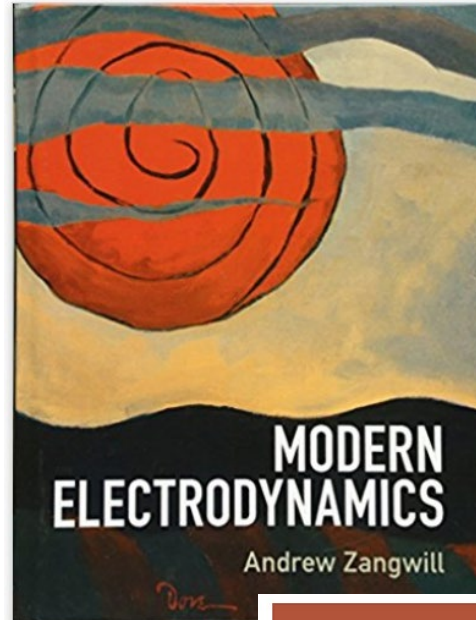
Last modfied: Tuesday, 03-Jan-2023 10:58:17 EST

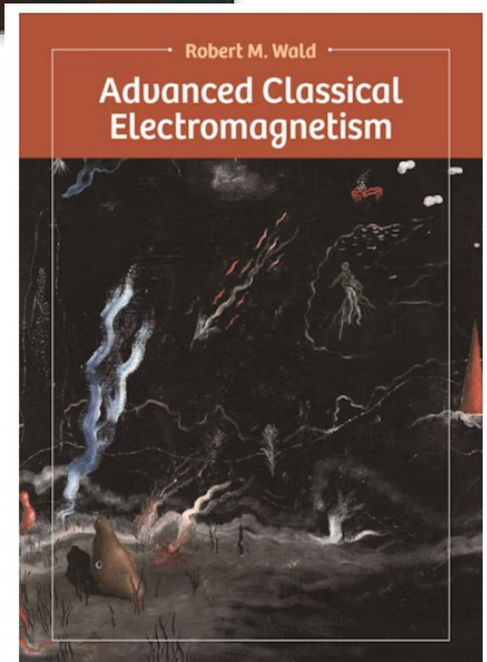
Optional supplements

Textbook



Third edition





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General Information

This course is a one semester survey of Electrodynamics at the graduate level, using the textbook: **Classical Electrodynamics**, 3rd edition, by John David Jackson (John Wiley & Sons, Inc., 1999) -- "**JDJ**". (link to errata for early printings) Note that it is necessary to get the **third** edition in order to synchronize with the class lectures and homework. The more recent textbook: **Modern Electrodynamics**, by Andrew Zangwill (Cambridge University Press, 2013) will be used as a supplement. LINK An even more recent textbook: **Advanced Classical Electromagnetism**, by Robert M. Wald (Princeton University Press, 2022) may be of interest to some of you. LINK

The course will consist of the following components:

- In person meetings in Olin 103 MWF 10-10:50 AM. Zoom connections can be made available if requested, but not on a regular basis. The class sessions will focus on discussion of the material, particularly answering your prepared and spontaneous questions.
- Asynchronous review of annotated lecture notes and corresponding textbook sections. The reading assignment and annotated lecture notes will be available one day before the corresponding synchronous online discussion. For each class meeting, students will be expected to submit (by email) at least one question for class discussion at least 3 hours before the class meeting.
- Homework sets. Typically there will be one homework problem associated with each class meeting.
- There will be two take-home exams, one at mid-term and the other during finals week.
- There will be one project on a chosen topic related to electrodynamics.
- It is highly recommended that each student arrange for weekly one-on-one meetings with the instructor to discuss the course material, homework, and/or projects. These may be face-to-face or online as appropriate.

It is likely that your grade for the course will depend upon the following factors:

Class participation	15%
Problem sets*	35%
Project	15%
Exams	35%

*In general, there will a new assignment after each lecture, so that for optimal learning, it would be best to complete each assignment before the next scheduled lecture. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts.

➔ Email your questions >= 1 hour before each class, schedule weekly one-on-one meetings, and/or attend office hours

Some Ideas for Computational Project

The purpose of the "Computational Project" is to provide an opportunity for you to study a topic of your choice in greater depth. The general guideline for your choice of project is that it should have something to do with electrodynamics, and there should be some degree of computation or analysis with the project. The completed project will include a short write-up and a ~20min presentation to the class. You may design your own project or use one of the following list (which will be updated throughout the term).

- Evaluate the Ewald sum of various ionic crystals using Maple or a programing language. (Template available in Fortran code.)
- Work out the details of the finite difference or finite element methods.
- Work out the details of the hyperfine Hamiltonian as discussed in Chapter 5 of Jackson.
- Work out the details of Jackson problem 7.2 and related problems.
- Work out the details of reflection and refraction from birefringent materials.
- Analyze the Kramers-Kronig transform of some optical data or calculations.
- Determine the classical electrodynamics associated with an infrared or optical laser.
- Analyze the radiation intensity and spectrum from an interesting source such as an atomic or molecular transition, a free electron laser, etc.
- Work out the details of Jackson problem 14.15.

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Course schedule for Spring 2023

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/9/2023	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/13/2023
2	Wed: 01/11/2023	Chap. 1	Electrostatic energy calculations		
3	Fri: 01/13/2023	Chap. 1	Electrostatic energy calculations		
	Mon: 01/16/2023		MLK Holiday -- no class		

PHY 712 -- Assignment #1

January 9, 2023

Read Chapters I and 1 and Appendix 1 in **Jackson**.

1. Jackson Problem #1.5. Be careful to take into account the behavior of $\Phi(\mathbf{r})$ for r-->0.

Comment about HW #1: (Jackson problem 1.5)

The time-averaged potential of a neutral hydrogen atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-2r/a_0}}{r}\left(1+\frac{r}{a_0}\right)$$

where q denotes the magnitude of the elementary charge of an electron or proton and where a_0 denotes the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your results physically.

Be careful to take into account the behavior of the potential for r→0.

Tentative additional information –

Mon Jan 16 – MLK Holiday

Spring break March 6-10

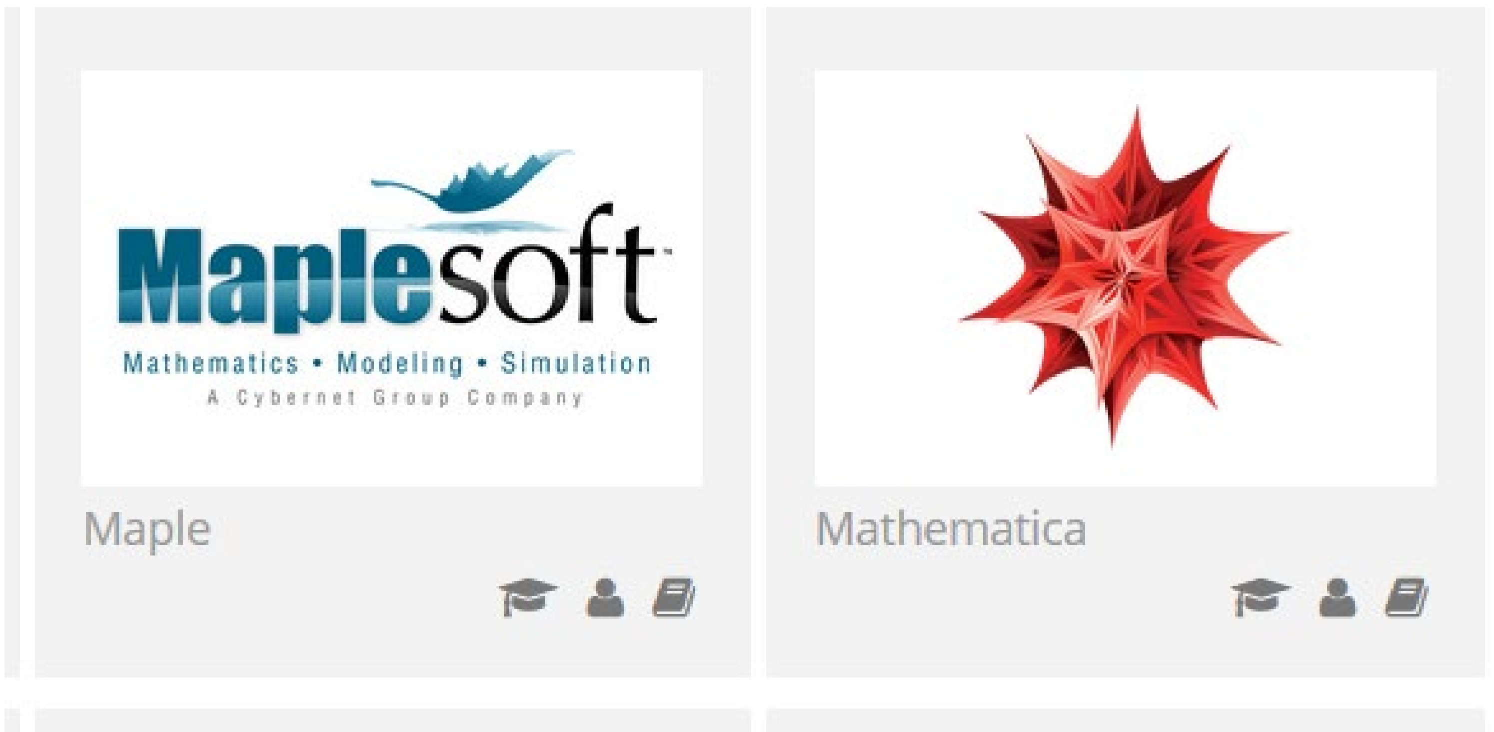
Mid term grades due March 9

Wed Apr 26 – Last day of class

May 1-5 – Final exams

Remember to check your algebraic manipulation software --

https://software.wfu.edu/audience/students/



Your questions –

From Sam: I remember field vectors D and H being related to E and B by dielectric and permittivity constants when not in a vacuum. Is This correct?
Short answer: Yes, in many cases

From Banasree: I didn't understand how we end up with the expression in page 24, after introducing a small radius a. And if you could discuss how we get power of 5/2, that would be great.
Short answer: Details to follow

Material discussed in Appendix of textbook --

Units - SI vs Gaussian

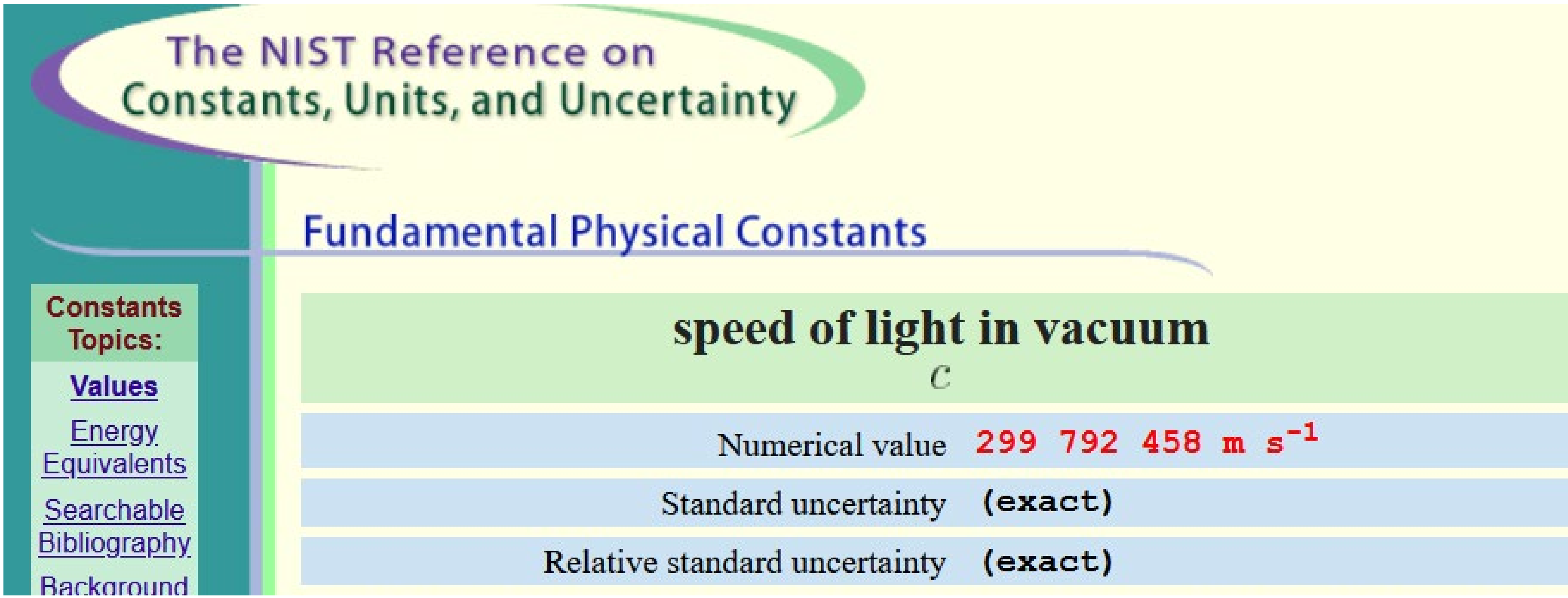
Coulomb's Law

$$F = K_C \frac{q_1 q_2}{r_{12}^2}. \tag{1}$$

Ampere's Law

$$F = K_A \frac{i_1 i_2}{r_{12}^2}\, d\mathbf{s_1} \times d\mathbf{s_2} \times \hat{\mathbf{r}}_{\mathbf{12}}, \tag{2}$$

In the equations above, the current and charge are related by $i_1 = dq_1/dt$ for all unit systems. The two constants K_C and K_A are related so that their ratio K_C/K_A has the units of $(m/s)^2$ and it is *experimentally* known that the ratio has the value $K_C/K_A = c^2$, where c is the speed of light.


The NIST Reference on
Constants, Units, and Uncertainty
Fundamental Physical Constants
Constants Topics:
Values
Energy Equivalents
Searchable Bibliography
Background
speed of light in vacuum
c
Numerical value 299 792 458 m s-1
Standard uncertainty (exact)
Relative standard uncertainty (exact)

Units - SI vs Gaussian – continued

The choices for these constants in the SI and Gaussian units are given below:

	CGS (Gaussian)	SI
K_C	1	$\frac{1}{4\pi\epsilon_0}$
K_A	$\frac{1}{c^2}$	$\frac{\mu_0}{4\pi}$

Here, $\frac{\mu_0}{4\pi} \equiv 10^{-7} N/A^2$ and $\frac{1}{4\pi\epsilon_0} = c^2 \cdot 10^{-7} N/A^2 = 8.98755 \times 10^9 N \cdot m^2/C^2$.

Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m^2/s$	$dyne$	$gm \cdot cm^2/s$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$, $\mathbf{H}$, $\mathbf{P}$, $\mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

As noted by Sam, form many materials in the linear response approximation,

$\mathbf{D}=\epsilon\mathbf{E}$ in vacuum $\epsilon = 1$ (for cgs Gaussian); $\epsilon = \epsilon_0$ (for SI)

$\mathbf{B}=\mu\mathbf{H}$ in vacuum $\mu = 1$ (for cgs Gaussian); $\mu = \mu_0$ (for SI)

Rectangular Snip

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

As we will see throughout the course, the E and B fields represent the basic electric and magnetic fields while the other fields include or represent electric and magnetic effects of matter.

Basic equations of electrodynamics

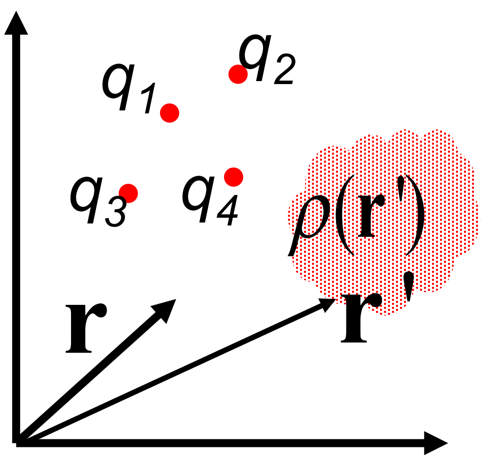
CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$	$u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

Units choice for this course:

SI units for Jackson in Chapters 1-10

Gaussian units for Jackson in Chapters 11-16

Electrostatics



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i q_i \frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3}$$

$$= \frac{1}{4\pi\epsilon_0} \int d^3r' \rho(\mathbf{r}') \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

Electrostatics

Discrete versus continuous charge distributions

In terms of Dirac delta function:

$$\rho(\mathbf{r}) = \sum_i q_i \ \delta(\mathbf{r} - \mathbf{r}_i)$$

Digression: Note that in cartesian coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \delta(x - x_i)\delta(y - y_i)\delta(z - z_i)$$

in spherical polar coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \frac{1}{r^2}\delta(r - r_i)\delta(\cos\theta - \cos\theta_i)\delta(\phi - \phi_i)$$

Differential equations --

Electrostatics

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{E} = 0$$

Electrostatic potential

$$\mathbf{E} = -\nabla\Phi(r).$$

$$\nabla^2\Phi(r) = -\rho(r)/\epsilon_0.$$

Relationship between integral and differential forms of electrostatics --

Differential form

$$\nabla^2 \Phi(\mathbf{r}) = -\rho(\mathbf{r}) / \epsilon_0$$

Integral form

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Relationship between integral and differential forms of electrostatics --

Need to show: $$\nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -4\pi \delta^3(\mathbf{r} - \mathbf{r}').$$

Noting that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \delta^3(\mathbf{r} - \mathbf{r}') f(\mathbf{r}) = f(\mathbf{r}'),$$

we see that we must show that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) = -4\pi f(\mathbf{r}').$$

We introduce a small radius a such that:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \lim_{a\to 0} \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}}.$$

For a fixed value of a,

$$\nabla^2 \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}} = \frac{-3a^2}{(|\mathbf{r}-\mathbf{r}'|^2 + a^2)^{5/2}}.$$

Some details --

Let $|\mathbf{r}-\mathbf{r}'| \equiv u$ $\qquad \nabla^2 = \frac{\partial^2}{\partial u^2} + \frac{2}{u}\frac{\partial}{\partial u}$

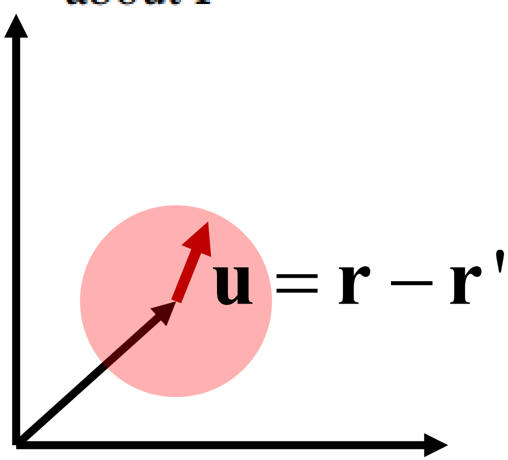
$$\nabla^2 \frac{1}{\sqrt{u^2+a^2}} = \left(\frac{\partial^2}{\partial u^2} + \frac{2}{u}\frac{\partial}{\partial u}\right)\frac{1}{\sqrt{u^2+a^2}} = -\frac{1}{\left(u^2+a^2\right)^{3/2}} + \frac{3u^2}{\left(u^2+a^2\right)^{5/2}} - \frac{2}{\left(u^2+a^2\right)^{3/2}}$$

$$= -\frac{3a^2}{\left(u^2+a^2\right)^{5/2}}$$

If the function $f(\mathbf{r})$ is continuous, we can make a Taylor expansion of it about the point $\mathbf{r} = \mathbf{r}'$, keeping only the first term. The integral over the small sphere about $\mathbf{r}'$ can be carried out analytically, by changing to a coordinate system centered at $\mathbf{r}'$;

so that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \, \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) \approx f(\mathbf{r}') \lim_{a \to 0} \int_{u<R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}}.$$



$$\mathbf{r} = \mathbf{r}' + \mathbf{u} \qquad f(\mathbf{r}) \approx f(\mathbf{r}')$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}} = 4\pi \int_0^R du \, \frac{-3a^2u^2}{(u^2 + a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2 + a^2)^{3/2}}.$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du \, \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

For $a \ll R$, $4\pi \frac{-R^3}{\left(R^2+a^2\right)^{3/2}} \approx -4\pi$

$$\rightarrow \int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \, \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) f(\mathbf{r}) \approx f(\mathbf{r}')(-4\pi),$$

$$\rightarrow \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$$

Example in HW1

The electrostatic potential of a neutral H atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right).$$

Find the charge density (both continuous and discrete) for this potential.

Hint #1: For continuous contribution you can use the identity: $\nabla^2\Phi(r) = \frac{1}{r}\frac{\partial^2(r\Phi(r))}{\partial r^2}$

Hint #2: Don't forget to consider possible discrete contributions.