PHY 712 Electrodynamics
10-10:50 AM Online

Notes for Lecture 22:

Chap. 8 in Jackson – Wave Guides

-- second shortened lecture

1. Rectangular wave guides
2. Rectangular resonant cavity

17	Fri: 02/17/2023	Chap. 7	Electromagnetic plane waves	#15	02/20/2023
18	Mon: 02/20/2023	Chap. 7	Electromagnetic plane waves	#16	02/22/2023
19	Wed: 02/22/2023	Chap. 7	Optical effects of refractive indices		
20	Fri: 02/24/2023	Chap. 1-7	Review		
21	Mon: 02/27/2023	Chap. 8	Short lectures on waveguides	Exam	
22	Wed: 03/01/2023	Chap. 8	Short lectures on waveguides	Exam	
23	Fri: 03/03/2023	Chap. 8	Short lectures on waveguides	Exam	03/03/2023
	Mon: 03/06/2023	No class	*Spring Break*		
	Wed: 03/08/2023	No class	*Spring Break*		
	Fri: 03/10/2023	No class	*Spring Break*		
24	Mon: 03/13/2023	Chap. 9	Radiation from localized oscillating sources		

For 2/27/2023-3/03/2023:

- Individual work on take home exam
- Shortened class lectures on Chapter 8 of **Jackson**

Physics Colloquium

Thursday

.

March 2, 2023

Organic transistors: Simple solutions to their complex problems

Plastic semiconductors incorporated into transistors have shown enormous potential for low-cost, flexible, printable electronics and bioelectronics. In my talk, I will discuss their history, operating mechanisms, and potential applications. I will highlight key challenges to these applications, and discuss some of the approaches I've taken to overcome them. I will show how these simple solutions can work towards the broad realization of organic transistors.

Alexandra (Zan) Paterson, Ph.D.
Assistant Professor,
University of Kentucky

4:00 pm - Olin 101*

Note: For additional information on the seminar, contact wfuphys@wfu.edu

Reception at 3:30pm - Olin Entrance

Boundary values for ideal conductor

Inside the conductor:

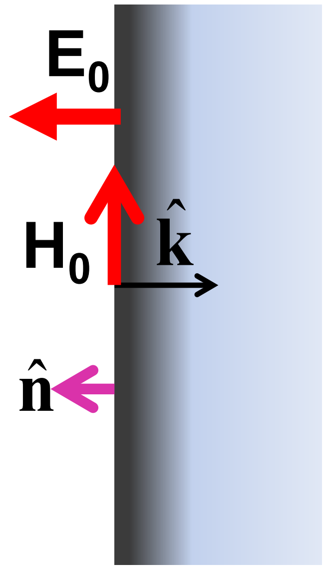
$$\mathbf{H}(\mathbf{r},t) = e^{-\hat{\mathbf{k}}\cdot\mathbf{r}/\delta}\Re\left(\mathbf{H}_0 e^{i\hat{\mathbf{k}}\cdot\mathbf{r}/\delta - i\omega t}\right)$$

$$\mathbf{E}(\mathbf{r},t) = \delta\mu\omega\frac{1-i}{2}\hat{\mathbf{k}}\times\mathbf{H}(\mathbf{r},t)$$

At the boundary of an ideal conductor, the **E** and **H** fields decay in the direction normal to the interface.

Boundary conditions for fields outside ideal conductor:

$$\hat{\mathbf{n}}\times\mathbf{E}\Big|_S = 0 \qquad \hat{\mathbf{n}}\cdot\mathbf{H}\Big|_S = 0$$

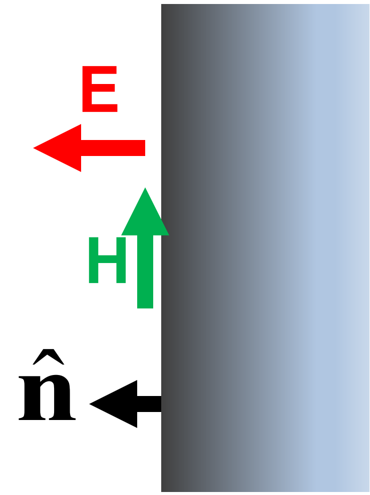


Wave guides – dielectric media with one or more metal boundary

Continuity conditions for fields near metal boundaries --

Ideal conductor boundary conditions:

$$\hat{\mathbf{n}} \times \mathbf{E}\Big|_S = 0 \qquad \hat{\mathbf{n}} \cdot \mathbf{H}\Big|_S = 0$$



Waveguide terminology

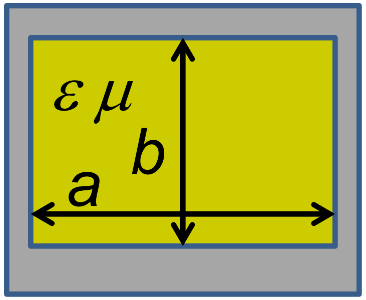
- TEM: transverse electric and magnetic (both E and H fields are perpendicular to wave propagation direction)
- TM: transverse magnetic (H field is perpendicular to wave propagation direction)
- TE: transverse electric (E field is perpendicular to wave propagation direction)

Analysis of rectangular waveguide

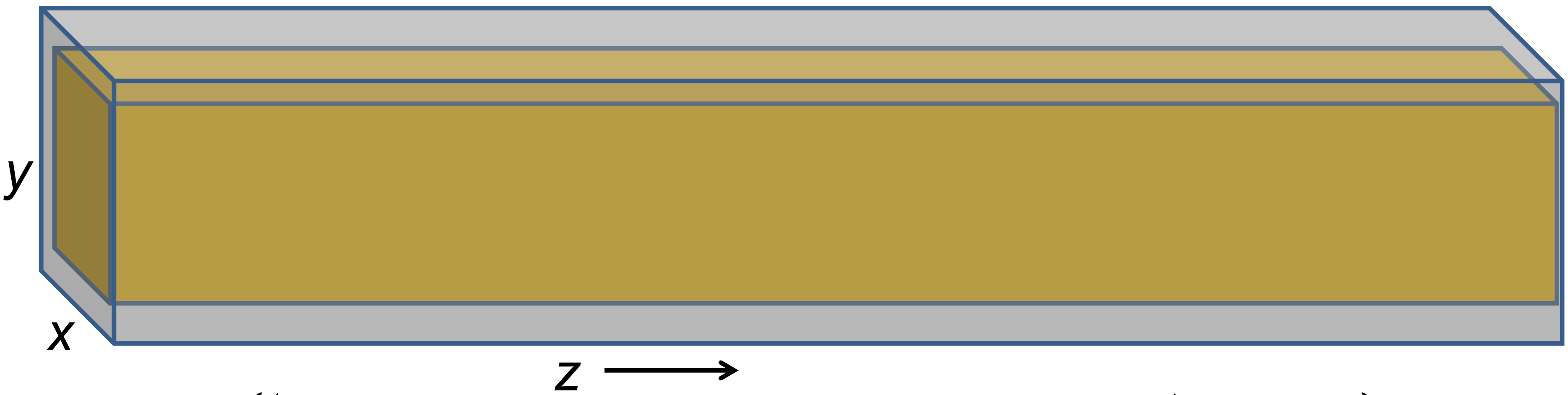
Boundary conditions at surface of waveguide:

$\mathbf{E}_{\text{tangential}}=0, \quad \mathbf{B}_{\text{normal}}=0$

Cross section view



Analysis of rectangular waveguide



$$\mathbf{B} = \Re\left\{ \left(B_x(x,y)\hat{\mathbf{x}} + B_y(x,y)\hat{\mathbf{y}} + B_z(x,y)\hat{\mathbf{z}} \right) e^{ikz - i\omega t} \right\}$$

$$\mathbf{E} = \Re\left\{ \left(E_x(x,y)\hat{\mathbf{x}} + E_y(x,y)\hat{\mathbf{y}} + E_z(x,y)\hat{\mathbf{z}} \right) e^{ikz - i\omega t} \right\}$$

Inside the dielectric medium: (assume ε to be real)

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \times \mathbf{B} - \varepsilon\mu \frac{\partial \mathbf{E}}{\partial t} = 0$$

Solution of Maxwell's equations within the pipe:

Combining Faraday's Law and Ampere's Law, we find that each field component must satisfy a two-dimensional Helmholz equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-k^2+\mu\varepsilon\omega^2\right)\mathbf{F}(x,y) = 0.$$

F = **E** or **H**
propagation along *z*.

For the rectangular wave guide discussed in Section 8.4 of your text a solution for a TE mode can have:

$$E_z(x,y)\equiv 0 \quad \text{and} \quad B_z(x,y)=B_0\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right),$$

$$\text{with } k^2 \equiv k_{mn}^2 = \mu\varepsilon\omega^2 - \left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]$$

Maxwell's equations within the pipe in terms of all 6 components:

For TE mode with $E_z \equiv 0$

$$B_x = -\frac{k}{\omega} E_y$$

$$B_y = \frac{k}{\omega} E_x$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + ikB_z = 0.$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + ikE_z = 0.$$

$$\frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x.$$

$$\frac{\partial B_z}{\partial y} - ikB_y = -i\mu\varepsilon\omega E_x.$$

$$ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y.$$

$$ikB_x - \frac{\partial B_z}{\partial x} = -i\mu\varepsilon\omega E_y.$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z.$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -i\mu\varepsilon\omega E_z.$$

TE modes for rectangular wave guide continued:

$$E_z(x,y) \equiv 0 \quad \text{and} \quad B_z(x,y) = B_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right),$$

$$E_x = \frac{\omega}{k} B_y = \frac{-i\omega}{k^2 - \mu\varepsilon\omega^2}\frac{\partial B_z}{\partial y} = \frac{-i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}\frac{n\pi}{b} B_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right),$$

$$E_y = -\frac{\omega}{k} B_x = \frac{i\omega}{k^2 - \mu\varepsilon\omega^2}\frac{\partial B_z}{\partial x} = \frac{i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}\frac{m\pi}{a} B_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right).$$

Check boundary conditions:

$$\mathbf{E}_{\text{tangential}} = 0 \ \text{ because:} \quad E_z(x,y) \equiv 0, \ \ E_x(x,0) = E_x(x,b) = 0$$

$$\text{and } \ E_y(0,y) = E_y(a,y) = 0.$$

$$\mathbf{B}_{\text{normal}} = 0 \quad \text{because:} \quad B_y(x,0) = B_y(x,b) = 0$$

$$\text{and } \ B_x(0,y) = B_x(a,y) = 0.$$

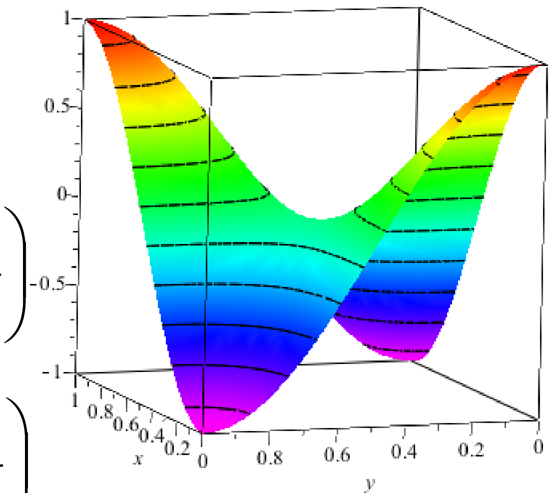
Solution for m=n=1

$$B_z(x,y) = B_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

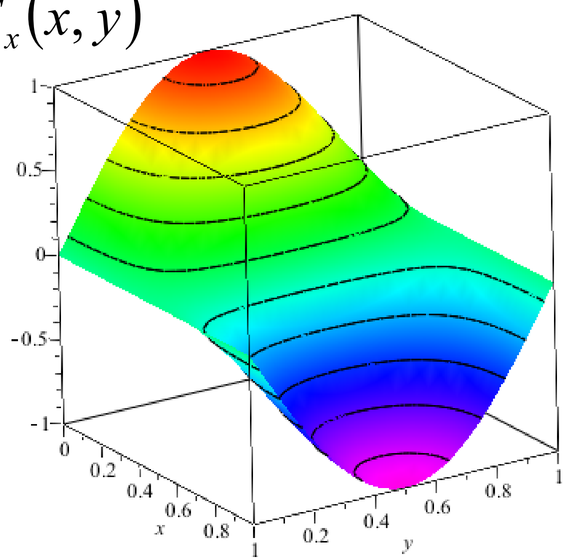
$$iE_x(x,y) = B_0\left(\frac{\omega n\pi / b}{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right)\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$iE_y(x,y) = B_0\left(\frac{-\omega m\pi / a}{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

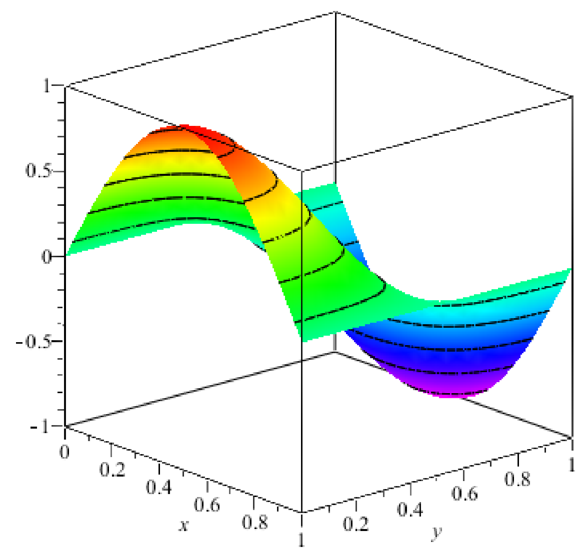
$B_z(x,y)$



$iE_x(x,y)$

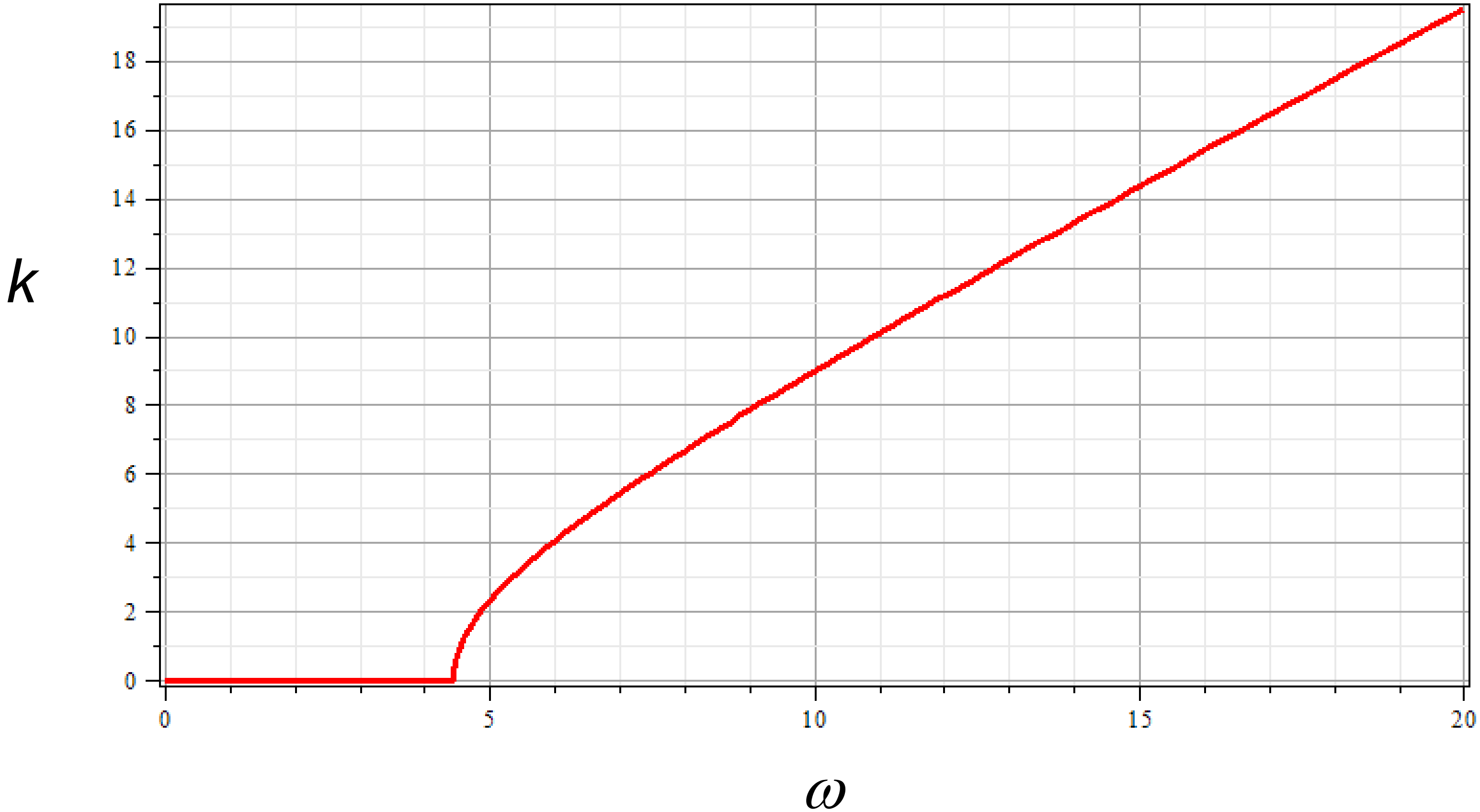


$iE_y(x,y)$



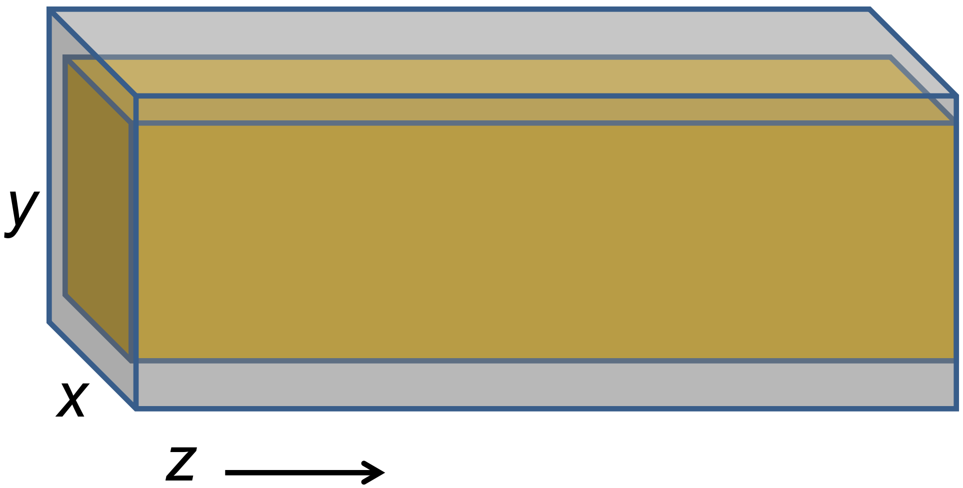
Solution for m=n=1

$$k^2 \equiv k_{mn}^2 = \mu\varepsilon\omega^2 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$



Now consider the case of a rectangular box bounded by an ideal conductor --

Resonant cavity



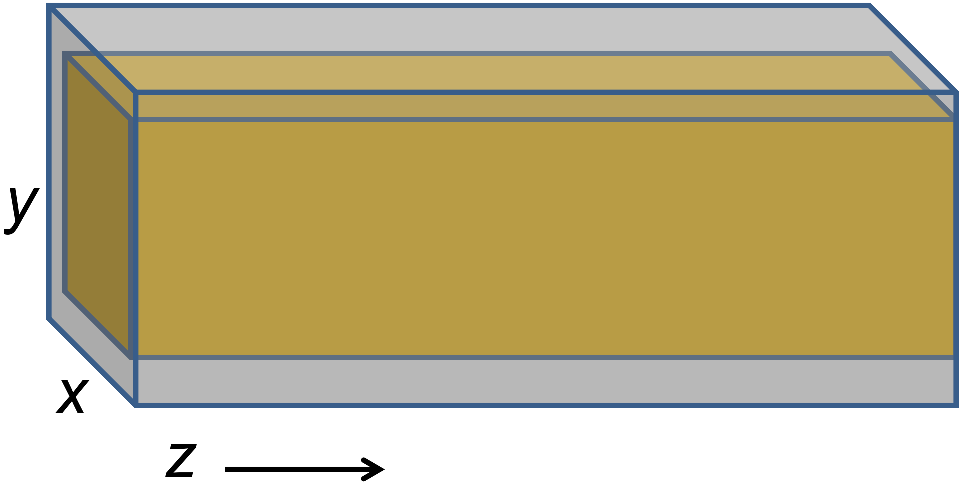
$$0 \le x \le a$$

$$0 \le y \le b$$

$$0 \le z \le d$$

Propagating wave form becomes a standing wave along *z*.

Resonant cavity



$$0 \le x \le a$$

$$0 \le y \le b$$

$$0 \le z \le d$$

$$\mathbf{B} = \Re\left\{\left(B_x(x,y,z)\hat{\mathbf{x}} + B_y(x,y,z)\hat{\mathbf{y}} + B_z(x,y,z)\hat{\mathbf{z}}\right)e^{-i\omega t}\right\}$$

$$\mathbf{E} = \Re\left\{\left(E_x(x,y,z)\hat{\mathbf{x}} + E_y(x,y,z)\hat{\mathbf{y}} + E_z(x,y,z)\hat{\mathbf{z}}\right)e^{-i\omega t}\right\}$$

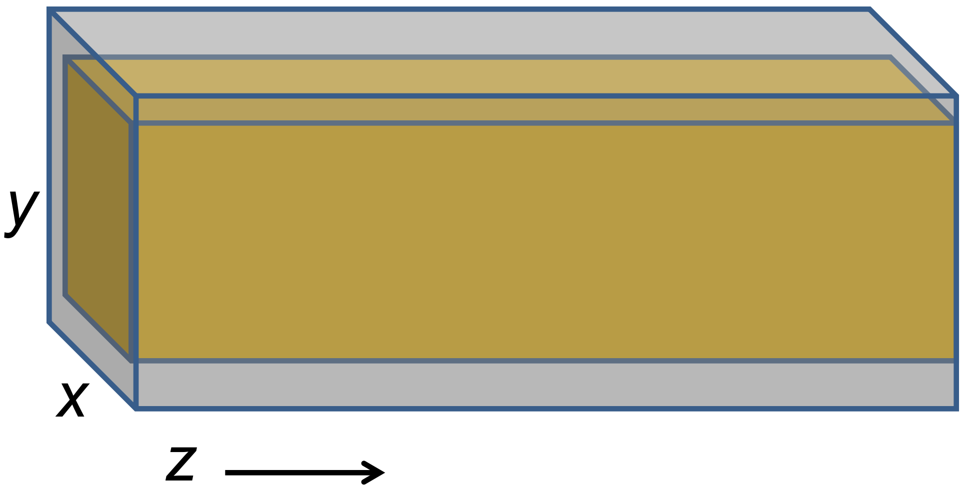
In general: $E_i(x,y,z) = E_i(x,y)\sin(kz)$ or $E_i(x,y)\cos(kz)$

$$B_i(x,y,z) = B_i(x,y)\sin(kz) \text{ or } B_i(x,y)\cos(kz)$$

$$\Rightarrow k = \frac{p\pi}{d}$$

Now consider the case of a rectangular box bounded by an ideal conductor --

Resonant cavity



$$0 \le x \le a$$

$$0 \le y \le b$$

$$0 \le z \le d$$

$$k^2 = \left(\frac{p\pi}{d}\right)^2 = \mu\varepsilon\omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow \omega^2 = \frac{1}{\mu\varepsilon}\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right)$$

For example --

$$B_z(x,y,z) = B_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)$$

$$\Rightarrow \omega^2 = \frac{1}{\mu\varepsilon}\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right)$$

Typically, microwave oven use frequencies of 2.45 GHz and the wavelength is ~12 cm