PHY 712 Electrodynamics
10-10:50 AM in Olin 103

Notes for Lecture 23:

Chap. 8 in Jackson – Wave Guides

-- third shortened lecture

1. Coaxial cable
2. Cylindrical wave guides

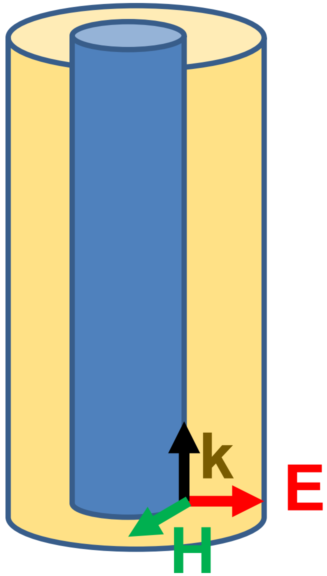
17	Fri: 02/17/2023	Chap. 7	Electromagnetic plane waves	#15	02/20/2023
18	Mon: 02/20/2023	Chap. 7	Electromagnetic plane waves	#16	02/22/2023
19	Wed: 02/22/2023	Chap. 7	Optical effects of refractive indices		
20	Fri: 02/24/2023	Chap. 1-7	Review		
21	Mon: 02/27/2023	Chap. 8	Short lectures on waveguides	Exam	
22	Wed: 03/01/2023	Chap. 8	Short lectures on waveguides	Exam	
23	Fri: 03/03/2023	Chap. 8	Short lectures on waveguides	Exam	03/03/2023
	Mon: 03/06/2023	No class	*Spring Break*		
	Wed: 03/08/2023	No class	*Spring Break*		
	Fri: 03/10/2023	No class	*Spring Break*		
24	Mon: 03/13/2023	Chap. 9	Radiation from localized oscillating sources		

For 2/27/2023-3/03/2023:

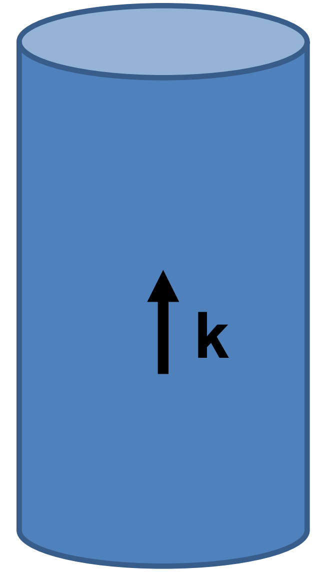
- Individual work on take home exam
- Shortened class lectures on Chapter 8 of **Jackson**

Wave guides – dielectric media with one or more metal boundary

Coaxial cable
TEM modes



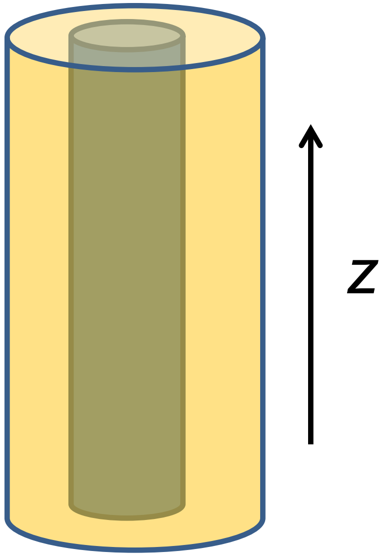
Simple optical pipe
TE or TM modes



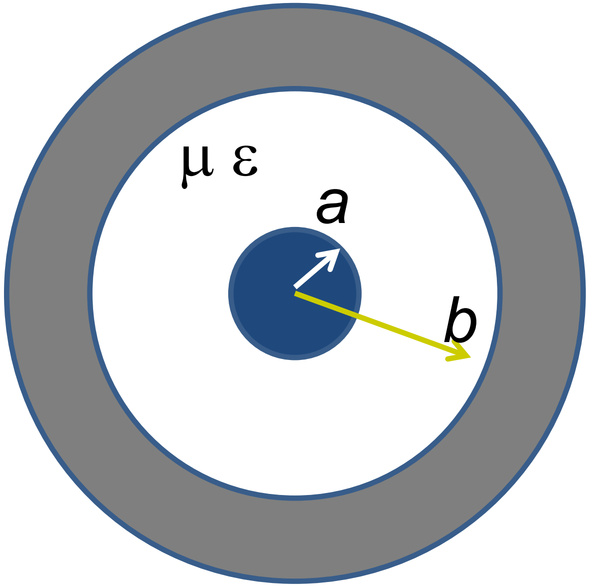
Waveguide terminology

- TEM: transverse electric and magnetic (both E and H fields are perpendicular to wave propagation direction)
- TM: transverse magnetic (H field is perpendicular to wave propagation direction)
- TE: transverse electric (E field is perpendicular to wave propagation direction)

Wave guides



Top view:



Inside medium, μ ε assumed to be real

Coaxial cable
TEM modes

(following problem 8.2 in Jackson's text)

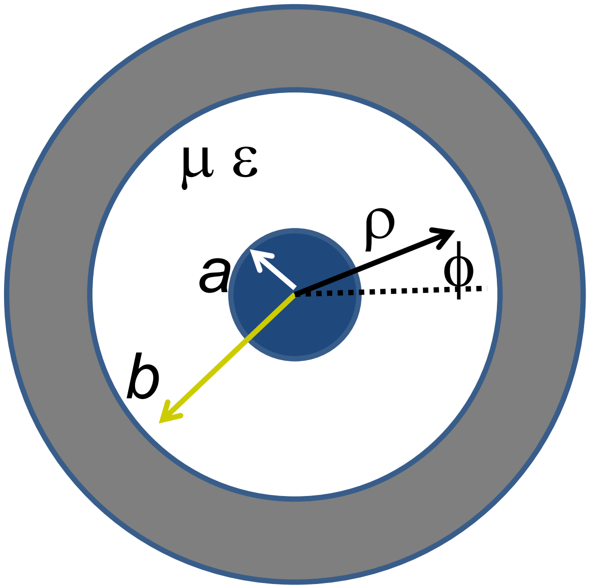
Maxwell's equations inside medium: for $a \le \rho \le b$

$$\nabla \times \mathbf{E} = i\omega\mathbf{B} \qquad \nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{B} = -i\omega\mu\varepsilon\mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0$$

Electromagnetic waves in a coaxial cable -- continued

Top view:



Example solution for $a \le \rho \le b$

$$\mathbf{E} = \hat{\boldsymbol{\rho}}\Re\left(\frac{E_0 a}{\rho}e^{ikz-i\omega t}\right)$$

$$\mathbf{B} = \hat{\boldsymbol{\varphi}}\Re\left(\frac{B_0 a}{\rho}e^{ikz-i\omega t}\right)$$

$$\hat{\boldsymbol{\rho}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}}$$

$$\hat{\boldsymbol{\varphi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

Find:

$$k = \omega\sqrt{\mu\varepsilon}$$

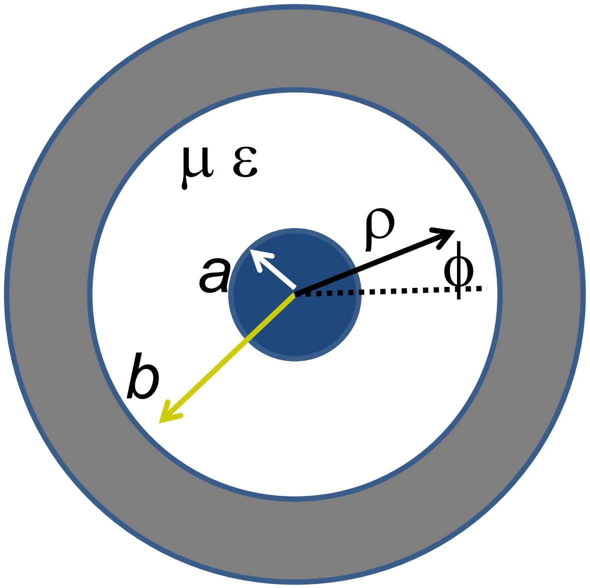
$$E_0 = \frac{B_0}{\sqrt{\mu\varepsilon}}$$

Poynting vector within cable medium (with μ, ε):

$$\langle \mathbf{S} \rangle_{avg} = \frac{1}{2\mu}\Re\left(\mathbf{E}\times\mathbf{B}^*\right) = \frac{|B_0|^2}{2\mu\sqrt{\mu\varepsilon}}\left(\frac{a}{\rho}\right)^2 \hat{\mathbf{z}}$$

Electromagnetic waves in a coaxial cable -- continued

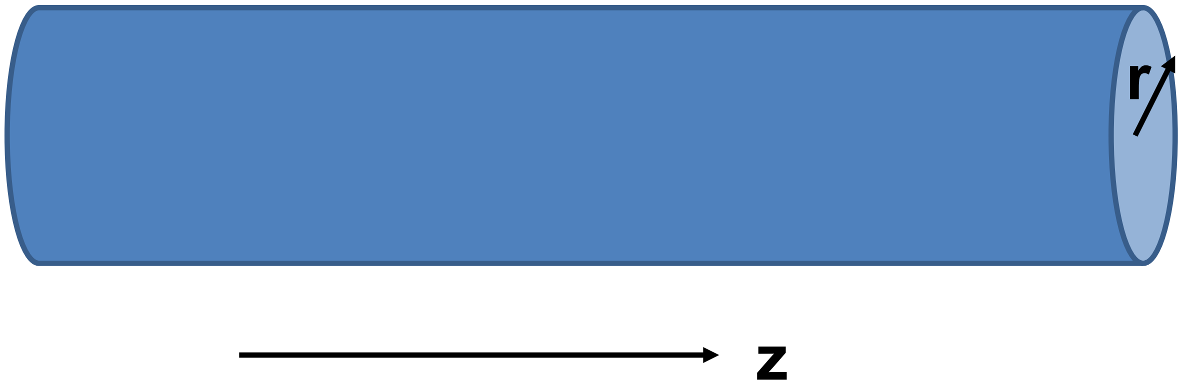
Top view:



Time averaged power in cable material:

$$\int_0^{2\pi} d\phi \int_a^b \rho d\rho \left(\langle \mathbf{S} \rangle_{avg} \cdot \hat{\mathbf{z}} \right) = \frac{|B_0|^2 \pi a^2}{\mu\sqrt{\mu\varepsilon}} \ln\left(\frac{b}{a}\right)$$

Simple cylindrical waveguide



For the rectangular geometry, Maxwell's equations within the pipe:

$$\mathbf{B} = \Re\left\{\left(B_x(x,y)\hat{\mathbf{x}} + B_y(x,y)\hat{\mathbf{y}} + B_z(x,y)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

$$\mathbf{E} = \Re\left\{\left(E_x(x,y)\hat{\mathbf{x}} + E_y(x,y)\hat{\mathbf{y}} + E_z(x,y)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

Inside the dielectric medium: (assume ε, μ to be real)

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \times \mathbf{B} - \varepsilon\mu \frac{\partial \mathbf{E}}{\partial t} = 0$$

Simple cylindrical waveguide -- continued

For the cylindrical geometry, Maxwell's equations within the pipe:

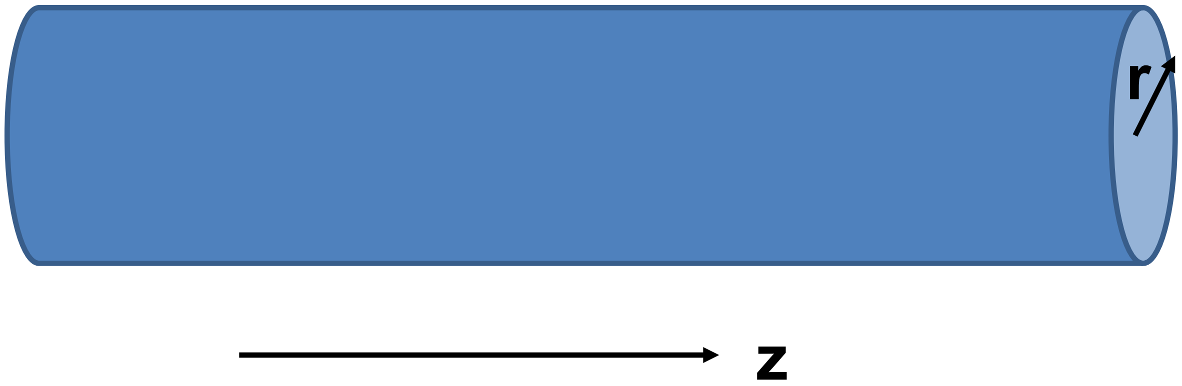
$$\mathbf{B} = \Re\left\{\left(\mathbf{B}_T(\mathbf{r}_T) + B_z(\mathbf{r}_T)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

$$\mathbf{E} = \Re\left\{\left(\mathbf{E}_T(\mathbf{r}_T) + E_z(\mathbf{r}_T)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

Using Maxwell's equations within medium:

$$\nabla\cdot\mathbf{E} = 0 \quad \nabla\cdot\mathbf{B} = 0 \quad \nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0 \quad \nabla\times\mathbf{B} - \varepsilon\mu\frac{\partial\mathbf{E}}{\partial t} = 0$$

Simple cylindrical waveguide



$$\mathbf{B} = \Re\left\{\left(\mathbf{B}_T(\mathbf{r}_T) + B_z(\mathbf{r}_T)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

$$\mathbf{E} = \Re\left\{\left(\mathbf{E}_T(\mathbf{r}_T) + E_z(\mathbf{r}_T)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

Combining equations: $\left(\nabla_T^2 - k^2 + \mu\varepsilon\omega^2\right) F(\mathbf{r}_T) = 0.$ for each field amplitude

For the cylindrical case, $F(\mathbf{r}_T) \to F(r,\varphi)$

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} - k^2 + \mu\varepsilon\omega^2\right)\mathbf{F}(r,\varphi) = 0.$$

Simple cylindrical waveguide -- continued

Evaluating $F(r,\varphi) = B_z(\mathbf{r}_T)$ for TE mode or $F(r,\varphi) = E_z(\mathbf{r}_T)$ for TM mode:

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} - k^2 + \mu\varepsilon\omega^2\right) F(r,\varphi) \;=\; 0$$

The components take form $F(r,\varphi) = F_m(r)e^{im\varphi}$

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2}{r^2} - k^2 + \mu\varepsilon\omega^2\right) F_m(r) = 0$$

For $\kappa^2 \equiv k^2 - \mu\varepsilon\omega^2$ $\quad \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2}{r^2} - \kappa^2\right) F_m(r) = 0$

$\Rightarrow F_m(r) = J_m(\kappa r)$ Bessel functions

Boundary values will be applied for $r = a$

In some cases, for zeroes of Bessel function $J_m(\kappa a) = 0$

$\Rightarrow \kappa = \frac{x_{mn}}{a}$ for $J_m(x_{mn}) = 0$

Have a very enjoyable spring break.

See you March 13, 2023.