PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

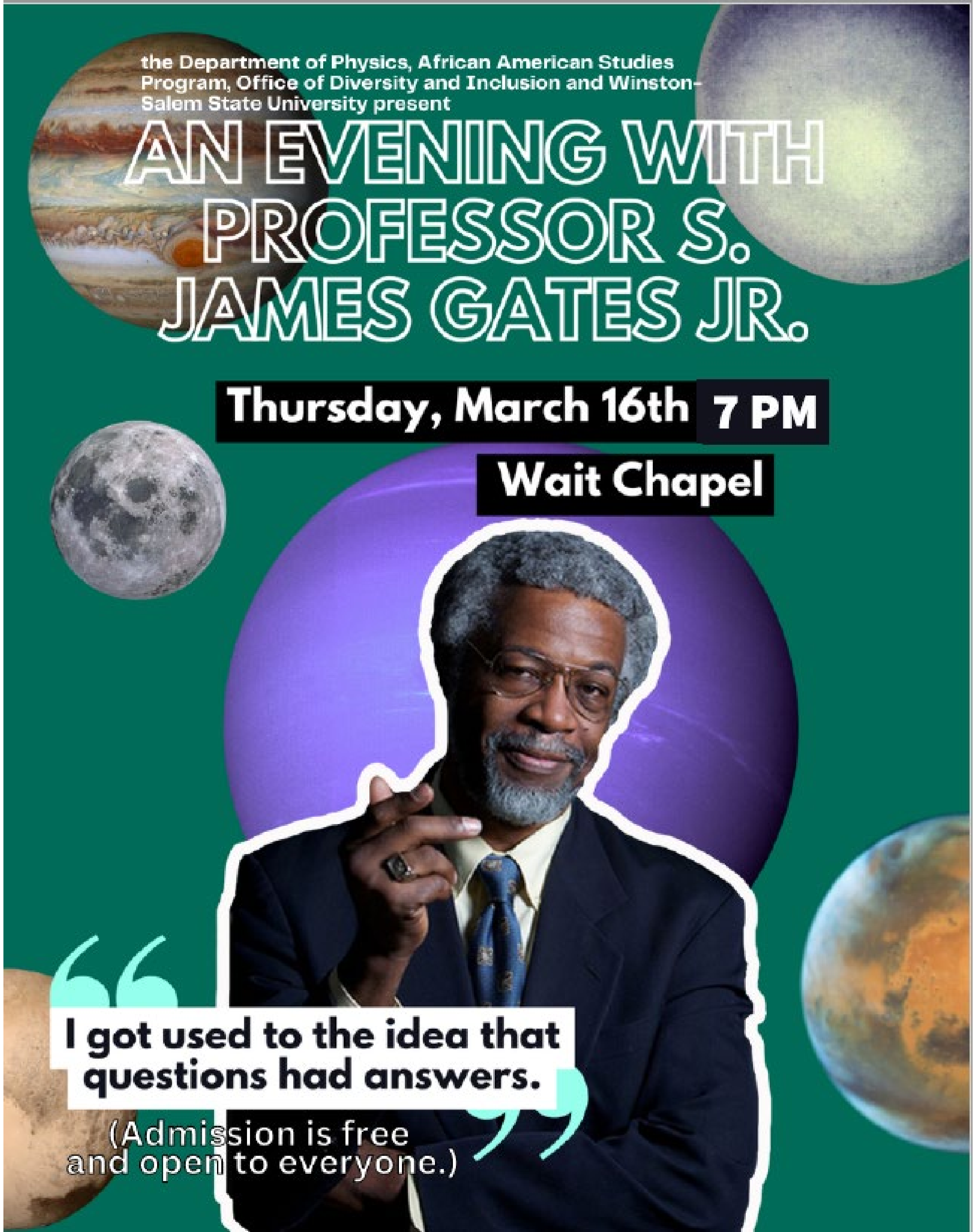
Notes for Lecture 25:

Continue reading Chap. 9

A. Electromagnetic waves due to specific sources

B. Dipole radiation examples

C. Radiation from antennas


the Department of Physics, African American Studies Program, Office of Diversity and Inclusion and Winston-Salem State University present
AN EVENING WITH PROFESSOR S. JAMES GATES JR.
Thursday, March 16th 7 PM
Wait Chapel
I got used to the idea that questions had answers.
(Admission is free and open to everyone.)

PHYSICS COLLOQUIUM

FRIDAY

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MARCH 17, 2023

A Surprising Connection: Mathematical Coloring Problems & Supersymmetry!

The Four Color Map Conjecture was apparently the first example in the mathematical literature where a theorem was created that depended heavily on the existence of computer algorithms and IT applications. A related problem is the Graph Vertex Coloring Problem and recent study shows evidence that supersymmetry is connected with solutions of the Graph Vertex Coloring Problem.

3:00 pm - Olin 101

Reception at 2:30pm - Olin Entrance

Professor S. James Gates, Jr.

Department of Physics and School of Public Policy

University of Maryland

20	Fri: 02/24/2023	Chap. 1-7	Review		
21	Mon: 02/27/2023	Chap. 8	Short lectures on waveguides	Exam	
22	Wed: 03/01/2023	Chap. 8	Short lectures on waveguides	Exam	
23	Fri: 03/03/2023	Chap. 8	Short lectures on waveguides	Exam	03/03/2023
	Mon: 03/06/2023	No class	*Spring Break*		
	Wed: 03/08/2023	No class	*Spring Break*		
	Fri: 03/10/2023	No class	*Spring Break*		
24	Mon: 03/13/2023	Chap. 9	Radiation from localized oscillating sources	#17	03/17/2023
25	Wed: 03/15/2023	Chap. 9	Radiation from oscillating sources		
26	Fri: 03/17/2023	Chap. 9 & 10	Radiation and scattering		

Discussion: Tentative plan for remainder of the course

Week of March 20	-- JDJ #11 Special relativity
Week of March 27	-- JDJ #14 especially synchrotron
Week of April 3	-- special topics
Week of April 10	-- special topics
Week of April 17	-- review
Week of April 24	-- presentations
Week of May 1	-- final take home

Final grades due May 9

Your questions –

From Arezoo: In slide 11, blue diagram and purple diagram are both exact results? What is the difference?

From Banasree: I don't understand what different values of kd in those graphs physically means for antenna.

Electromagnetic waves from time harmonic sources

Charge density: $\rho(\mathbf{r},t) = \Re\left(\tilde{\rho}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Current density: $\mathbf{J}(\mathbf{r},t) = \Re\left(\tilde{\mathbf{J}}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Note that the continuity condition applies:

$$\frac{\partial \rho(\mathbf{r},t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r},t) = 0 \quad \Rightarrow -i\omega\tilde{\rho}(\mathbf{r},\omega) + \nabla \cdot \tilde{\mathbf{J}}(\mathbf{r},\omega) = 0$$

General source: $f(\mathbf{r},t) = \Re\left(\tilde{f}(\mathbf{r},\omega)e^{-i\omega t}\right)$

For $\tilde{f}(\mathbf{r},\omega) = \frac{1}{4\pi\varepsilon_0}\tilde{\rho}(\mathbf{r},\omega)$

or $\tilde{f}(\mathbf{r},\omega) = \frac{\mu_0}{4\pi}\tilde{J}_i(\mathbf{r},\omega)$

Electromagnetic waves from time harmonic sources – continued:

$$\Psi(\mathbf{r},t) = \Psi_{f=0}(\mathbf{r},t) +$$

$$\int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) f(\mathbf{r}',t')$$

$$\widetilde{\Psi}(\mathbf{r},\omega)e^{-i\omega t} = \widetilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} +$$

$$\int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) \widetilde{f}(\mathbf{r}',\omega)e^{-i\omega t'}$$

$$= \widetilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \widetilde{f}(\mathbf{r}',\omega)e^{-i\omega t}$$

Important results from last time – EM waves from time harmonic sources – open isotropic boundaries

For scalar potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega),$$

where $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\Phi}_0(\mathbf{r},\omega) = 0$

For vector potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \frac{\mu_0}{4\pi}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega),$$

where $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\mathbf{A}}_0(\mathbf{r},\omega) = 0$

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr) = j_l(kr) + in_l(kr)$

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \sum_{lm} \tilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\phi}_{lm}(r,\omega) = \frac{ik}{\varepsilon_0}\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \sum_{lm} \tilde{\mathbf{a}}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega) = ik\mu_0\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

Example of dipole radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

Note that the continuity of charge and current must be satisfied. For the Fourier amplitudes, the relations are as bellow:

Recall continuity condition: $-i\omega\,\tilde{\rho}(\mathbf{r},\omega)+\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)=0$

$$-i\omega\mathbf{r}\,\tilde{\rho}(\mathbf{r},\omega)+\mathbf{r}\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)$$

Jackson's clever trick!

$$\int d^3r\ \mathbf{r}\ \tilde{\rho}(\mathbf{r},\omega)=\frac{1}{i\omega}\int d^3r\ \mathbf{r}\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)$$

$$=-\frac{1}{i\omega}\int d^3r\ \tilde{\mathbf{J}}(\mathbf{r},\omega)=\mathbf{p}(\omega)$$

Example of dipole radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\mathbf{p}(\omega)=\hat{\mathbf{z}}\frac{J_0}{-i\omega R}\int d^3r\ r\cos(\theta)\left(\cos(\theta)e^{-r/R}\right)$$

$$=\hat{\mathbf{z}}\frac{J_0}{-i\omega R}\frac{4\pi}{3}\int_0^\infty dr\ r^3 e^{-r/R}=\hat{\mathbf{z}}J_0\frac{8\pi R^3}{-i\omega}$$

$$=\frac{1}{-i\omega}\int d^3r\ \tilde{\mathbf{J}}(\mathbf{r},\omega)$$

From the analysis valid for *kr>>1* and *kR<<1*:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=-\frac{i\mu_0\omega}{4\pi}\mathbf{p}(\omega)\frac{e^{ikr}}{r}=\hat{\mathbf{z}}J_0\mu_0 2R^3\frac{e^{ikr}}{r}$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{ik}{4\pi\varepsilon_0}\mathbf{p}(\omega)\cdot\hat{\mathbf{r}}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}=\frac{J_0 2R^3}{\varepsilon_0 c}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}\cos\theta$$

Example of dipole radiation source -- exact results for r>>R:

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\,(ik\mu_0)\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0 k}{\varepsilon_0\omega R}\cos\theta\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

Evaluation for $r >> R$:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\;\frac{2R^3}{(1+k^2R^2)^2}$$

$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{J_0 k}{\varepsilon_0\omega}\cos\theta\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right)\;\frac{2R^3}{(1+k^2R^2)^2}$$

Agrees with dipole approximation for *kR<<1*.

More details --

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\left(ik\mu_0\right)\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0 k}{\varepsilon_0\omega R}\cos\theta\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

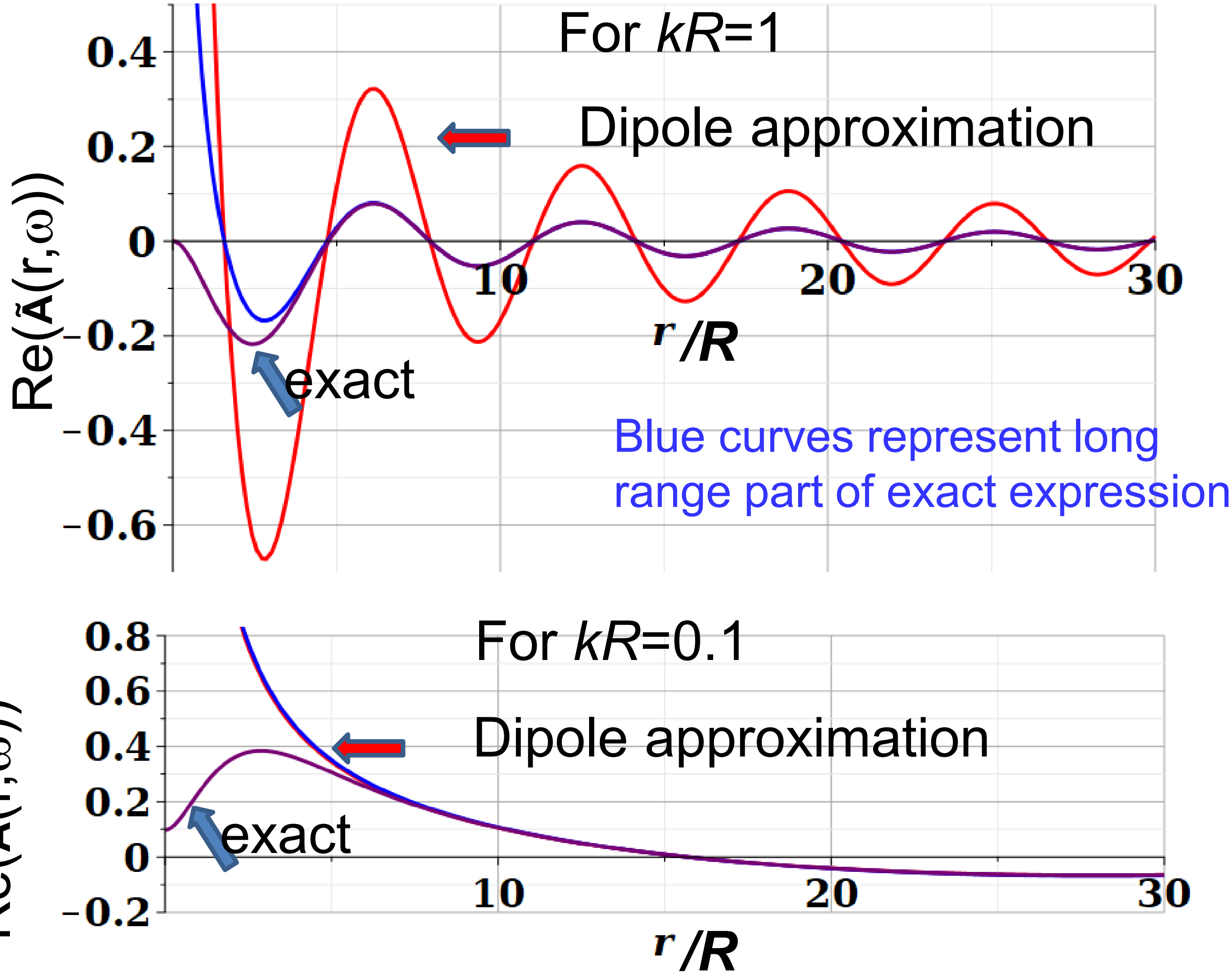
$$\tilde{\mathbf{A}}(r,\omega)=\hat{\mathbf{z}}J_0\mu_0\left(\frac{e^{ikr}}{kr}\int_0^r r'dr'e^{-r'/R}\sin(kr')+\frac{\sin(kr)}{kr}\int_r^\infty r'dr'e^{-r'/R+ikr'}\right)$$

$$\underset{r\gg R}{\approx}\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\frac{2R^3}{\left(k^2R^2+1\right)^2}$$

Correct when this term is negligible.

$$\tilde{\mathbf{A}}_{\text{dipole approx}}(r,\omega)=\hat{\mathbf{z}}J_0\mu_0\left(\frac{e^{ikr}}{kr}\int_0^\infty r'dr'e^{-r'/R}(kr')\right)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}2R^3$$

Example continued



Continued review of dipole results --

Power in the dipole approximation; Section 9.2 of Jackson

Here we use our notation with $\mathbf{n} \rightarrow \hat{\mathbf{r}}$ and $Z_0 \equiv \sqrt{\frac{\mu_0}{\epsilon_0}}$

$$\frac{dP}{d\Omega} = \frac{r^2}{2} \Re \left| \left(\hat{\mathbf{r}} \cdot \left(\mathbf{E} \times \mathbf{H}^* \right) \right) \right|^2$$

Using the expressions for the dipole fields far from the source:

$$\mathbf{H} = \frac{ck^2}{4\pi} \left(\hat{\mathbf{r}} \times \mathbf{p} \right) \frac{e^{ikr}}{r} \qquad \mathbf{E} = Z_0 \mathbf{H} \times \hat{\mathbf{r}}$$

The power can be written $\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2} k^4 \left| \left(\left(\hat{\mathbf{r}} \times \mathbf{p} \right) \times \hat{\mathbf{r}} \right) \right|^2$

Defining the angle θ by $\mathbf{p} \cdot \hat{\mathbf{r}} = \left| \mathbf{p} \right| \cos\theta$,

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2} k^4 \left| \mathbf{p} \right|^2 \sin^2\theta$$ integrating over solid angles $P = \frac{c^2 Z_0}{12\pi} k^4 \left| \mathbf{p} \right|^2$

Review:

Electromagnetic waves from time harmonic sources – continued:

Dipole radiation case:

Define dipole moment at frequency ω:

$$\mathbf{p}(\omega) \equiv \int d^3r \ \mathbf{r}\tilde{\rho}(\mathbf{r},\omega) = -\frac{1}{i\omega}\int d^3r \ \tilde{\mathbf{J}}(\mathbf{r},\omega)$$

For fields outside extent of source and $kr' << 1$ within the source:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = -\frac{i\mu_0\omega}{4\pi}\mathbf{p}(\omega)\frac{e^{ikr}}{r}$$

$$\tilde{\Phi}(\mathbf{r},\omega) = -\frac{ik}{4\pi\varepsilon_0}\mathbf{p}(\omega)\cdot\hat{\mathbf{r}}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}$$

Review:

Electromagnetic waves from time harmonic sources – continued:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = -\nabla\tilde{\Phi}(\mathbf{r},\omega) + i\omega\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\left(k^2\left(\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\times\hat{\mathbf{r}}\right) + \left(\frac{3\hat{\mathbf{r}}\left(\hat{\mathbf{r}}\cdot\mathbf{p}(\omega)\right)-\mathbf{p}(\omega)}{r^2}\right)(1-ikr)\right)$$

$$\tilde{\mathbf{B}}(\mathbf{r},\omega) = \nabla\times\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$= \frac{1}{4\pi\varepsilon_0 c}\frac{e^{ikr}}{r}k^2\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\left(1-\frac{1}{ikr}\right)$$

Power radiated for $kr >> 1$:

$$\frac{dP}{d\Omega} = r^2\hat{\mathbf{r}}\cdot\langle\mathbf{S}\rangle_{avg} = \frac{r^2}{2\mu_0}\hat{\mathbf{r}}\cdot\Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)\right)$$

$$= \frac{c^2k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left|\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\times\hat{\mathbf{r}}\right|^2$$

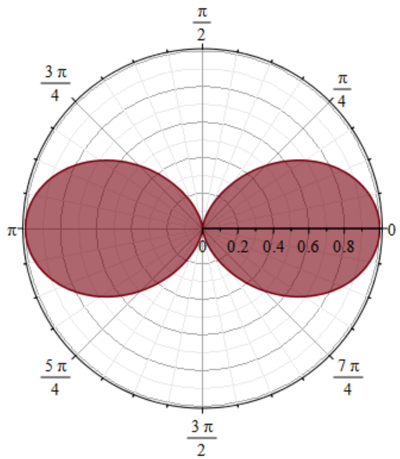
Properties of dipole radiation field for *kr* >>1:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega)=\frac{1}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\left(k^2\left(\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\times\hat{\mathbf{r}}\right)\right)$$

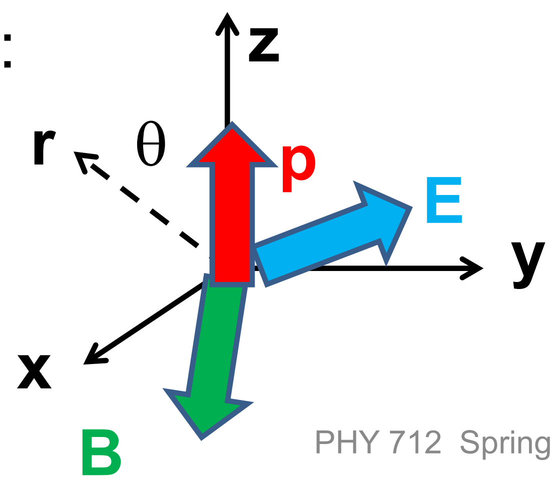
$$\tilde{\mathbf{B}}(\mathbf{r},\omega)=\frac{1}{4\pi\varepsilon_0 c^2}\frac{e^{ikr}}{r}k^2\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)$$

Power radiated for $kr>>1$:

$$\frac{dP}{d\Omega}=r^2\hat{\mathbf{r}}\cdot\langle\mathbf{S}\rangle_{avg}=\frac{c^2k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left|\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\times\hat{\mathbf{r}}\right|^2$$



Example:



Note that vectors **r, E, B** are mutually orthogonal

Alternative approach

Fields from time harmonic source:

$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{1}{4\pi\varepsilon_0}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\frac{\mu_0}{4\pi}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

For $r >> r'$: $|\mathbf{r}-\mathbf{r}'| \approx r-\hat{\mathbf{r}}\cdot\mathbf{r}'+\ldots$

$$\tilde{\Phi}(\mathbf{r},\omega)\approx\frac{1}{4\pi\epsilon_0}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\,\tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)\approx\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

For our example:

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

For $r >> r'$: $\quad |\mathbf{r}-\mathbf{r}'| \approx r-\hat{\mathbf{r}}\cdot\mathbf{r}'+\ldots$

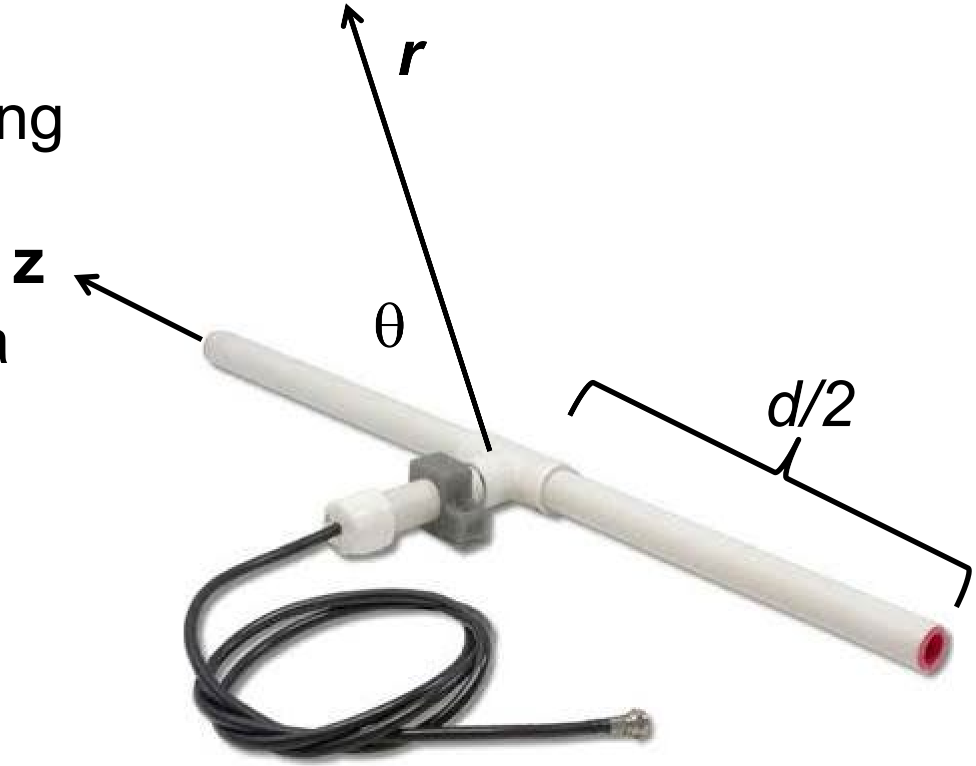
$$\tilde{\Phi}(\mathbf{r},\omega)\approx\frac{1}{4\pi\epsilon_0}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\,\tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)\approx\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

➔Results equivalent to Bessel function expansion in the limit $kr \rightarrow \infty$.

Other radiation sources using ``alternative approach”

Linear center-fed antenna



$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx$$

$$\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{J}}(\mathbf{r}',\omega) = I_0 \sin\left(\frac{kd}{2} - k|z|\right)\delta(x)\delta(y)\hat{\mathbf{z}}$$

Alternative approach – linear center-fed antenna continued

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx \hat{\mathbf{z}} \frac{\mu_0 I_0}{4\pi} \frac{e^{ikr}}{r} \int_{-d/2}^{d/2} dz' \, e^{-ik\cos(\theta) z'} \sin\left(\frac{kd}{2} - k|z'|\right)$$

$$= \hat{\mathbf{z}} \frac{\mu_0 I_0}{2\pi} \frac{e^{ikr}}{kr} \left(\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin^2\theta} \right)$$

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^{\,2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$

Alternative approach – linear center-fed antenna continued

Time averaged power:

$$\frac{dP}{d\Omega} = I_0{}^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$

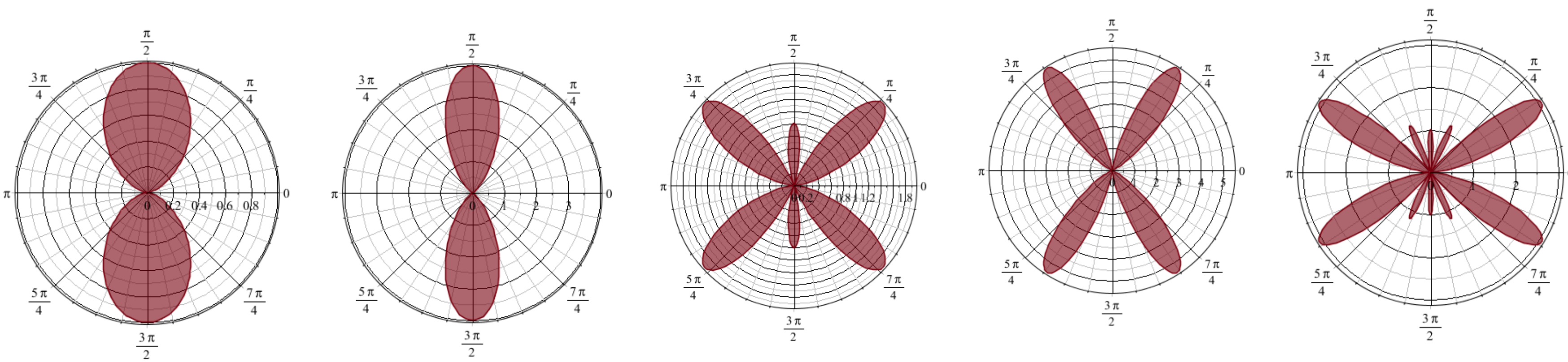
for $kd = \pi$: $$\frac{dP}{d\Omega} = I_0{}^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

for $kd = 2\pi$: $$\frac{dP}{d\Omega} = I_0{}^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{4}{8\pi^2} \frac{\cos^4\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

Alternative approach – linear center-fed antenna continued

Time averaged power:

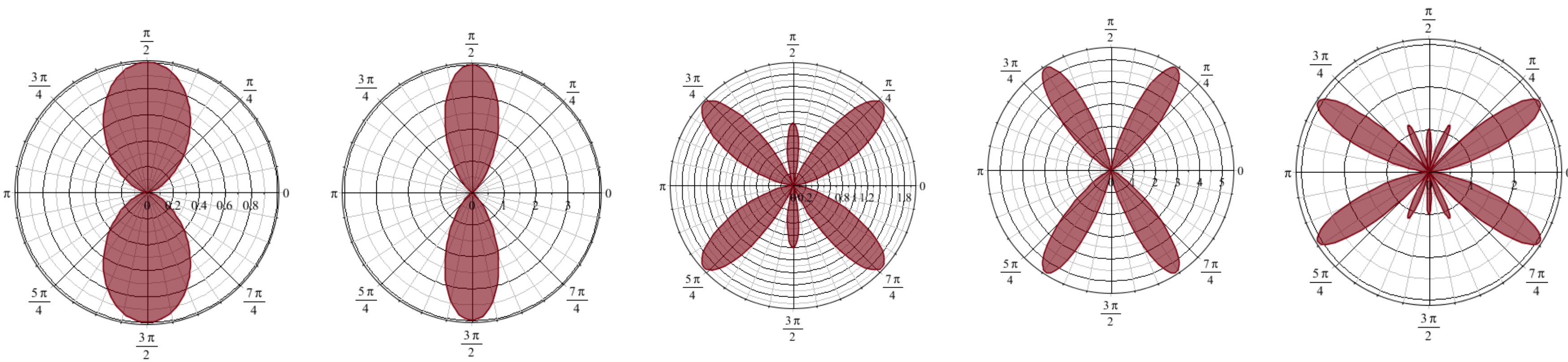
$$\frac{dP}{d\Omega} = I_0^{\ 2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$



kd= π 2π 3π 4π 5π

Interesting patterns for special values of *kd*

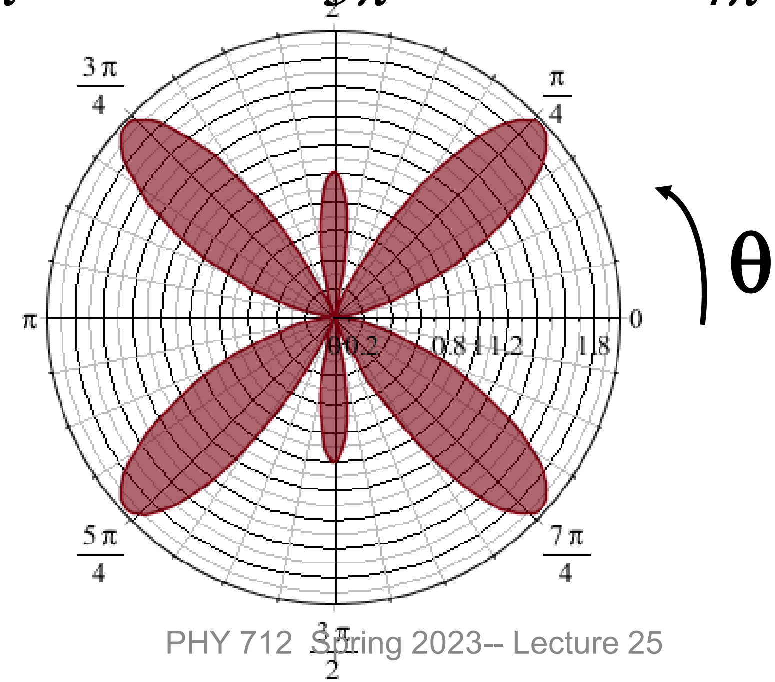
Some details --



kd=π 2π 3π 4π 5π

Polar plot:
Angle indicates values of theta

Radius indicates value scaled to 1.



Next time – we will consider the effects of multiple antennas (antenna arrays including interference effects) and radiation due to light scattering (from Chapter 10 of your textbook).