PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Discussion for Lecture 33:

Start reading Chap. 15 –

Radiation from collisions of charged particles

1. Overview
2. X-ray tube
3. Radiation from Rutherford scattering
4. Other collision models

PHYSICS COLLOQUIUM

4 PM in Olin 101

THURSDAY

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APRIL 6TH, 2023

An introduction to Honeywell's contributions to NASA's human space flight missions such as the Orion and Artemis missions to the Moon and to Mars

Honeywell contributes to several of NASA's human space programs including Orion, Artemis, and Mars missions. Specifically, Honeywell develops navigation tools which are commonly used in inertial measurement units. For example, these tools performed very well during a recent 26-day mission in the Artemis program involving a low earth orbit of an unmanned test space vehicle. This talk will present an overview of the inertial measurement units and discuss some of physics, engineering, and analyses needed for further improvements. Some of the discussion involves analysis of sensor noise signatures and dynamic calibrations using Kaman filtering. Next generation sensors may include Gaussian-Markov models and estimates of quantum and general relativistic effects.

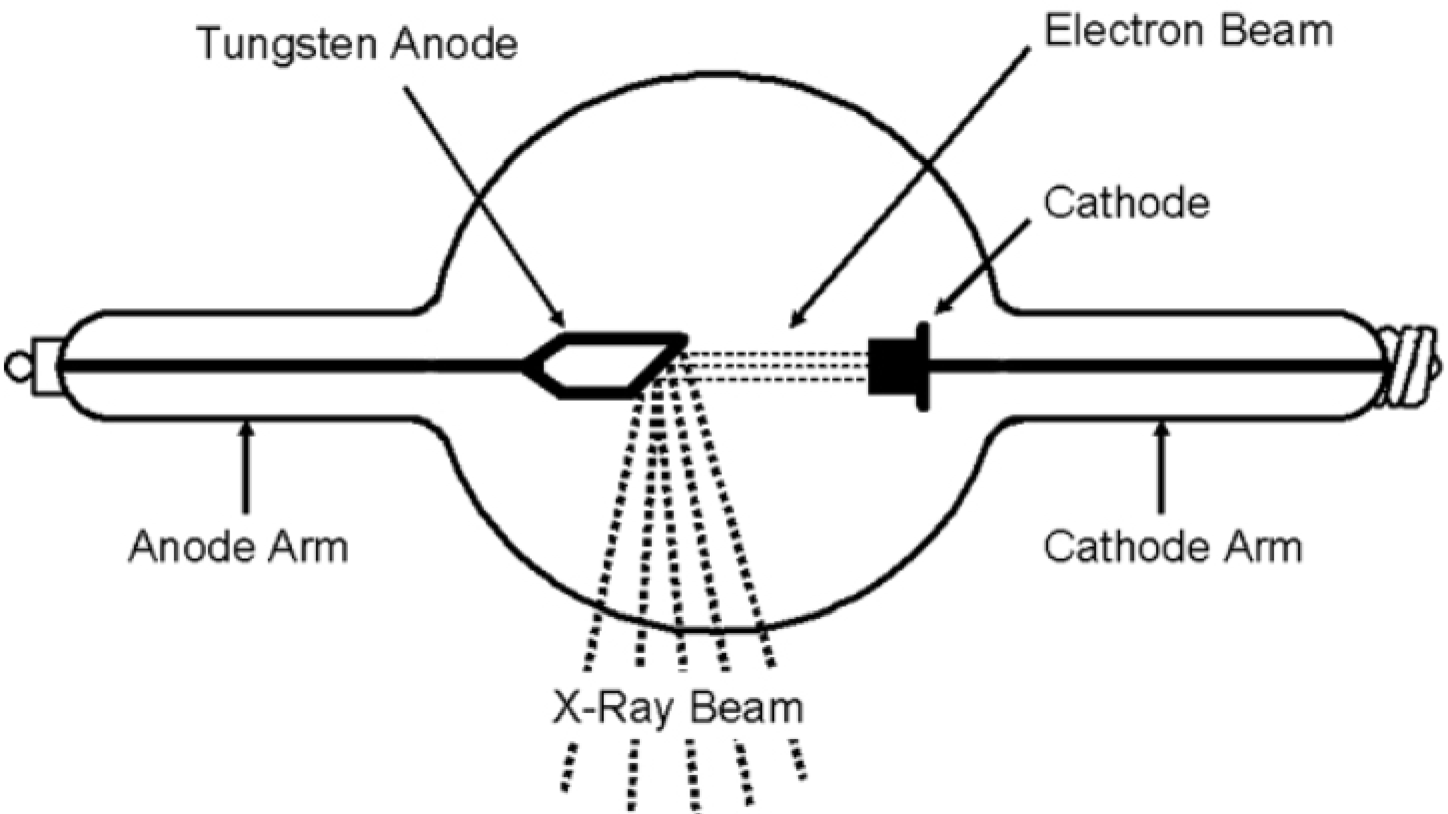
Dr. Yueping Zeng
SGNCOE
Honeywell Corporation
WFU Physics Alumnus


The Physics Department hosts:
&
Career and Internship
Opportunities at
Honeywell
Led by WFU Physics Alumnus and
Honeywell Lead Systems Engineer
Dr. Yueping Zeng
4/7 | Olin 105 | 11am-12:30pm
Honeywell International
develops tech for aerospace
missions and has employment
and internship opportunities
for undergraduates in STEM
fields. Pizza will be served!

24	Mon: 03/13/2023	Chap. 9	Radiation from localized oscillating sources	#17	03/17/2023
25	Wed: 03/15/2023	Chap. 9	Radiation from oscillating sources		
26	Fri: 03/17/2023	Chap. 9 & 10	Radiation and scattering	#18	03/20/2023
27	Mon: 03/20/2023	Chap. 11	Special Theory of Relativity	#19	03/24/2023
28	Wed: 03/22/2023	Chap. 11	Special Theory of Relativity		
29	Fri: 03/24/2023	Chap. 11	Special Theory of Relativity	#20	03/27/2023
30	Mon: 03/27/2023	Chap. 14	Radiation from moving charges	#21	03/29/2023
31	Wed: 03/29/2023	Chap. 14	Radiation from accelerating charged particles	#22	03/31/2023
32	Fri: 03/31/2023	Chap. 14	Synchrotron radiation and Compton scattering	#23	04/3/2023
33	Mon: 04/03/2023	Chap. 15	Radiation from collisions of charged particles		
34	Wed: 04/05/2023	Chap. 13	Cherenkov radiation		
35	Fri: 04/07/2023		Special topic: E & M aspects of superconductivity		
36	Mon: 04/10/2023		Special topic: Quantum Effects in E & M		
37	Wed: 04/12/2023		Special topic: Quantum Effects in E & M		
38	Fri: 04/14/2023		Special topic: Quantum Effects in E & M		
	Mon: 04/17/2023		Presentations I		
	Wed: 04/19/2023		Presentations II		
	Fri: 04/21/2023		Presentations III		
39	Mon: 04/24/2023		Review		
40	Wed: 04/26/2023		Review		

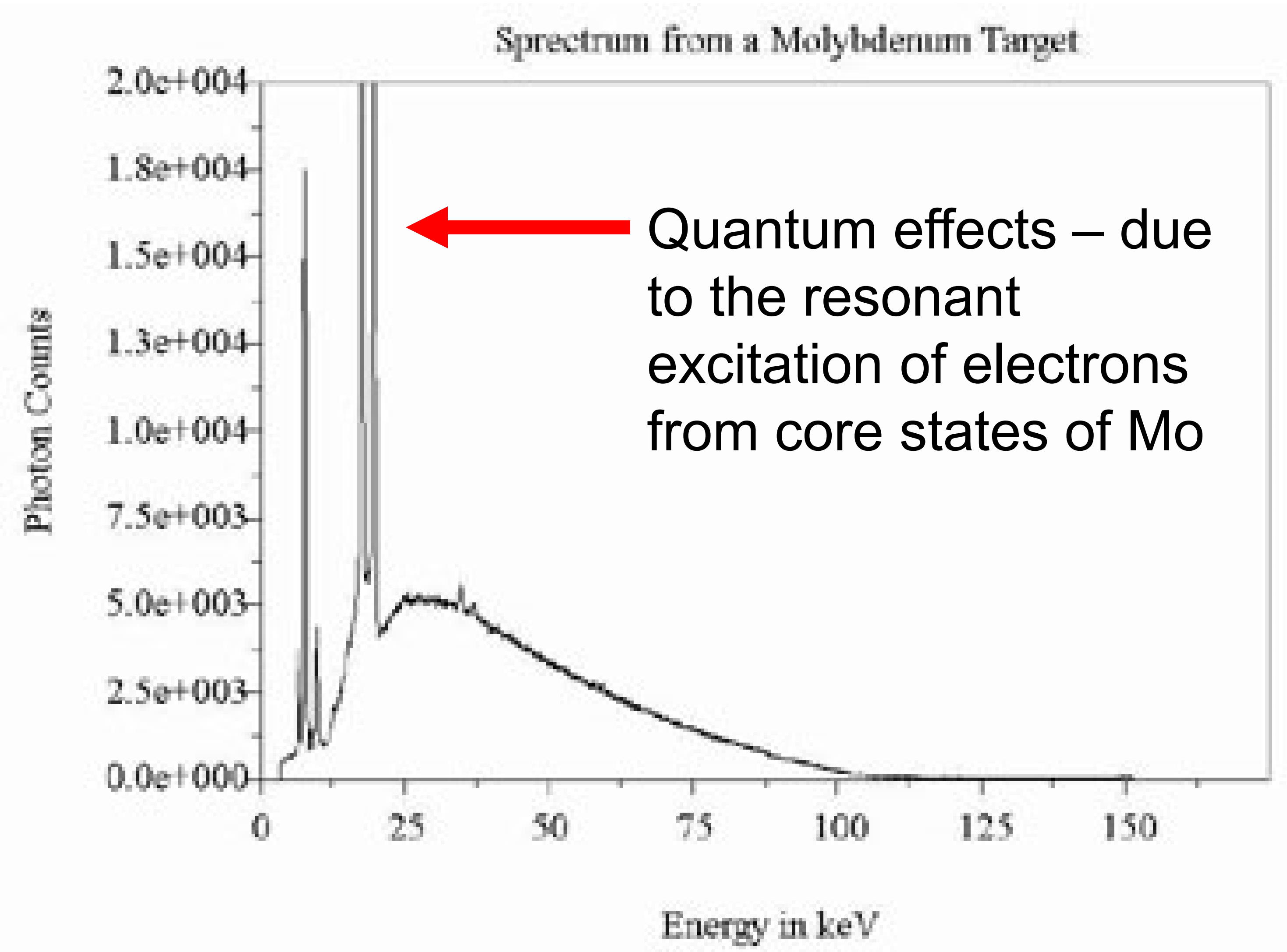
Generation of X-rays in a Coolidge tube

https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm



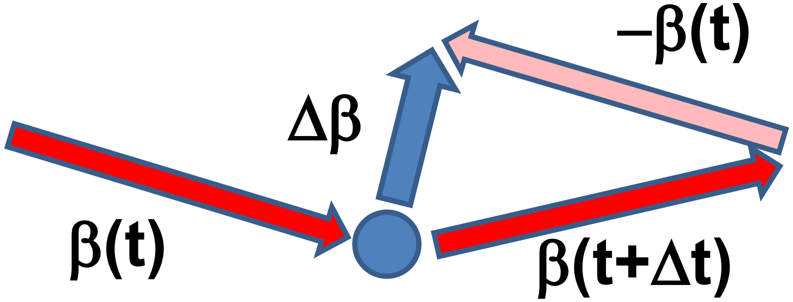
Invented in 1913. Associated with the German word "bremsstrahlung" – meaning breaking radiation.

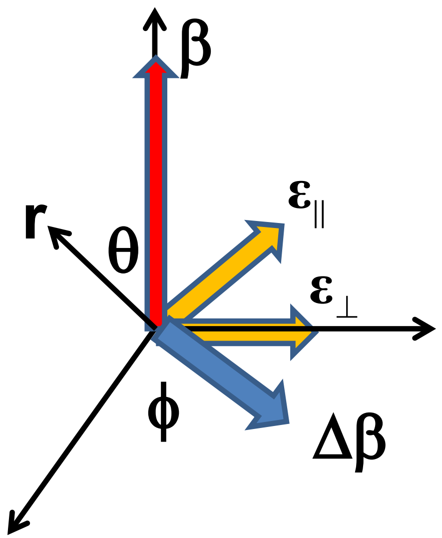
http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Quantum effects – due to the resonant excitation of electrons from core states of Mo

Radiation during collisions of charged particles





$\boldsymbol{\varepsilon}_{\parallel}$ is in the plane of $\boldsymbol{\beta}$ and $\mathbf{r}$

$\boldsymbol{\varepsilon}_{\perp}$ is perpendicular to the plane of $\boldsymbol{\beta}$ and $\mathbf{r}$

Results from previous analyses:

Spectral intensity of radiation from accelerating charged particle :

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\; e^{i\omega\left(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

Note that in the following slides we are taking the limit ω→0 but keeping the notation of the differential intensity….

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$;

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Note that $\boldsymbol{\varepsilon}$ is perpendicular to **r**.

Radiation during collisions -- continued

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \rightarrow 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

We will evaluate this expression for two cases:

Non-relativistic limit:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot(\Delta\boldsymbol{\beta})\right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$$

Relativistic collision with small $|\Delta\boldsymbol{\beta}| \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$:

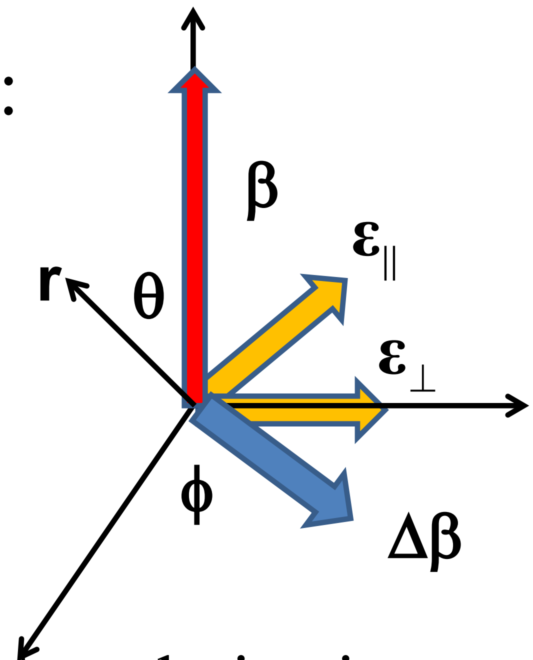
$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$

In the limit $\beta \rightarrow 0$, this is the same as the non-relativistic case.

Radiation during collisions -- continued

Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta}+\hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$



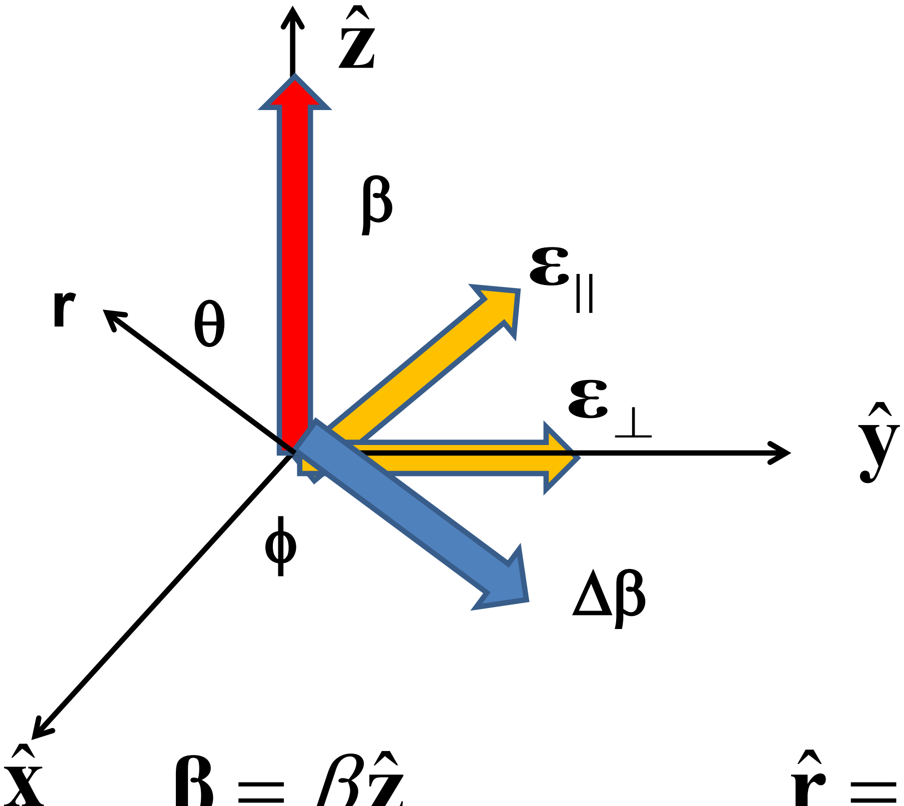
Also assume $\Delta\boldsymbol{\beta}$ is perpendicular to $\boldsymbol{\beta}$ direction

Expressions (averaging over φ) for || or $\perp$ polarization:

$$\frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\frac{(\beta-\cos\theta)^2}{(1-\beta\cos\theta)^4}$$ polarization in ***r*** and ***β*** plane

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\frac{1}{(1-\beta\cos\theta)^2}$$ polarization perpendicular to ***r*** and ***β*** plane

Some details:



(using geometry of Fig. 15.2 in Jackson)

$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}} \qquad \hat{\mathbf{r}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{||} = -\cos\theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{z}} \qquad \boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}}$$

$$\Delta\boldsymbol{\beta} = \Delta\beta\left(\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}\right)$$

Note: This is a wild assumption!

Some details -- continued:

$$\hat{\mathbf{r}} = \sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\parallel} = -\cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{z}}$$

Consistent with radiation from charged particles.

$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}}$$

Convenient geometry

$$\Delta\boldsymbol{\beta} = \Delta\beta\left(\cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}}\right)$$

Wild guess

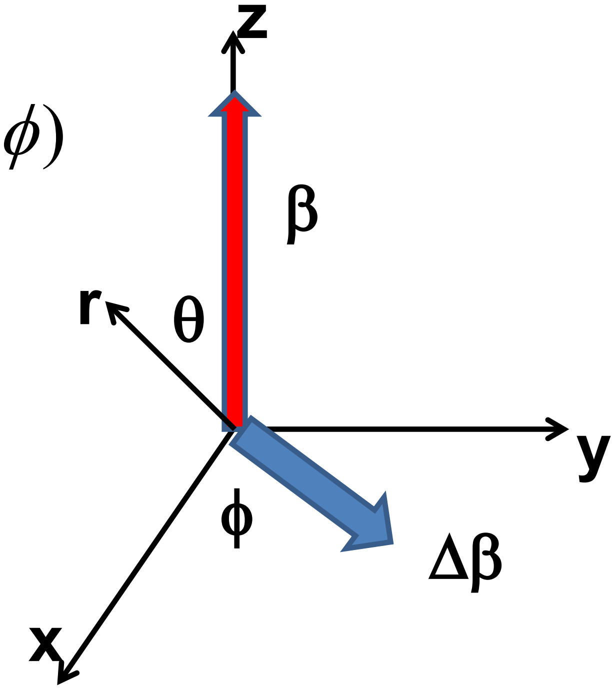
$$\Delta\boldsymbol{\beta} + \hat{\mathbf{r}} \times \left(\boldsymbol{\beta} \times \Delta\boldsymbol{\beta}\right) = \Delta\boldsymbol{\beta}\left(1 - \hat{\mathbf{r}} \cdot \boldsymbol{\beta}\right) + \boldsymbol{\beta}\left(\hat{\mathbf{r}} \cdot \Delta\boldsymbol{\beta}\right)$$

$$\boldsymbol{\varepsilon}_{\perp} \cdot \left(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}} \times \left(\boldsymbol{\beta} \times \Delta\boldsymbol{\beta}\right)\right) = \Delta\beta\sin\phi\left(1 - \beta\cos\theta\right)$$

$$\boldsymbol{\varepsilon}_{\parallel} \cdot \left(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}} \times \left(\boldsymbol{\beta} \times \Delta\boldsymbol{\beta}\right)\right) = \Delta\beta\cos\phi\left(\beta - \cos\theta\right)$$

Radiation during collisions -- continued

Intensity expressions: (averaging over ϕ)



$$\frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$

Relativistic collision at low ω and with small $|\Delta\boldsymbol{\beta}|$ and $\Delta\boldsymbol{\beta}$ perpendicular to plane of $\hat{\mathbf{r}}$ and $\boldsymbol{\beta}$, as a function of θ where $\hat{\mathbf{r}}\cdot\boldsymbol{\beta} = \beta\cos\theta$;

Integrating over solid angle:

$$\frac{dI}{d\omega} = \int d\Omega \left(\frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right) = \frac{2}{3\pi}\frac{q^2}{c}\gamma^2 \; |\Delta\boldsymbol{\beta}|^2$$

Some more details:

$$\int d\Omega \frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \, 2\pi \int_{-1}^{1} d\cos\theta \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$= \frac{q^2}{4\pi c} |\Delta\boldsymbol{\beta}|^2 \frac{2}{3} \frac{1}{(1-\beta^2)}$$

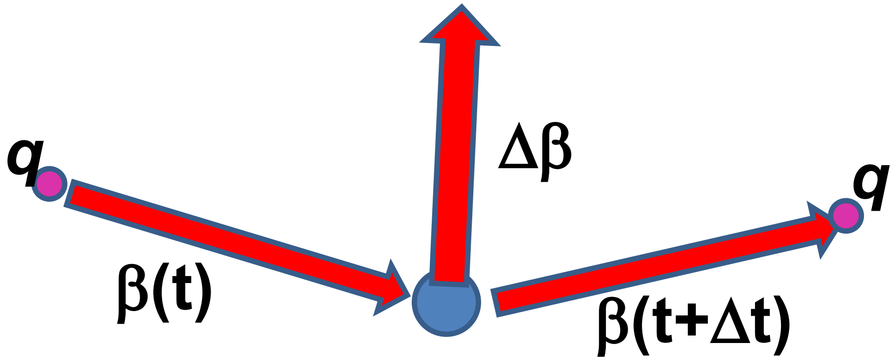
$$\int d\Omega \frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \int_{-1}^{1} d\cos\theta \frac{1}{(1-\beta\cos\theta)^2}$$

$$= \frac{q^2}{4\pi c} |\Delta\boldsymbol{\beta}|^2 \frac{2}{(1-\beta^2)}$$

$$\frac{dI}{d\omega} = \int d\Omega \left(\frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right) = \frac{2}{3\pi} \frac{q^2}{c} \gamma^2 \; |\Delta\boldsymbol{\beta}|^2$$

Estimation of $\Delta\beta$

Need to consider the mechanics of collision; it is convenient to parameterize in terms of momentum --



Momentum transfer:

$$Qc \equiv \left|\mathbf{p}\left(t+\tau\right)-\mathbf{p}\left(t\right)\right|c \approx \gamma Mc^2\left|\mathbf{\Delta\beta}\right|$$

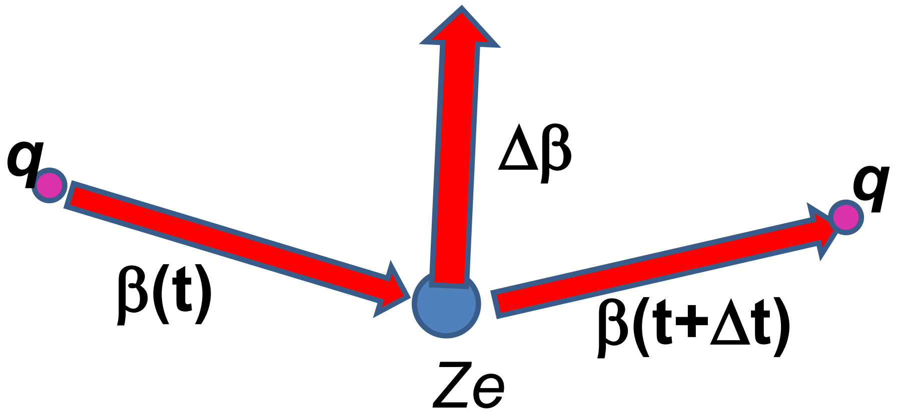
mass of particle having charge *q*

$$\frac{dI}{d\omega} = \frac{2}{3\pi}\ \frac{q^2}{c}\gamma^2\left|\mathbf{\Delta\beta}\right|^2 \approx \frac{2}{3\pi}\ \frac{q^2}{M^2c^3}Q^2$$

What are the conditions for the validity of this result?

What are possible mechanisms for the momentum transfer Q?

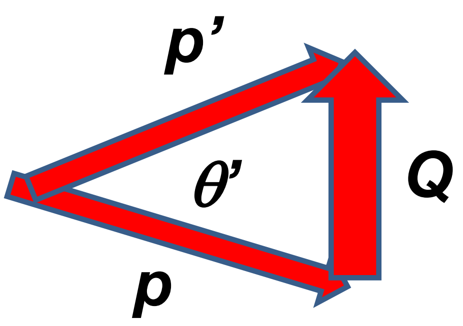
Estimation of Δβ or *Q* -- for the case of Rutherford scattering



Assume that target nucleus (charge *Ze*) has mass >>M;

Rutherford scattering cross-section in center of mass analysis:

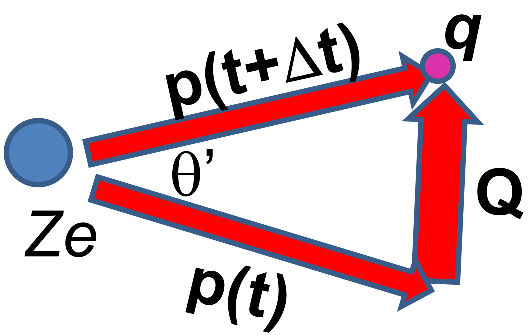
$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$



Assuming elastic scattering:

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2(1-\cos\theta')$$

Case of Rutherford scattering -- continued



Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin\left(\theta'/2\right)\right)^4}$$

$$\frac{d\sigma}{dQ} = \int_{\varphi'} \frac{d\sigma}{d\Omega} \left|\frac{d\Omega}{dQ}\right| d\varphi'$$

$$d\Omega = d\varphi' d\cos\theta'$$

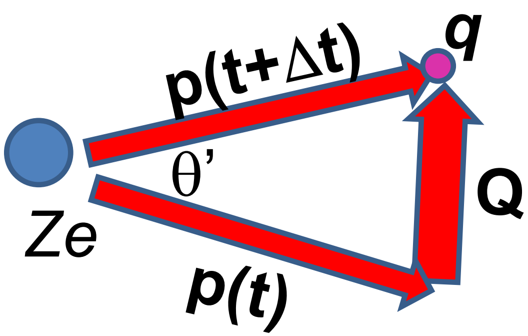
$$Q^2 = \left(2p\sin\left(\theta'/2\right)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

$$dQ = -\frac{p^2}{Q} d\cos\theta'$$

$$\Rightarrow \quad \frac{d\sigma}{dQ} = 8\pi\left(\frac{Zeq}{\beta c}\right)^2 \frac{1}{Q^3}$$

Does the algebra work out?

Case of Rutherford scattering -- continued



Differential radiation cross section :

$$\frac{d^2\chi}{d\omega dQ} = \frac{dI}{d\omega}\frac{d\sigma}{dQ} = \left(\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2\right)\left(8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}\right)$$

$$= \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\frac{1}{Q}$$

Differential radiation cross section -- continued

Integrating over momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

How do the limits of Q occur?

Jackson suggests that these come from the limits of validity of the analysis.

1. Seems like cheating?
2. Perhaps fair?

Comment on frequency dependence --

Original expression for radiation intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\ e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

In the previous derivations, we have assumed that

$$\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right) << 1.$$

$$\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right) = \omega\left(t-\hat{\mathbf{r}}\cdot\int_0^t dt'\boldsymbol{\beta}(t')\right) \approx \omega\tau\left(1-\hat{\mathbf{r}}\cdot\langle\boldsymbol{\beta}\rangle\right)$$

In the non-relativistic case, this means $\omega\tau \ll 1.$

Here τ is the effective collision time.

How to estimate the collision time?

Jackson uses the following analysis in terms of the impact parameter *b:*

Using classical mechanics and assuming $v \ll c$:

$$\tau \approx \frac{b}{v} \ll \frac{1}{\omega} \quad \text{and} \quad Q \approx \frac{2Zeq}{bv}$$

$$\text{Assume that } Q_{\min} = \frac{2Zeq}{b_{\max} v} = \frac{2Zeq\omega}{v^2}$$

Differential radiation cross section -- continued

Radiation cross section in terms of momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2} \ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Note that: $Q^2 = 2p^2(1-\cos\theta') \quad \Rightarrow Q_{\max} = 2p$

In general, $Q_{\min}$ is determined by the collision time condition $\omega\tau < 1 \quad \Rightarrow Q_{\min} \approx \frac{2Zeq\omega}{v^2}$

Radiation cross section for classical non-relativistic process

$$\frac{d\chi}{d\omega} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2} \ln\left(\frac{\lambda M v^3}{Zeq\omega}\right)$$

λ= “fudge factor” of order unity

What could be the origin of the fudge factor?

What do you take away from this analysis

1. Disgust?
2. Admiration?
3. Motivation to avoid charged particles?