PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Discussion for Lecture 34:

Special Topics in Electrodynamics:

Cherenkov radiation

References: Jackson Chapter 13.4

Zangwill Chapter 23.7

Smith Chapter 6.4

Physics Colloquium

4 PM in Olin 101

Thursday

.

April 6th, 2023

An introduction to Honeywell's contributions to NASA's human space flight missions such as the Orion and Artemis missions to the Moon and to Mars

Honeywell contributes to several of NASA's human space programs including Orion, Artemis, and Mars missions. Specifically, Honeywell develops navigation tools which are commonly used in inertial measurement units. For example, these tools performed very well during a recent 26-day mission in the Artemis program involving a low earth orbit of an unmanned test space vehicle. This talk will present an overview of the inertial measurement units and discuss some of physics, engineering, and analyses needed for further improvements. Some of the discussion involves analysis of sensor noise signatures and dynamic calibrations using Kaman filtering. Next generation sensors may include Gaussian-Markov models and estimates of quantum and general relativistic effects.

Dr. Yueping Zeng
SGNCOE
Honeywell Corporation
WFU Physics Alumnus


The Physics Department hosts:
&
Career and Internship
Opportunities at
Honeywell
Led by WFU Physics Alumnus and
Honeywell Lead Systems Engineer
Dr. Yueping Zeng
4/7 | Olin 105 | 11am-12:30pm
Honeywell International
develops tech for aerospace
missions and has employment
and internship opportunities
for undergraduates in STEM
fields. Pizza will be served!

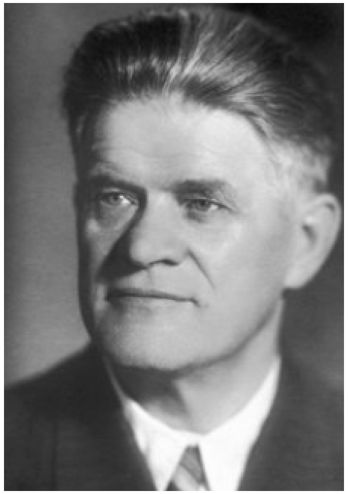
24	Mon: 03/13/2023	Chap. 9	Radiation from localized oscillating sources	#17	03/17/2023
25	Wed: 03/15/2023	Chap. 9	Radiation from oscillating sources		
26	Fri: 03/17/2023	Chap. 9 & 10	Radiation and scattering	#18	03/20/2023
27	Mon: 03/20/2023	Chap. 11	Special Theory of Relativity	#19	03/24/2023
28	Wed: 03/22/2023	Chap. 11	Special Theory of Relativity		
29	Fri: 03/24/2023	Chap. 11	Special Theory of Relativity	#20	03/27/2023
30	Mon: 03/27/2023	Chap. 14	Radiation from moving charges	#21	03/29/2023
31	Wed: 03/29/2023	Chap. 14	Radiation from accelerating charged particles	#22	03/31/2023
32	Fri: 03/31/2023	Chap. 14	Synchrotron radiation and Compton scattering	#23	04/3/2023
33	Mon: 04/03/2023	Chap. 15	Radiation from collisions of charged particles		
34	Wed: 04/05/2023	Chap. 13	Cherenkov radiation		
35	Fri: 04/07/2023		Special topic: E & M aspects of superconductivity		
36	Mon: 04/10/2023		Special topic: Quantum Effects in E & M		
37	Wed: 04/12/2023		Special topic: Quantum Effects in E & M		
38	Fri: 04/14/2023		Special topic: Quantum Effects in E & M		
	Mon: 04/17/2023		Presentations I		
	Wed: 04/19/2023		Presentations II		
	Fri: 04/21/2023		Presentations III		
39	Mon: 04/24/2023		Review		
40	Wed: 04/26/2023		Review		

Cherenkov radiation

Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013. http://www.britannica.com/EBchecked/media/174732

The Nobel Prize in Physics 1958

Pavel A. Cherenkov
Il´ja M. Frank
Igor Y. Tamm



Affiliation at the time of the award: P.N. Lebedev Physical Institute, Moscow, USSR

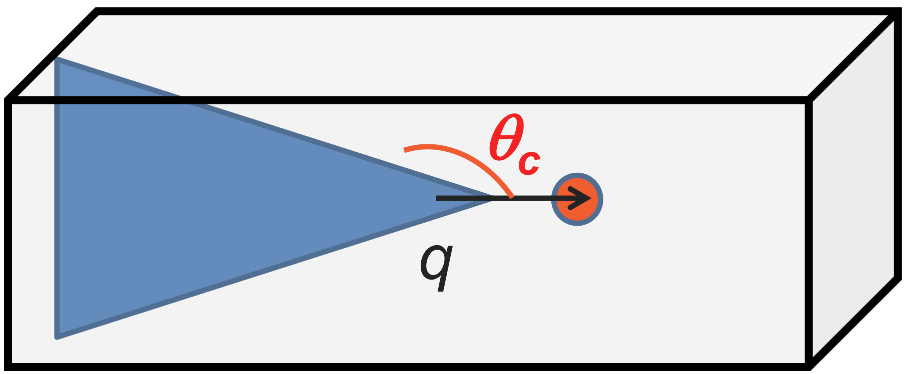
Prize motivation: "for the discovery and the interpretation of the Cherenkov effect."

https://www.nobelprize.org/prizes/physics/1958/ceremony-speech/

References for notes: Glenn S. Smith, *An Introduction to Electromagnetic Radiation* (Cambridge UP, 1997), Andrew Zangwill, Modern Electrodynamics (Cambridge UP, 2013)

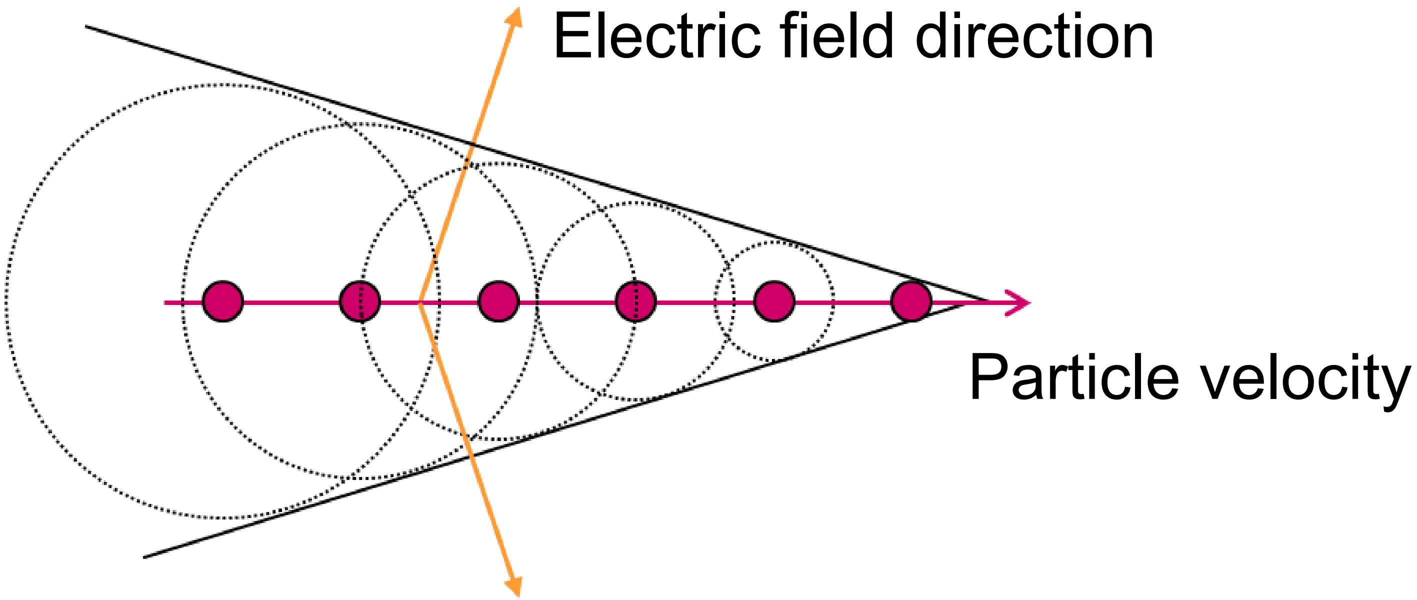
Cherenkov radiation

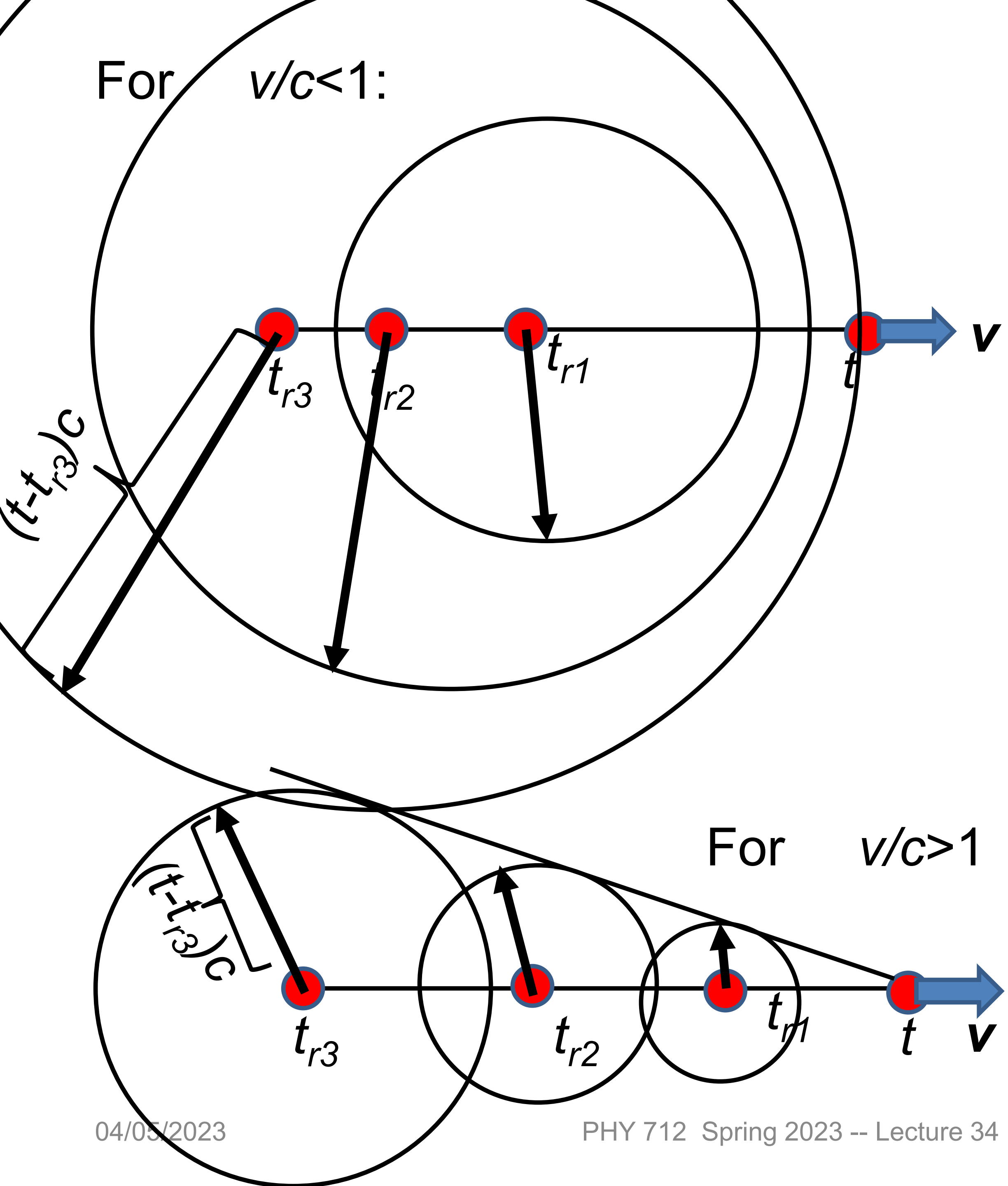
Discovered ~1930; bluish light emitted by energetic charged particles traveling within dielectric materials



Note that some treatments give the critical angle as $\theta_c - \pi/2$.

From: http://large.stanford.edu/courses/2014/ph241/alaeian2/




For v/c<1:
t_r3
t_r2
t_r1
t
v
(t-t_r3)c
For v/c>1
t_r3
t_r2
t_r1
t
v
(t-t_r3)c

Maxwell's potential equations within a material having permittivity and permeability (Lorentz gauge; cgs Gaussian units)

$$\nabla^2\Phi - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\rho$$

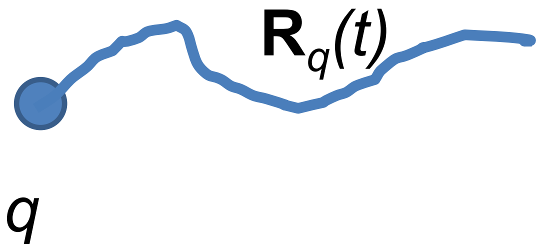
$$\nabla^2\mathbf{A} - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\mathbf{J}$$

Here the values of μ and ε depend on the material and on frequency.

Source: charged particle moving on trajectory $\mathbf{R}_q(t)$:

$$\rho(\mathbf{r},t) = q\delta\left(\mathbf{r}-\mathbf{R}_q(t)\right)$$

$$\mathbf{J}(\mathbf{r},t) = q\dot{\mathbf{R}}_q(t)\delta\left(\mathbf{r}-\mathbf{R}_q(t)\right)$$



Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon} \frac{1}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

$$\mathbf{A}(\mathbf{r},t) = q\mu \frac{\boldsymbol{\beta}_n}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r)$$

$$\boldsymbol{\beta}_n(t_r) \equiv \frac{\dot{\mathbf{R}}_q(t_r)}{c_n} \qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

$$t_r = t - \frac{R(t_r)}{c_n}$$

Example --

$$\beta_n \equiv \frac{v}{c_n} \qquad\qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

Consider water with $n \approx 1.3$

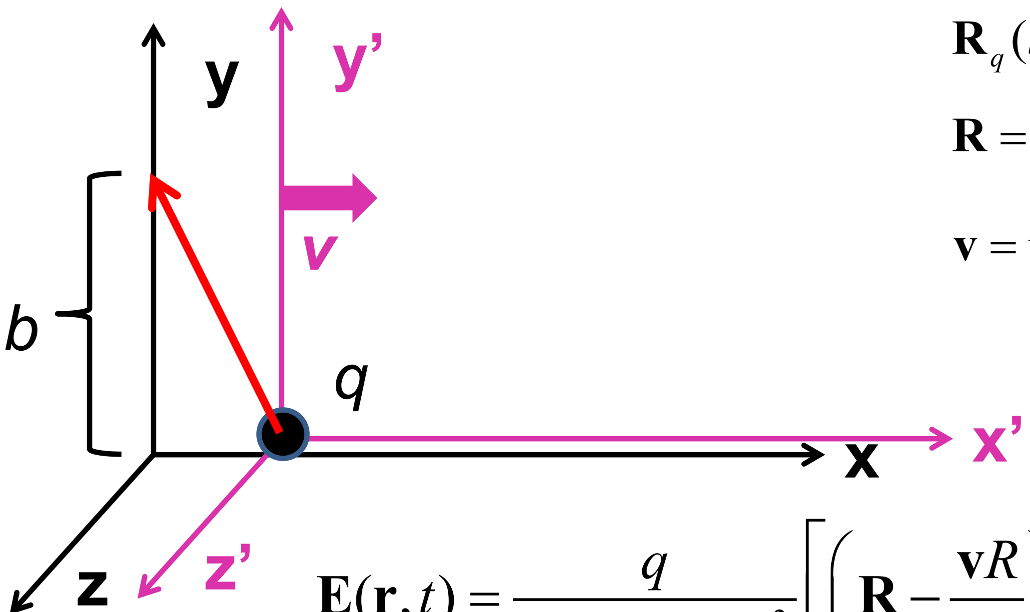
Which of these particles could produce Cherenkov radiation?

1. A neutron with speed c?
2. An electron with speed 0.6c?
3. A proton with speed 0.6c?
4. An electron with speed 0.8c?
5. An alpha particle with speed 0.8c?
6. None of these?

Further comment –

As discussed particularly in Chap. 13 of Jackson, a particle moving within a medium is likely to be slowed down so that the Cherenkov effect will only happen while $\beta_n>1$.

Recall – in Lecture 29, we considered a particle moving at constant velocity v in vacuum:



$$\mathbf{R}_q(t_r) = vt_r\hat{\mathbf{x}} \qquad \mathbf{r} = b\hat{\mathbf{y}}$$

$$\mathbf{R} = b\hat{\mathbf{y}} - vt_r\hat{\mathbf{x}} \qquad R = \sqrt{v^2 t_r^2 + b^2}$$

$$\mathbf{v} = v\hat{\mathbf{x}} \qquad t_r = t - \frac{R}{c}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\left(1 - \frac{v^2}{c^2}\right)\right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left(1 - \frac{v^2}{c^2}\right)\right]$$

Some details

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\left(1 - \frac{v^2}{c^2}\right)\right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left(1 - \frac{v^2}{c^2}\right)\right]$$

For our example:

$$\mathbf{R}_q(t_r) = vt_r\hat{\mathbf{x}} \qquad \mathbf{r} = b\hat{\mathbf{y}}$$

$$\mathbf{R} = b\hat{\mathbf{y}} - vt_r\hat{\mathbf{x}} \qquad R = \sqrt{v^2t_r^2 + b^2}$$

$$\mathbf{v} = v\hat{\mathbf{x}} \qquad t_r = t - \frac{R}{c}$$

t_r must be a solution to a quadratic equation: where $\frac{v}{c} \leq 1; \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

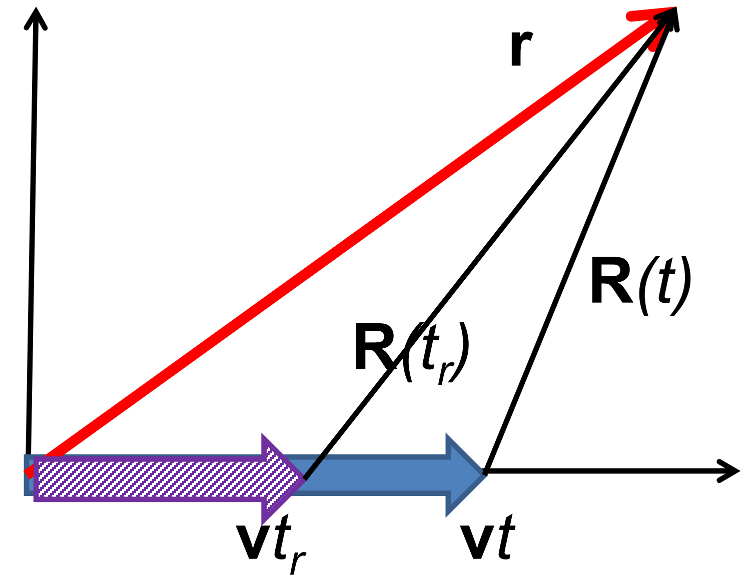
$$t_r - t = -\frac{R}{c} \qquad \Rightarrow \qquad t_r^2 - 2\gamma^2 t t_r + \gamma^2 t^2 - \gamma^2 b^2 / c^2 = 0$$

with the physical solution:

$$t_r = \gamma\left(\gamma t - \sqrt{(\gamma^2 - 1)t^2 + b^2 / c^2}\right) = \gamma\left(\gamma t - \frac{\sqrt{(v\gamma t)^2 + b^2}}{c}\right)$$

For Cherenkov case --

Consider a particle moving at constant velocity **v**; $v > c_n$



Some algebra

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r) = \left|\mathbf{R}(t) + \mathbf{v}(t - t_r)\right|$$

Quadratic equation for $(t - t_r)c_n$:

$$\left((t-t_r)c_n\right)^2 = R^2(t) + 2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t-t_r)c_n + \beta_n^{\,2}\left((t-t_r)c_n\right)^2$$

$$\left(\beta_n^{\,2} - 1\right)\left((t-t_r)c_n\right)^2 + 2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t-t_r)c_n + R^2(t) = 0$$

Quadratic equation for $(t-t_r)c_n$:

$$\left(\beta_n{}^2-1\right)\left((t-t_r)c_n\right)^2+2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t-t_r)c_n+R^2(t)=0$$

For β_n>1, how can the equality be satisfied?

1. No problem
2. It cannot be satisfied.
3. It can only be satisfied for special conditions

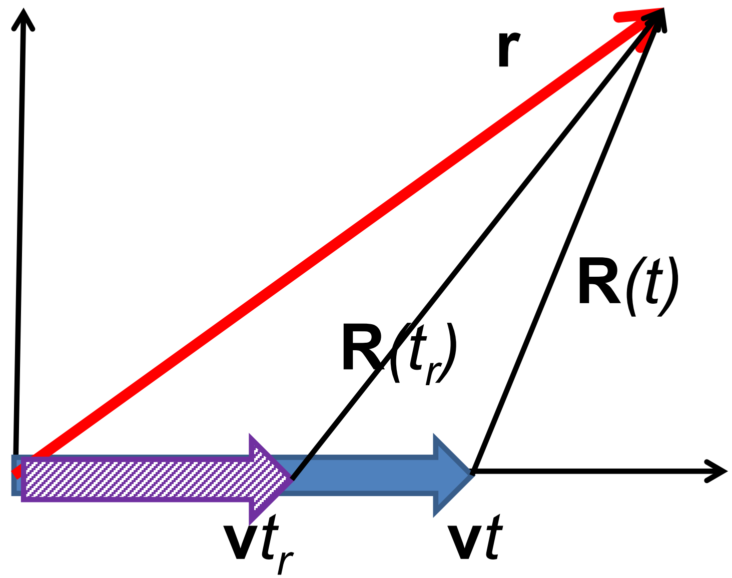
From solution of quadratic equation:

$$(t-t_r)c_n=\frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\pm\sqrt{\left(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\right)^2-\left(\beta_n{}^2-1\right)R^2(t)}}{\beta_n{}^2-1}$$

$\Rightarrow \mathbf{R}(t)\cdot\boldsymbol{\beta}_n<0$ (initial diagram is incorrect!)

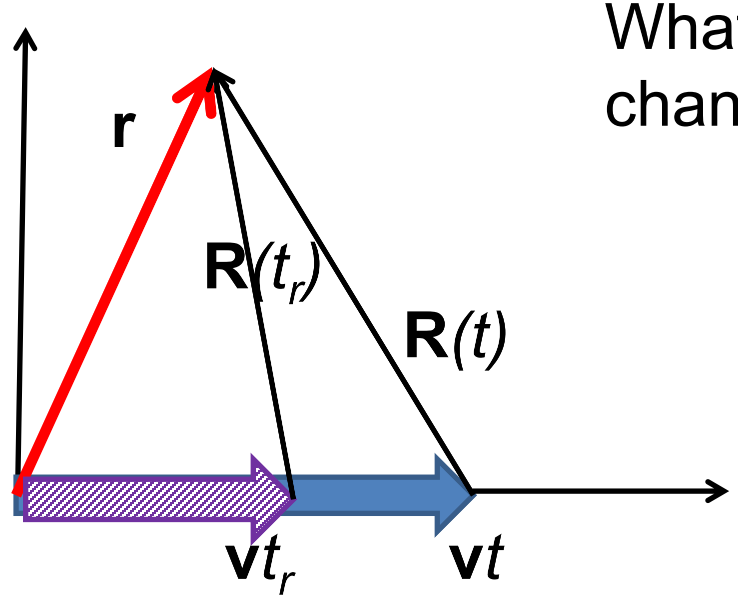
Moreover, there are two retarded time solutions!

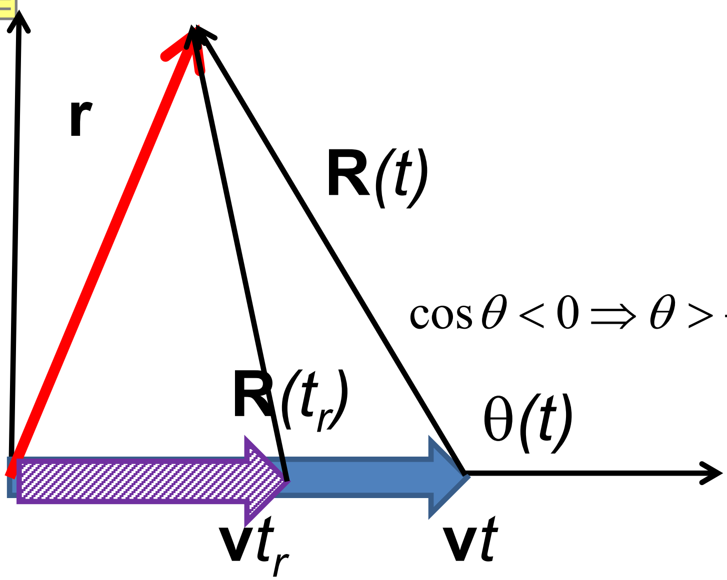
Original diagram:



What is the significance of changing the diagram?

New diagram:





$$\cos\theta \equiv \hat{\mathbf{R}}(t)\cdot\hat{\mathbf{v}}$$

$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r)$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n =$$

$$(t - t_r)c_n\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$= R(t_r)\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$R(t_r) = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm \sqrt{\left(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\right)^2 - \left(\beta_n{}^2 - 1\right)R^2(t)}}{\beta_n{}^2 - 1}$$

$$R(t_r) = \frac{R(t)}{\beta_n{}^2 - 1}\left(-\beta_n\cos\theta \pm \sqrt{1 - \beta_n{}^2\sin^2\theta}\right) = (t - t_r)c_n$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n = \mp R(t)\sqrt{1 - \beta_n{}^2\sin^2\theta}$$

Recall the Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon} \frac{1}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

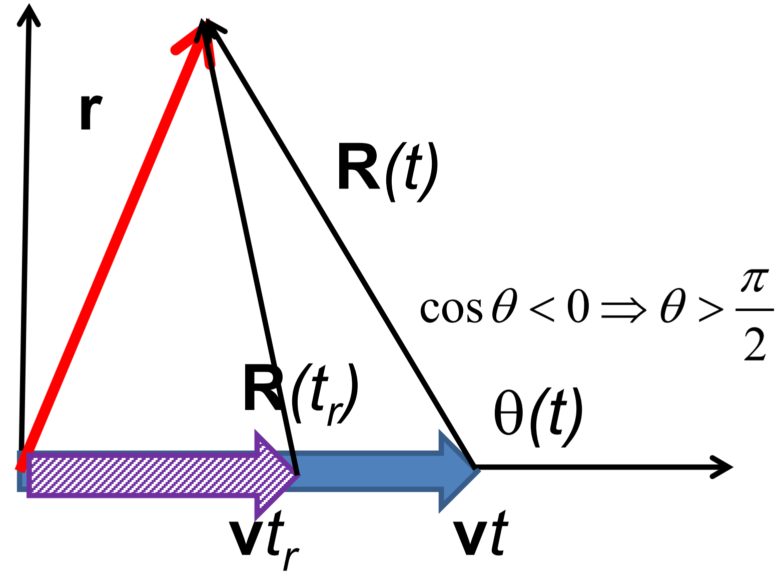
$$\mathbf{A}(\mathbf{r},t) = q\mu \frac{\boldsymbol{\beta}_n}{\left| R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r) \right|}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r)$$

$$\boldsymbol{\beta}_n(t_r) \equiv \frac{\dot{\mathbf{R}}_q(t_r)}{c_n} \qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

$$t_r = t - \frac{R(t_r)}{c_n}$$

Liénard-Wiechert potentials for two solutions:



$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon}\frac{1}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$

$$\mathbf{A}(\mathbf{r},t) = q\mu\frac{\boldsymbol{\beta}_n}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$

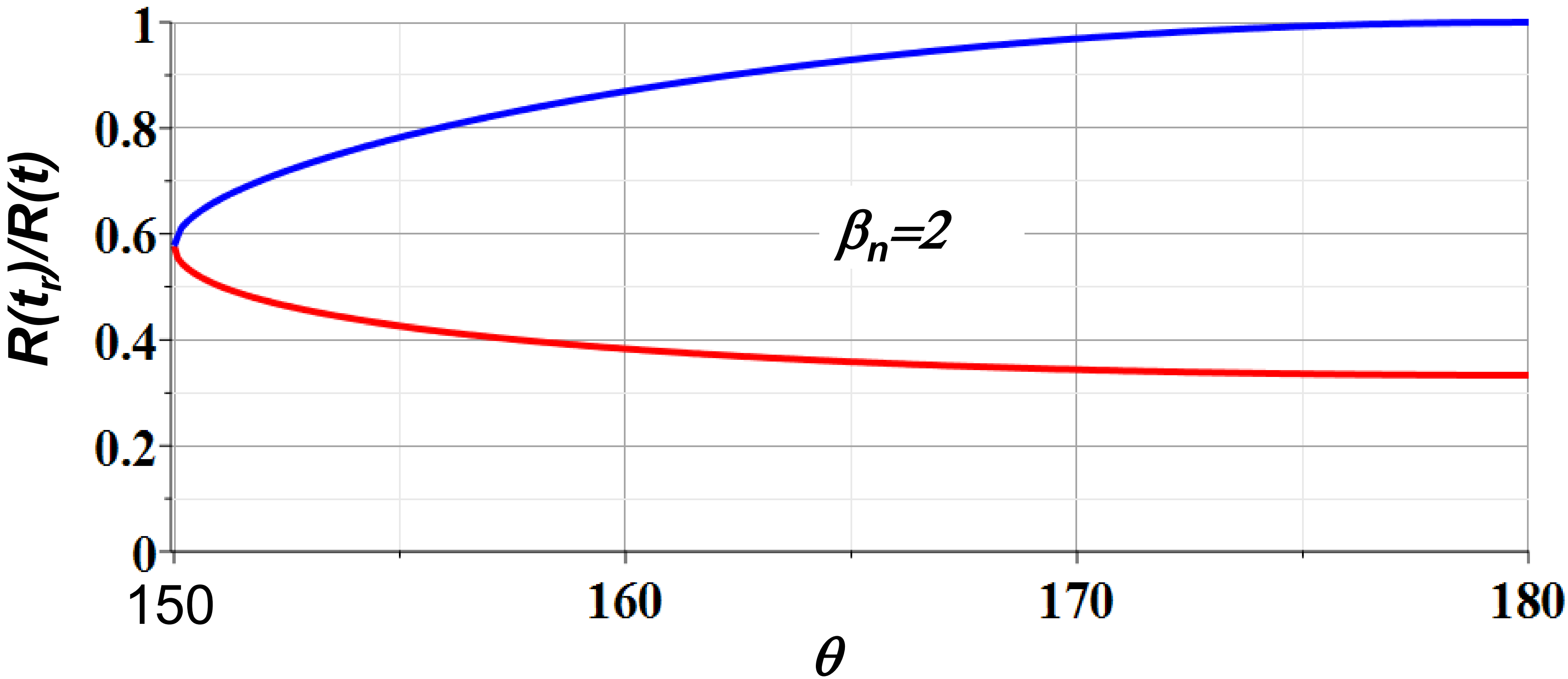
For $\beta_n > 1$, the range of θ is limited further:

$$R(t_r) = \frac{R(t)}{\beta_n{}^2 - 1}\left(-\beta_n\cos\theta \pm \sqrt{1-\beta_n{}^2\sin^2\theta}\right) \geq 0$$

$$\Rightarrow \left|\sin\theta\right| \leq \frac{1}{\beta_n} \equiv \left|\sin\theta_c\right| \quad \text{and} \quad \pi \geq \theta_c \geq \pi/2 \qquad \cos\,\theta_c = -\sqrt{1-\frac{1}{\beta_n^2}}$$

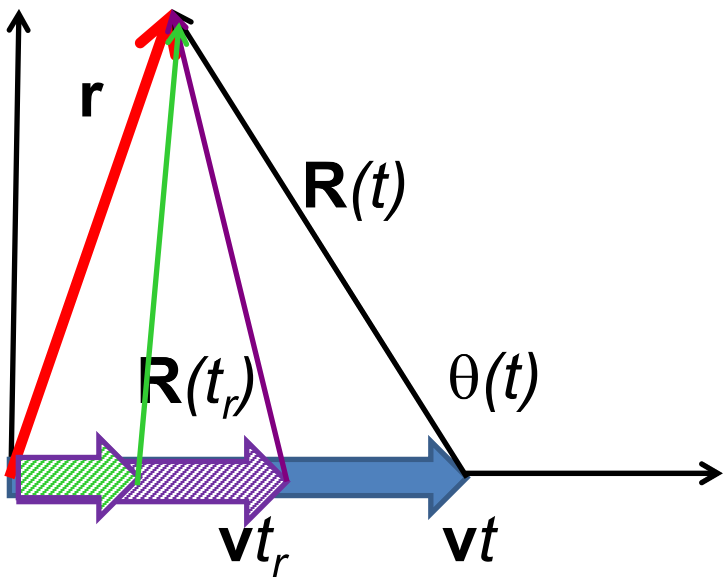
In this range, $\theta \geq \theta_c$

$$R(t_r) = \frac{R(t)}{\beta_n{}^2 - 1}\left(-\beta_n \cos\theta \pm \sqrt{1 - \beta_n{}^2 \sin^2\theta}\right)$$



$$\theta_c = 150^o \text{ for this case}$$

Physical fields for $\beta_n > 1$ -- two retarded solutions contribute



$$\theta \leq \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

$$\text{Define} \quad \cos\theta_C \equiv -\sqrt{1-\frac{1}{\beta_n{}^2}}$$

$$\Rightarrow \cos\theta \leq \cos\theta_C$$

Adding two solutions; in terms of Heaviside $\Theta(x)$:

$$\Phi\left(\mathbf{r},t\right) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}\left(\mathbf{r},t\right) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

Physical fields for $\beta_n > 1$

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

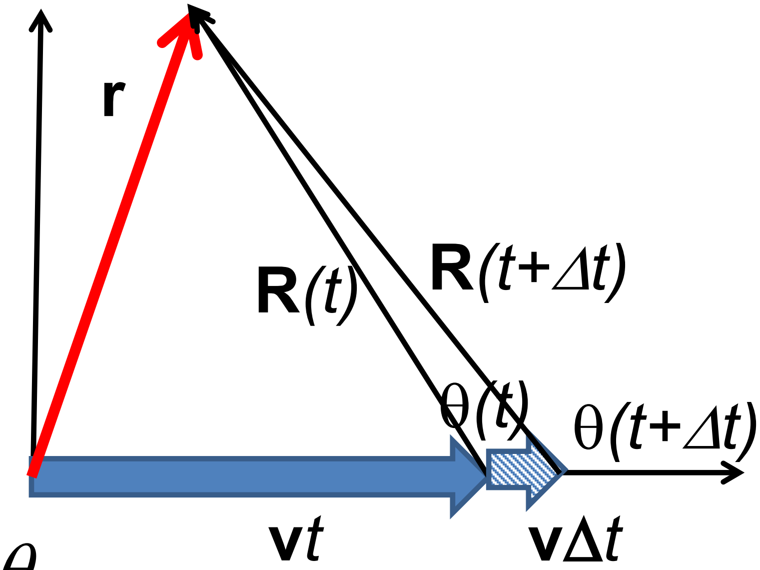
$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi - \frac{1}{c_n}\frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{\left(R(t)\right)^2\sqrt{1-\beta_n{}^2\sin^2\theta}} \quad \times$$

$$\left(-\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n{}^2-1}\;\delta(\cos\theta_C - \cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Intermediate steps:



$$\frac{d\theta}{dt} = \frac{v \sin\theta}{R} \qquad \frac{dR}{dt} = -v\cos\theta$$

Using instantaneous polar coordinates: $\nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial \theta}$

$$\nabla\Theta\left(\cos\theta_C - \cos\theta(t)\right) = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta\left(\cos\theta_C - \cos\theta(t)\right)}{\partial t} = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{v\sin^2\theta(t)}{R(t)}$$

Power radiated:

$$\frac{dP(t)}{d\Omega} = \left(R(t)\right)^2 \hat{\mathbf{R}} \cdot \mathbf{S}(t) = \left(R(t)\right)^2 \frac{c_n}{4\pi} \left|\mathbf{E}\left(\mathbf{R}(t), t\right)\right|^2 = \left|\boldsymbol{a}(t)\right|^2$$

where

$$\mathbf{E}(\mathbf{r}, t) = \frac{2q}{\varepsilon} \frac{\hat{\mathbf{R}}}{\left(R(t)\right)^2 \sqrt{1-\beta_n{}^2 \sin^2\theta}} \quad \times$$

$$\left(\frac{\beta_n{}^2 - 1}{1-\beta_n{}^2 \sin^2\theta} \Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n{}^2 - 1}\; \delta(\cos\theta_C - \cos\theta(t)) \right)$$

Spectral analysis using Parseval's theorem:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} \left|\boldsymbol{a}(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|\tilde{\boldsymbol{a}}(\omega)\right|^2 d\omega = \int_0^{\infty} \left(\left|\tilde{\boldsymbol{a}}(\omega)\right|^2 + \left|\tilde{\boldsymbol{a}}(-\omega)\right|^2 \right) d\omega = \int_0^{\infty} \frac{\partial^2 I}{\partial\Omega \partial\omega} d\omega$$

$$\tilde{\boldsymbol{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\; \boldsymbol{a}(t)\; e^{i\omega t}$$

From these results, we arrive at an approximate expression in terms of a constant *K* and critical angle θ_c:

$$\boldsymbol{a}(t)=\frac{K}{R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}}\left(-\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C-\cos\theta(t))+\sqrt{\beta_n{}^2-1}\;\delta(\cos\theta_C-\cos\theta(t))\right)$$

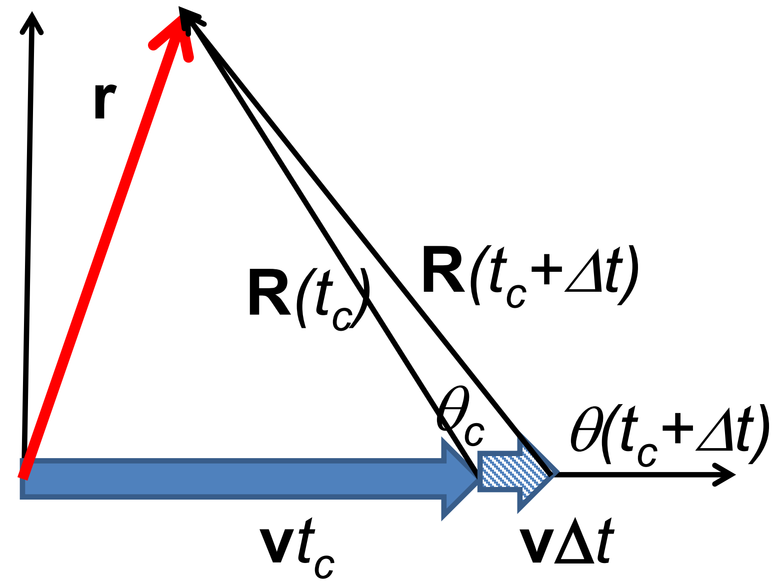
Analysis near the critical angle θ_c

Denote t_c as the time that the trajectory $\mathbf{R}(t_c)$ has the critical angle θ_c. For $t=t_c+\Delta t$:

$$\theta(t_c+\Delta t)=\theta_c+\Delta\theta(\Delta t) \text{ where } \Delta\theta(\Delta t)\approx v\Delta t\frac{\sin\theta_c}{R(t_c)}$$

$$\cos\theta_c-\cos\theta(t_c+\Delta t)\approx\frac{c_n\Delta t}{\beta_n R(t_c)}$$

$$1-\beta_n^2\sin^2\theta(t_c+\Delta t)\approx-\frac{2c_n\Delta\sqrt{\beta_n^2-1}}{R(t_c)}$$



Approximate amplitude near $t\approx t_c; t=t_c+\Delta t$:

$$\boldsymbol{a}(\Delta t)\approx K\frac{\left(\beta_n^2-1\right)^{1/4}}{(2c_n)^{3/2}\sqrt{R(t_c)}}\left(\frac{\delta(\Delta t)}{\sqrt{\Delta t}}-\frac{\Theta(\Delta t)}{2\sqrt{(\Delta t)^3}}\right)$$

Analysis continued --

Approximate amplitude near $t \approx t_c + \Delta t$:

$$\boldsymbol{a}(\Delta t) \approx K \frac{\left(\beta_n^2 - 1\right)^{1/4}}{(2c_n)^{3/2}\sqrt{R(t_c)}} \left(\frac{\delta(\Delta t)}{\sqrt{\Delta t}} - \frac{\Theta(\Delta t)}{2\sqrt{\left(\Delta t\right)^3}} \right)$$

Approximate Fourier amplitude :

$$\tilde{\boldsymbol{a}}(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \boldsymbol{a}(t) e^{i\omega\Delta t} d\Delta t \approx K \sqrt{\frac{\pi}{2}} \frac{\left(\beta_n^2 - 1\right)^{1/4} \left(1 - i\right)}{(c_n)^{3/2}\sqrt{R(t_c)}} \sqrt{\omega}$$

$$\frac{d^2I}{d\Omega d\omega} = \left|\tilde{\mathbf{a}}(\omega)\right|^2 + \left|\tilde{\mathbf{a}}^*(-\omega)\right|^2$$

$$\tilde{\mathbf{a}}(\omega) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \mathbf{a}(t)e^{i\omega\Delta t}d\Delta t \approx K\sqrt{\frac{\pi}{2}}\frac{\left(\beta_n^2-1\right)^{1/4}\left(1-i\right)}{(c_n)^{3/2}\sqrt{R(t_c)}}\sqrt{\omega}$$

$$\frac{d^2I}{d\Omega d\omega} \propto \omega\left(\beta_n^2-1\right)^{1/2}$$

Converting from intensity per unit solid angle

$\frac{d^2I}{d\ell d\omega} \propto \omega\left(\beta_n^2-1\right)$ to intensity per unit path length:

$$\frac{d^2I}{d\ell d\omega} \propto \omega\left(\beta_n^2-1\right)$$

Noting that $c_n = \frac{c}{n(\omega)} = \frac{c}{\sqrt{\epsilon(\omega)}}$ $\qquad \beta_n = \frac{v}{c_n}$

$$\frac{d^2I}{d\ell d\omega} \propto \omega\left(\epsilon(\omega)\frac{v^2}{c^2}-1\right) = \frac{2\pi}{\lambda}\left(\epsilon(\omega)\frac{v^2}{c^2}-1\right)$$

From this expression, how would you explain that Cherenkov radiation is typically observed as a blue glow?

1. It is still a mystery.
2. It is obvious from the result.

If $\varepsilon \approx 1.8$ for water, what is the slowest particle speed that can generate Cherenkov radiation?

a. $v = 0.9c$

b. $v = 0.8c$

c. $v = 0.7c$

c. $v = 0.6c$