PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Notes for Lecture 38:

Quantum effects in electrodynamics

Connections to experiment

a. Coherent states

b. Squeezed states

c. More complicated states

24	Mon: 03/13/2023	Chap. 9	Radiation from localized oscillating sources	#17	03/17/2023
25	Wed: 03/15/2023	Chap. 9	Radiation from oscillating sources		
26	Fri: 03/17/2023	Chap. 9 & 10	Radiation and scattering	#18	03/20/2023
27	Mon: 03/20/2023	Chap. 11	Special Theory of Relativity	#19	03/24/2023
28	Wed: 03/22/2023	Chap. 11	Special Theory of Relativity		
29	Fri: 03/24/2023	Chap. 11	Special Theory of Relativity	#20	03/27/2023
30	Mon: 03/27/2023	Chap. 14	Radiation from moving charges	#21	03/29/2023
31	Wed: 03/29/2023	Chap. 14	Radiation from accelerating charged particles	#22	03/31/2023
32	Fri: 03/31/2023	Chap. 14	Synchrotron radiation and Compton scattering	#23	04/3/2023
33	Mon: 04/03/2023	Chap. 15	Radiation from collisions of charged particles		
34	Wed: 04/05/2023	Chap. 13	Cherenkov radiation		
35	Fri: 04/07/2023		Special topic: E & M aspects of superconductivity		
36	Mon: 04/10/2023		Special topic: Quantum Effects in E & M		
37	Wed: 04/12/2023		Special topic: Quantum Effects in E & M		
38	Fri: 04/14/2023		Special topic: Quantum Effects in E & M		
	Mon: 04/17/2023		Presentations I		
	Wed: 04/19/2023		Presentations II		
	Fri: 04/21/2023		Presentations III		
39	Mon: 04/24/2023		Review		
40	Wed: 04/26/2023		Review		

PHY 712 Presentation Schedule
There will be 3 or 4 ~12-minute presentations each day

Monday, April 17, 2023

	Name	Presentation topic
1	Arezoo Nameny	Scanning electron microscopy
2	Lee Pryor	Jackson Problem 11.5
3	Moti Mirhosseini	Jackson Problem 7.22
4		

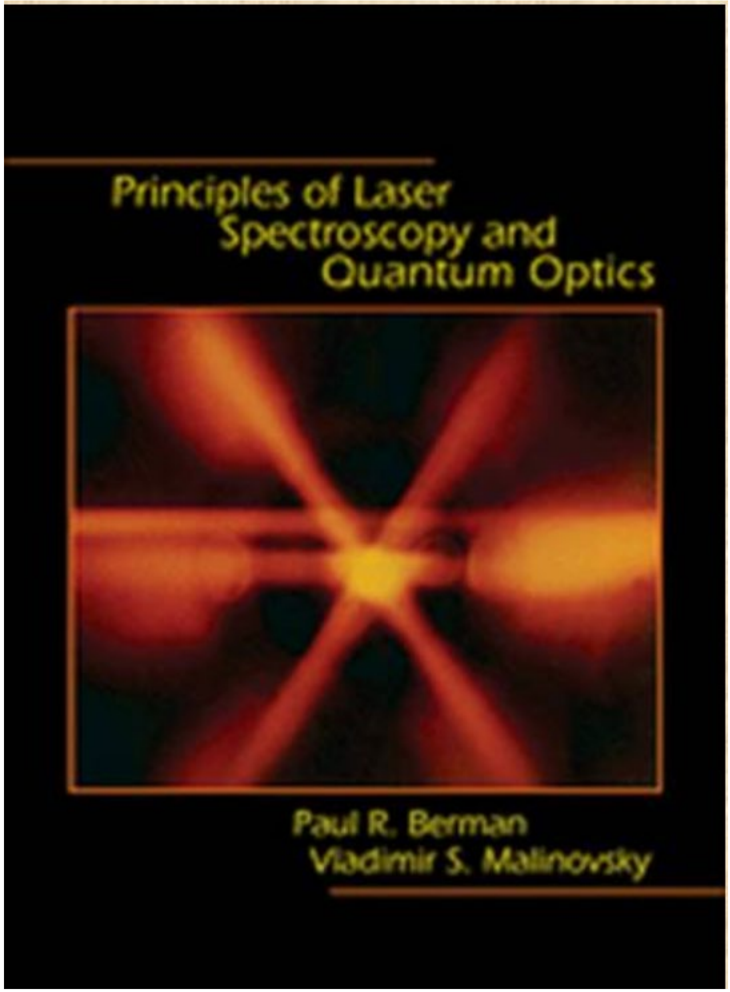
Wednesday, April 19, 2023

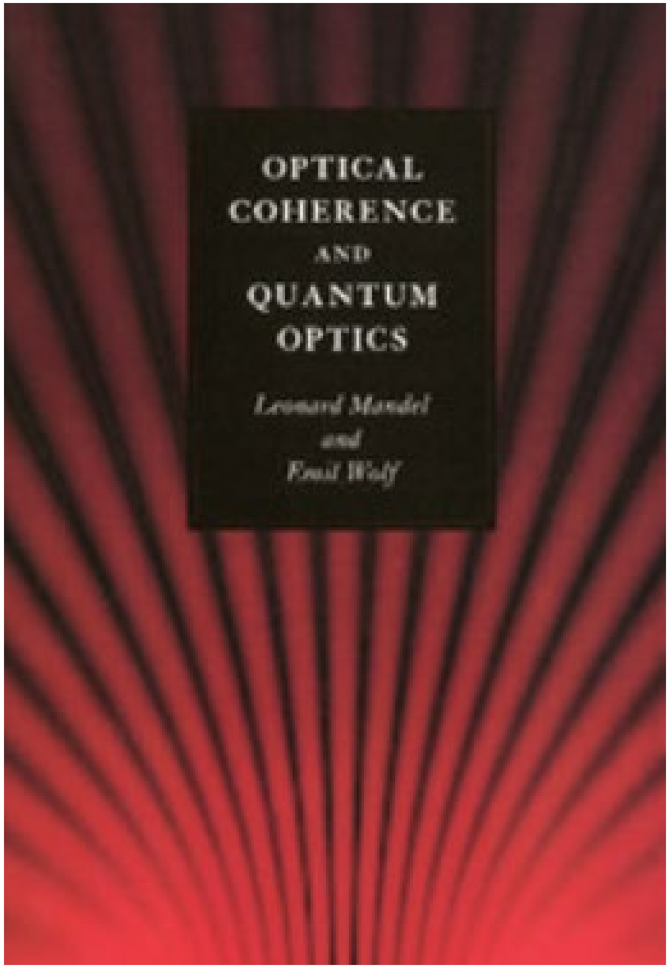
	Name	Presentation topic
1		
2	Katie Koch	NOPAs
3	Evan Kumar	Spectral Analysis of the Free Electron Laser
4	Banasree Sarkar Mou	2D computational Optical imaging with FDTD in photonics and electrodynamics

Friday, April 21, 2023

	Name	Presentation topic
1	David Carchipulla-Morales	Ewald Summation
2	Zezhong Zhang	Electromagnetism of the Earth and other stars
3	Samuel Griffith	Hyperfine Hamiltonian
4	Caela Flake	

Recommended reading





Princeton U Press 2010

Cambridge U Press 1995

More comments about the coherent state

Previously we discussed Glauber's coherent state (which Prof. Kandada assures me is well realized as the output of continuous wave (CW) lasers) formed as a superposition of single photon eigenstates:

$$\left|c_\alpha\right\rangle \equiv \sum_{n=0}^{\infty} \frac{\alpha^n e^{-|\alpha|^2/2}}{\sqrt{n!}}\left|n\right\rangle$$ based on a single mode $n \rightarrow n_{\mathbf{k}\sigma}$ where α

is a complex number. It turns out these states do not form a complete representation of the system; in fact they are "over complete".

It is sometimes convenient to represent these coherent states as a shift of the vacuum state $\left|0\right\rangle$:

$$\left|c_\alpha\right\rangle \equiv e^{-|\alpha|^2/2} \exp\left(\alpha a^\dagger\right)\left|0\right\rangle$$

Note that the operator $D\left(\alpha,\alpha^*\right) \equiv e^{-|\alpha|^2/2} \exp\left(\alpha a^\dagger\right)$ is unitary

and $D^\dagger\left(\beta,\beta^*\right) a\, D\left(\beta,\beta^*\right) = a + \beta$ and $D^\dagger\left(\beta,\beta^*\right) a^\dagger\, D\left(\beta,\beta^*\right) = a^\dagger + \beta^*$

Further analysis and modifications of the "coherent state"

Recall that we can write the EM Hamiltonian for a single mode $\omega_{\mathbf{k}} \equiv \omega$ --

$$H = \frac{1}{2}\hbar\omega\left(a^\dagger a + a a^\dagger\right) \quad \text{where } \left[a, a^\dagger\right] = 1$$

Define convenient unitless operators

$$\hat{X} \equiv \frac{1}{2}\left(a^\dagger + a\right) \quad \text{and} \quad \hat{Y} \equiv \frac{i}{2}\left(a^\dagger - a\right) \quad \Rightarrow \left[\hat{X}, \hat{Y}\right] = \frac{i}{2}$$

$$H = \hbar\omega\left(\hat{X}^2 + \hat{Y}^2\right)$$

From the Heisenberg uncertainty ideas applied to the standard deviations:

$$\Delta\hat{X}\,\Delta\hat{Y} \geq \frac{1}{4}$$

In terms of the eigenstates of the EM Hamiltonian:

$$H\left|n\right\rangle = \hbar\omega\left(n+\frac{1}{2}\right)\left|n\right\rangle$$

$$\Delta\hat{X} = \sqrt{\left\langle n\left|\hat{X}^2\right|n\right\rangle - \left|\left\langle n\left|\hat{X}\right|n\right\rangle\right|^2} = \sqrt{\frac{1}{2}\left(n+\frac{1}{2}\right)} = \Delta\hat{Y}$$

$$\Rightarrow \Delta\hat{X}\Delta\hat{Y} = \frac{1}{2}\left(n+\frac{1}{2}\right)$$

For the coherent state:

$$\left|c_\alpha\right\rangle \equiv \sum_{n=0}^{\infty} \frac{\alpha^n e^{-|\alpha|^2/2}}{\sqrt{n!}} \left|n\right\rangle$$

$$\Delta\hat{X} = \sqrt{\left\langle c_\alpha \left| \hat{X}^2 \right| c_\alpha \right\rangle - \left| \left\langle c_\alpha \left| \hat{X} \right| c_\alpha \right\rangle \right|^2} = \frac{1}{2} = \Delta\hat{Y}$$

$$\Rightarrow \Delta\hat{X}\Delta\hat{Y} = \frac{1}{4}$$

In this sense, the coherent state represents the minimum uncertainty process. For the pure phonon eigenstates, only the vacuum state has this minimum uncertainty.

We can use the operators $\hat{X}$ and $\hat{Y}$ to describe the EM fields:

Electric field:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad \Rightarrow \mathbf{E}(\mathbf{r},t) = i\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

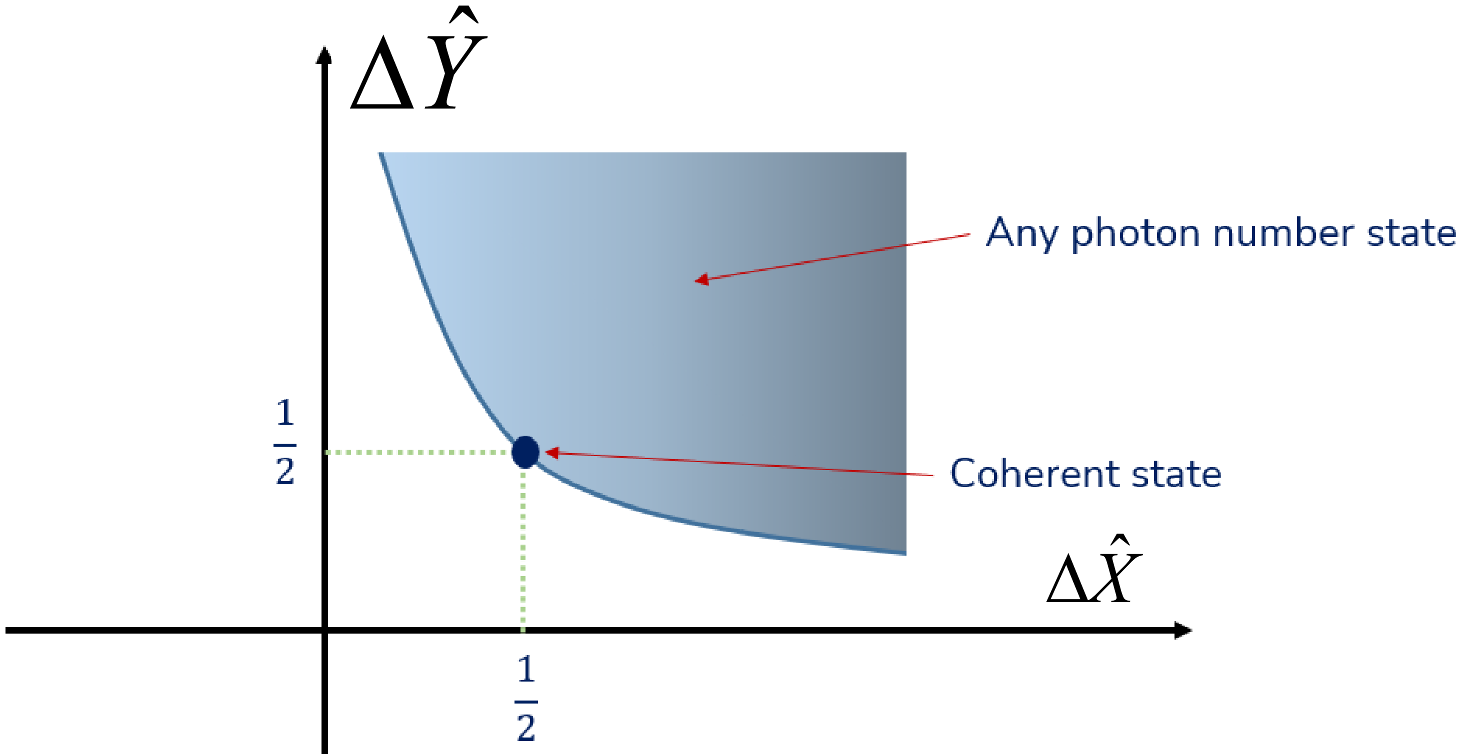
$$= -2\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V\epsilon_0}}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(\hat{X}_{\mathbf{k}\sigma}\sin\left(\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t\right)+i\hat{Y}_{\mathbf{k}\sigma}\cos\left(\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t\right)\right)$$

Magnetic field:

$$\mathbf{B} = \nabla\times\mathbf{A} \quad \Rightarrow \mathbf{B}(\mathbf{r},t) = i\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(a_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t} - a^{\dagger}_{\mathbf{k}\sigma}e^{-(i\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t)}\right)$$

$$= -2\sum_{\mathbf{k}\sigma}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}}\mathbf{k}\times\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}\left(\hat{X}_{\mathbf{k}\sigma}\sin\left(\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t\right)+i\hat{Y}_{\mathbf{k}\sigma}\cos\left(\mathbf{k}\cdot\mathbf{r}-i\omega_{\mathbf{k}}t\right)\right)$$

Plot of possible standard deviations (Figure from Prof. A. Kandada)



It is possible to modify the coherent state to produce fields with other properties within the blue region of the plot.

The hyperbolic curve represents $\Delta\hat{X}\,\Delta\hat{Y} = \frac{1}{4}$

$\Rightarrow$ Can reduce/increase $\Delta\hat{X}$ and increase/reduce $\Delta\hat{Y}$

Notion of a "squeezed" state --

First, note that the pure coherent state can be written:

$$\left|c_\alpha\right\rangle \equiv \sum_{n=0}^{\infty} \frac{\alpha^n e^{-|\alpha|^2/2}}{\sqrt{n!}} \left|n\right\rangle = e^{\alpha a^\dagger - \frac{1}{2}|\alpha|^2} \left|0\right\rangle$$

where $\left|0\right\rangle$ is the vacuum state and $\alpha = \Lambda e^{i\Psi}$ is a complex number

To form a "squeezed" state we introduce a multiplicative operator

$$S(\zeta) \equiv e^{\frac{1}{2}\zeta^* a^2 - \frac{1}{2}\zeta a^{\dagger 2}} \quad \text{where } \zeta = s e^{i\vartheta} \text{ is a complex number}$$

Better(?) description of squeezed states thanks to Berman ad Malinovsky ➔ two-photon coherent state

Model Hamiltonian for such a state --

$H = \lambda a^{\dagger 2} + \lambda^* a^2$ for some given parameter λ

Eigenstates of this Hamiltonian are called "squeezed" states and can be written as a product of a coherent state $|\alpha\rangle$ and a "squeezing" operator with parameter z:

$$|\alpha, z\rangle \equiv \underbrace{e^{-|\alpha|^2/2} \exp\left(\alpha a^\dagger\right)}_{\text{Coherent state operator}} \underbrace{\exp\left(\frac{1}{2}\left(-z a^{\dagger 2} + z^* a^2\right)\right)}_{\text{“Squeezing” operator}} \underbrace{|0\rangle}_{\text{Vacuum state}}$$

Properties of the squeezed state continued --

$$\left|\alpha, z\right\rangle \equiv e^{-|\alpha|^2/2} \exp\left(\alpha a^\dagger\right)\exp\left(\frac{1}{2}\left(-z a^{\dagger 2} + z^* a^2\right)\right)\left|0\right\rangle$$

For $z \equiv re^{i\theta}$, it is possible to show that

$$\left\langle \alpha, z\right| a^\dagger a \left|\alpha, z\right\rangle \equiv \left\langle \alpha, z\right| N \left|\alpha, z\right\rangle = \sinh^2 r + |\alpha|^2$$

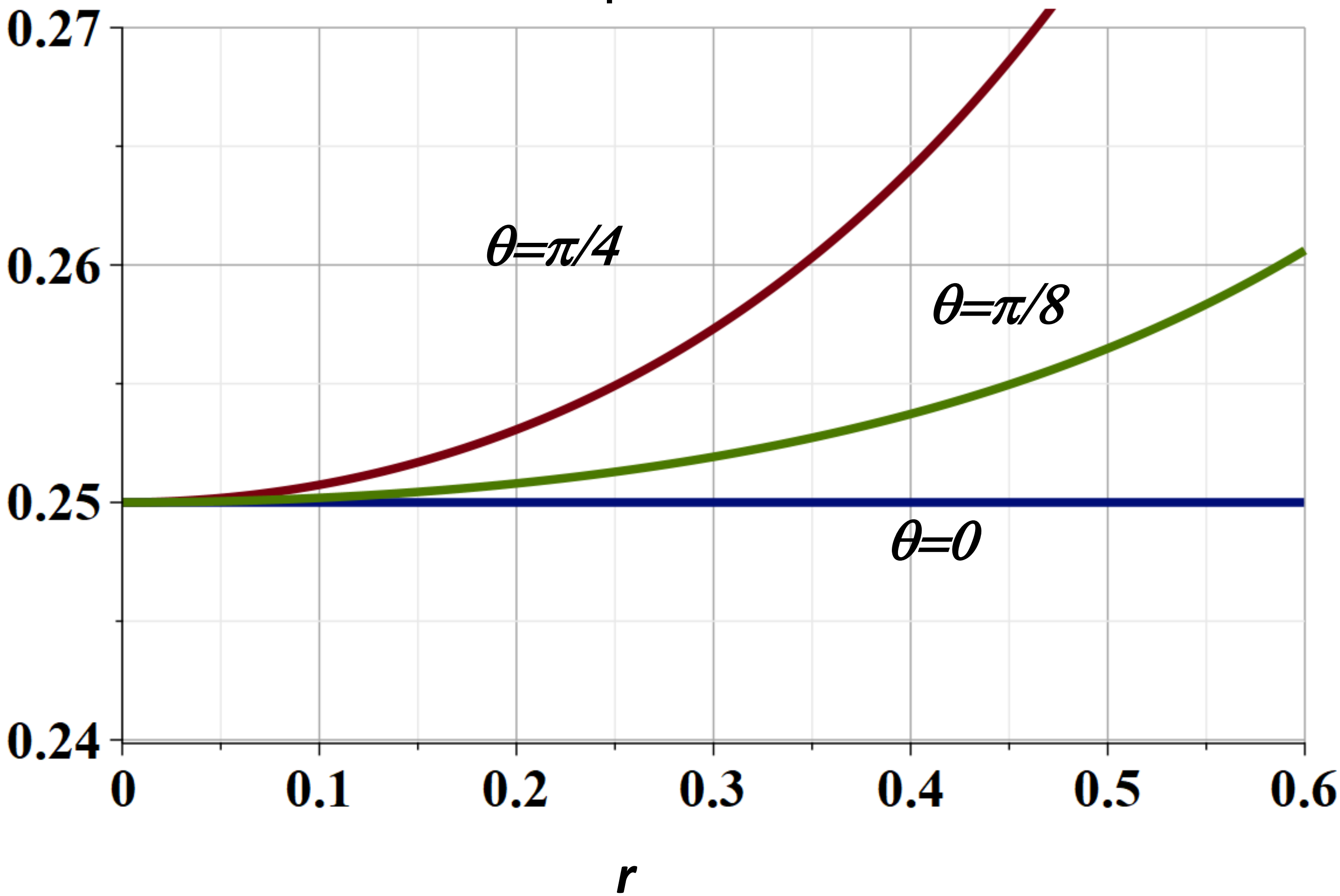
$$\left\langle \alpha, z\right| \left(\Delta N\right)^2 \left|\alpha, z\right\rangle = \left|\alpha \cosh r - \alpha^* \sinh r\ e^{i\theta}\right|^2 + 2\sinh^2 r \cosh^2 r$$

Also, for the field operators $\hat{X}$ and $\hat{Y}$, it is possible to show:

$$\left(\Delta\hat{X}\right)^2 = \frac{1}{4}\left|\cosh r - \sinh r\ e^{i\theta}\right|^2 \quad \left(\Delta\hat{Y}\right)^2 = \frac{1}{4}\left|\cosh r + \sinh r\ e^{i\theta}\right|^2$$

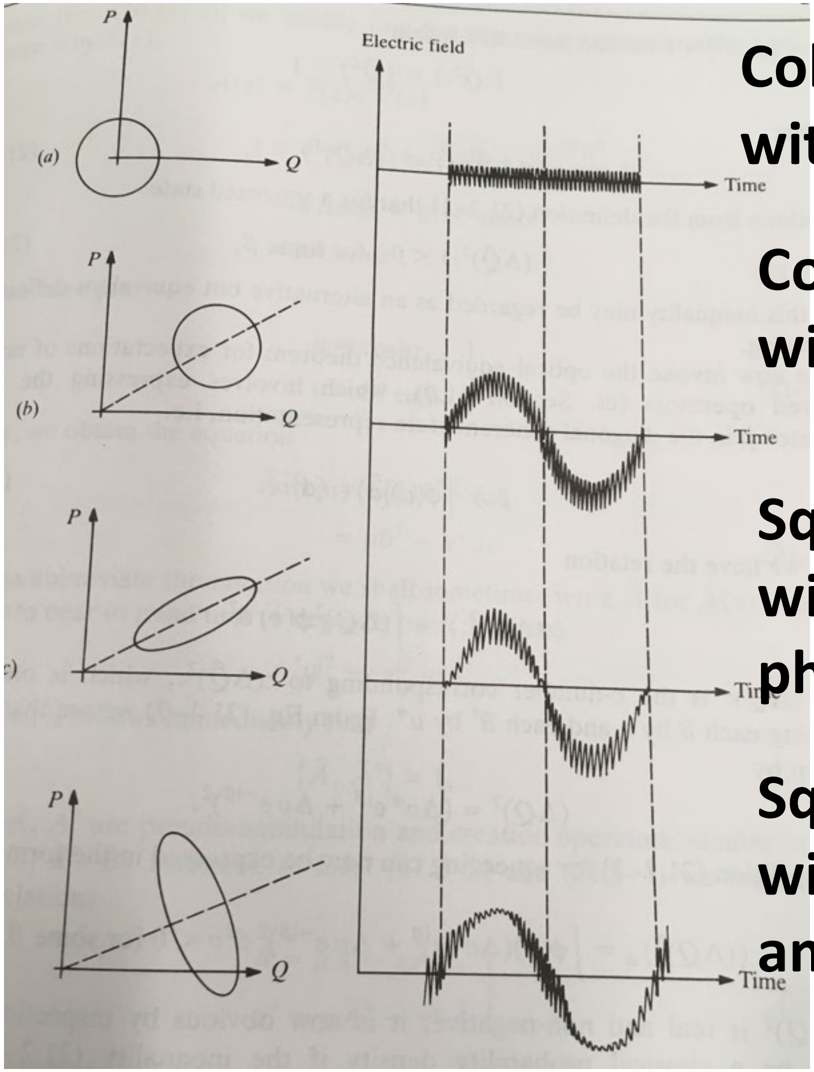
$$\left(\Delta\hat{X}\right)\left(\Delta\hat{Y}\right) = \frac{1}{4}\left(\left|\cosh^2 r - \sinh^2 r\ e^{2i\theta}\right|\right)$$

Plot of $\left(\Delta\hat{X}\right)\left(\Delta\hat{Y}\right)=\frac{1}{4}\left(\left|\cosh^2 r-\sinh^2 r\; e^{2i\theta}\right|\right)$



Squeezed states can be generated by taking a photon of frequency ω and using devices to generate two photons of frequency $\omega/2$.

From Mandel & Wolf



Coherent state with $\alpha=0$

Coherent state with $|\alpha|>0$

Squeezed state with controlled phase

Squeezed state with controlled amplitude

More complicated states

Notion of "entangled" states (according to Berman and Malinovsky) –

Example involving two particles

Suppose we have component 1 which can have states a and b

component 2 which can have states c and d

$$\Psi_{\text{separable}} = \left|\left(a_1 + c_2\right)\left(b_1 + d_2\right)\right\rangle = \left|\left(a_1 + c_2\right)\right\rangle\left|\left(b_1 + d_2\right)\right\rangle = \left|a_1 b_1 + a_1 d_2 + c_2 b_1 + c_2 d_2\right\rangle$$

$$\Psi_{\text{entangled}} = \left|a_1 c_2 + b_1 d_2\right\rangle$$

Cannot be written as two factors;

also discussed as correlation effects