



PHY 712 Electrodynamics

10-10:50 AM MWF in Olin 103

Class notes for Lecture 6:

Reading: Chapter 1 - 3 in JDJ

Introduction to numerical methods

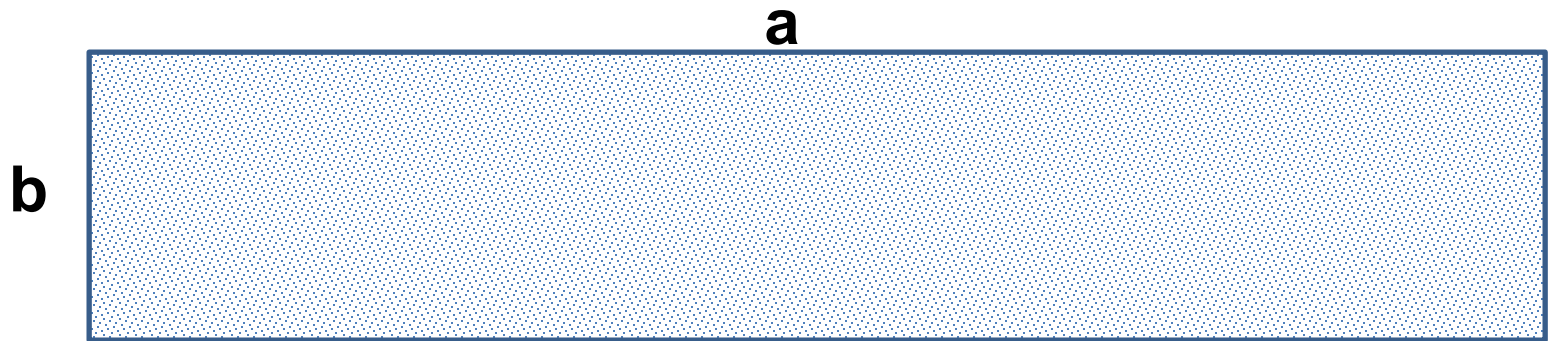
- 1. Finite difference**
- 2. Finite element**

Course schedule for Spring 2023

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/9/2023	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/13/2023
2	Wed: 01/11/2023	Chap. 1	Electrostatic energy calculations	#2	01/18/2023
3	Fri: 01/13/2023	Chap. 1	Electrostatic energy calculations thanks to Ewald	#3	01/18/2023
	Mon: 01/16/2023		MLK Holiday -- no class		
4	Wed: 01/18/2023	Chap. 1 & 2	Electrostatic potentials and fields	#4	01/20/2023
5	Fri: 01/20/2023	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions	#5	01/23/2023
6	Mon: 01/23/2023	Chap. 1 - 3	Brief introduction to numerical methods	#6	01/25/2023

Recall example from Lecture 5:



$$\rho(x, y) = \rho_0 \quad \text{for } 0 \leq x \leq a \text{ and } 0 \leq y \leq b$$

With boundary values $\Phi(0, y) = 0, \Phi(a, y) = 0, \Phi(x, 0) = 0, \Phi(x, b) = 0$

Solution: $\Phi(x, y) =$

$$\frac{16\rho_0 a}{4\pi^2 \epsilon_0} \sum_{n \text{ (odd)}} \frac{\sin\left(\frac{n\pi x}{a}\right)}{n^2 \sinh\left(\frac{n\pi b}{a}\right)} \left(\sinh\left(\frac{n\pi(b-y)}{a}\right) \int_0^y dy' \sinh\left(\frac{n\pi y'}{a}\right) + \sinh\left(\frac{n\pi y}{a}\right) \int_y^b dy' \sinh\left(\frac{n\pi(b-y')}{a}\right) \right)$$

$$= \frac{16\rho_0 a^2}{4\pi^3 \epsilon_0} \sum_{n \text{ (odd)}} \frac{\sin\left(\frac{n\pi x}{a}\right)}{n^3 \sinh\left(\frac{n\pi b}{a}\right)} \left(\sinh\left(\frac{n\pi b}{a}\right) - \sinh\left(\frac{n\pi y}{a}\right) - \sinh\left(\frac{n\pi(b-y)}{a}\right) \right)$$

=

PHY 712 -- Assignment #6

January 23, 2023

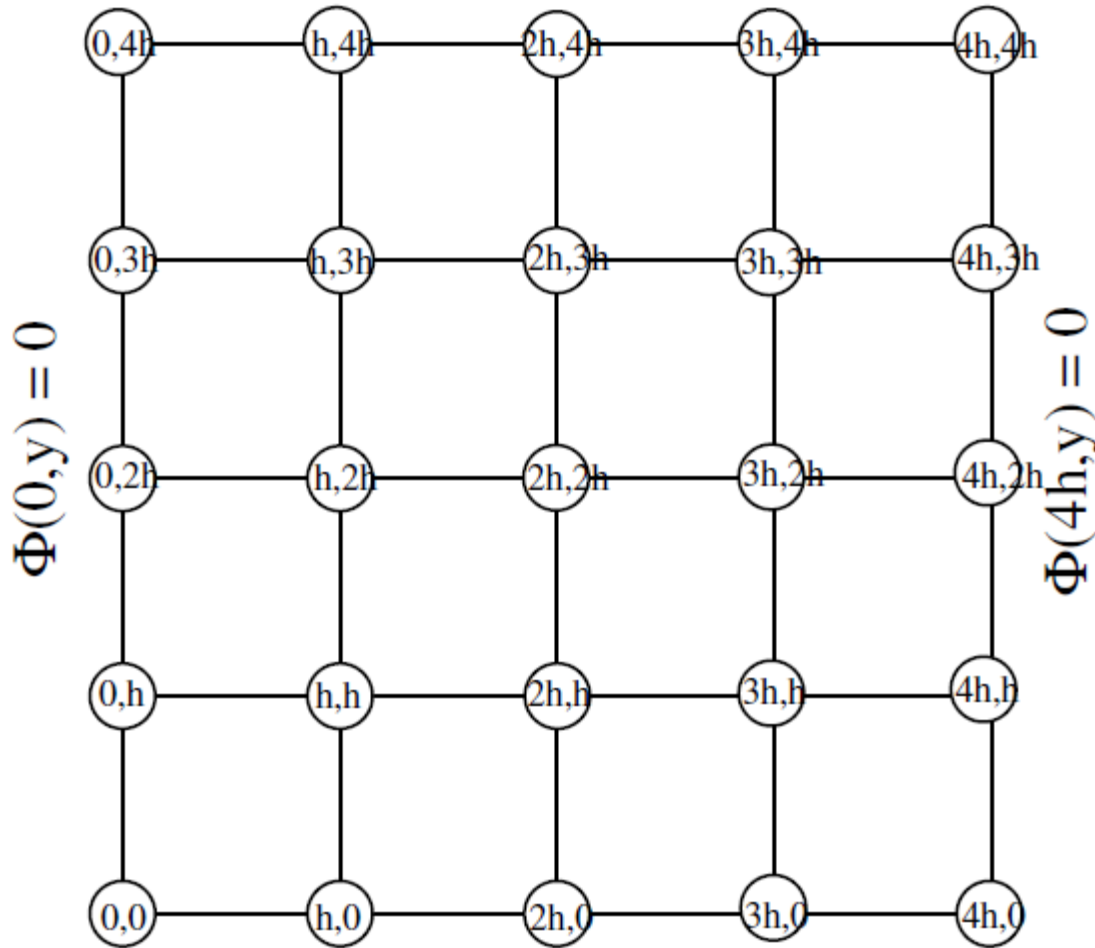
Continue reading Chap. 1-3 in **Jackson**.

Consider a two-dimensional square system, with $0 \leq x \leq a$ and $0 \leq y \leq a$ where the electrostatic potential $\Phi(x,y)$ vanishes on the boundaries -- $\Phi(x,0)=0$, $\Phi(x,a)=0$, $\Phi(0,y)=0$, $\Phi(a,y)=0$. The charge density within the square is constant (ρ_0); $\rho(x,y)=\rho_0$.

1. Using the results derived in Lecture 5 (and mentioned in Lecture 6), numerically evaluate $\Phi(x,y)$ at the grid points discussed in Lecture 6.
2. Now, using *either* the finite difference or finite element method for the two grids discussed in class, find $\Phi(x,y)$ on the grid points and compare the numerical solutions with the numerical answers you determined.

Some details for fine grid --

$$\Phi(x, 4h) = 0$$



9 interior grid points

→ reduced to 6 evaluation points using symmetry

$$\Phi(x, 0) = 0$$