

> *restart, assume(a,'positive');* with(*LinearAlgebra*);

[&x, *Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA\_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip*]

(1)

Define lattice translation

>  $T1 := \text{Vector}(3, [a, 0, 0]); T2 := \text{Vector}(3, [0, a, 0]); T3 := \text{Vector}(3, [0, 0, a]);$

$$T1 := \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$$T2 := \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix}$$

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$$T3 := \begin{bmatrix} 0 \\ 0 \\ a\sim \end{bmatrix} \quad (2)$$

Define reciprocal lattice translation

$$\begin{aligned} > G1 := \text{Vector}\left(3, \left[\frac{2 \cdot \text{Pi}}{a}, 0, 0\right]\right); G2 := \text{Vector}\left(3, \left[0, \frac{2 \cdot \text{Pi}}{a}, 0\right]\right); \\ G3 := \text{Vector}\left(3, \left[0, 0, \frac{2 \cdot \text{Pi}}{a}\right]\right); \end{aligned}$$

$$G1 := \begin{bmatrix} \frac{2 \pi}{a\sim} \\ 0 \\ 0 \end{bmatrix}$$

$$G2 := \begin{bmatrix} 0 \\ \frac{2 \pi}{a\sim} \\ 0 \end{bmatrix}$$

$$G3 := \begin{bmatrix} 0 \\ 0 \\ \frac{2 \pi}{a\sim} \end{bmatrix} \quad (3)$$

$$> \text{tau} := \text{Vector}(3, [0.5 \cdot a, 0.5 \cdot a, 0.5 \cdot a]);$$

$$\tau := \begin{bmatrix} 0.5 a\sim \\ 0.5 a\sim \\ 0.5 a\sim \end{bmatrix} \quad (4)$$

>

$$> \eta := \frac{4}{a^2}; \Omega := a^3; \text{con1} := \frac{4 \cdot \text{Pi}}{\Omega}; \text{con2} := \text{sqrt}\left(\frac{\eta}{\text{Pi}}\right);$$

$$\eta := \frac{4}{a\sim^2}$$

$$\Omega := a\sim^3$$

$$\text{con1} := \frac{4 \pi}{a\sim^3}$$

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$$con2 := \frac{2}{a \sim \sqrt{\pi}} \quad (5)$$

Initial terms — —  $C_l - C_l$  and  $C_s - C_s$

>

>  $tot := -evalf(con2 \cdot 2);$

$$tot := -\frac{2.256758334}{a \sim} \quad (6)$$

> **for**  $n$  **from**  $-8$  **by**  $1$  **while**  $n < 8$  **do** **for**  $m$  **from**  $-8$  **by**  $1$   
**while**  $m < 8$  **do** **for**  $l$  **from**  $-8$  **by**  $1$  **while**  $l < 8$   
**do** **if**  $(n \neq 0$  **or**  $m \neq 0$  **or**  $l \neq 0)$  **then**  $g :=$

$$(n \cdot G1 + m \cdot G2 + l \cdot G3) ; \quad tot := tot + evalf \left( 2 \cdot con1 \cdot \left( 1 - \exp(-I \cdot DotProduct(g, \tau)) \right) \cdot \frac{\exp\left(-\frac{DotProduct(g, g)}{\eta}\right)}{DotProduct(g, g)} \right)$$

**end if end do end do end do;**  $evalf(tot);$

$$-\frac{2.256758334}{a \sim} + \frac{0.0003951360355 + 1.907345200 \times 10^{-24} I}{a \sim} \quad (7)$$

> **for**  $n$  **from**  $-8$  **by**  $1$  **while**  $n < 8$  **do** **for**  $m$  **from**  $-8$  **by**  $1$   
**while**  $m < 8$  **do** **for**  $l$  **from**  $-8$  **by**  $1$  **while**  $l < 8$   
**do**  $t := (n \cdot T1 + m \cdot T2 + l \cdot T3) ;$

$tot := tot$

$$-evalf \left( \frac{2 \cdot \left( erfc\left(\frac{\sqrt{\eta}}{2}\right) \cdot VectorNorm(\tau + t, 2) \right) \right)}{VectorNorm(\tau + t, 2)} \right) ;$$

**if**  $(n \neq 0$  **or**  $m \neq 0$  **or**  $l \neq 0)$

**then**

$tot := tot$

$$+ evalf \left( \frac{2 \cdot erfc\left(\frac{\sqrt{\eta}}{2}\right) \cdot VectorNorm(t, 2)}{VectorNorm(t, 2)} \right) \quad \text{end if end do}$$

**end do end do;**  $evalf(tot);$

$$-\frac{4.071118106}{a \sim} + \frac{0.0003951360355 + 1.907345200 \times 10^{-24} I}{a \sim} \quad (8)$$

> Re(%);  
=>

$$- \frac{4.070722970}{a \sim}$$

(9)