PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Notes for Lecture 1:

Reading: Appendix 1 and Chapters I&1

1. Course structure and expectations
2. Units – SI vs Gaussian
3. Electrostatics – Poisson equation

Comment on upcoming physics colloquia –
https://physics.wfu.edu/wfu-phy-news/colloquium/seminar-spring-2024/

Physics Colloquium Schedule – Spring 2024

Previous and Future Colloquia

All colloquia will be held at 4 PM in Olin 101 (unless noted otherwise). Refreshments will be served at 3:30 PM in Olin Lobby prior to each seminar. For additional information contact wfuphys@wfu.edu.

Thurs. Jan. 18, 2024 — Dr. Alejandro Cárdenas Avendaño, Princeton U, "A Panoptic Study of Strong Gravitational Phenomena", host: D. Kim-Shapiro

Thurs. Jan. 25, 2024 — Dr. Jiang-Xiazi Lin, Brown University, "Exploring Emergent Quantum Phases in Two-Dimensional Flat Band Systems", (host: D. Kim-Shapiro)

Thurs. Feb. 1, 2024 — Dr. Emilie Huffman, Perimeter Institute for Theoretical Physics,"Principles for Modeling Physically-Relevant Quantum Systems of Many Particles with Computers", (host: D. Kim-Shapiro)

Thurs. Feb. 8, 2024 — Dr. Samantha Barrick, Washington University, "Molecular mechanisms of mechanotransduction at cell junctions and altered contractility in cardiomyopathy" (host: D. Kim-Shapiro)

Physics Colloquium

Thursday

4 PM Olin 101

January 18th, 2024

A Panoptic Study of Strong Gravitational Phenomena

Recent novel data channels have been instrumental in exploring various predictions of Einstein's general relativity. From the direct detection of gravitational waves to imaging supermassive black holes, these groundbreaking observations have shed light on the behavior of spacetime under extreme conditions, confirmed core predictions of the theory, and opened up new frontiers that bridge fundamental physics and astrophysics. Our ability to learn about the underlying physics depends heavily on our understanding of the gravity theory that describes the geometry around these compact objects and for the electromagnetic observations, also on the complex astrophysics that produces the observed radiation. In this talk, I will discuss our upcoming capability to study general relativity in the strong gravity regime using (i) the electromagnetic radiation from a black hole's accretion disk and (ii) the gravitational radiation when a small compact object falls into a supermassive one. Emphasizing the significance of multi-messenger approaches across diverse astrophysical systems, I will highlight their pivotal role in unraveling new physics.

Alejandro Cárdenas-Avendaño, Ph.D.
Princeton University
Physics Department

Spring 2024 Schedule
for N. A. W. Holzwarth

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Lecture Preparation / Office Hours	Physics Research	Lecture Preparation / Office Hours	Physics Research	Lecture Preparation / Office Hours
10:00-11:00	Electrodynamics: PHY 712		Electrodynamics: PHY 712		Electrodynamics: PHY 712
11:00-12:00	Office Hours		Office Hours		Condensed Matter Seminar
12:00-4:00	Physics Research		Physics Research		Physics Research
4:00-5:00				Physics Department Colloquium	

http://users.wfu.edu/natalie/s24phy712/

PHY 712 Electrodynamics

MWF 10-10:50 AM	Olin 103	Webpage: http://www.wfu.edu/~natalie/s24phy712/

Instructor: Natalie Holzwarth	Office:300 OPL	e-mail:natalie@wfu.edu

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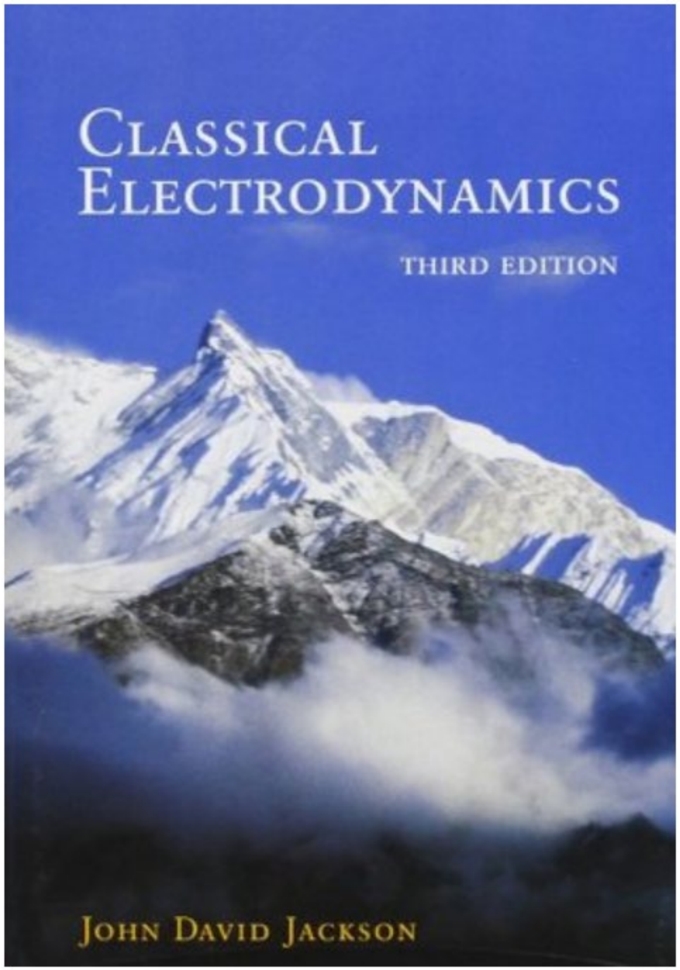
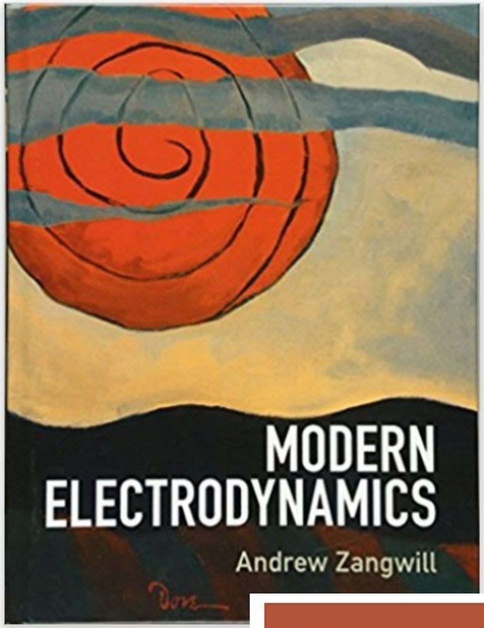
- General information
- Syllabus and homework assignments
- Lecture notes
- Some presentation ideas

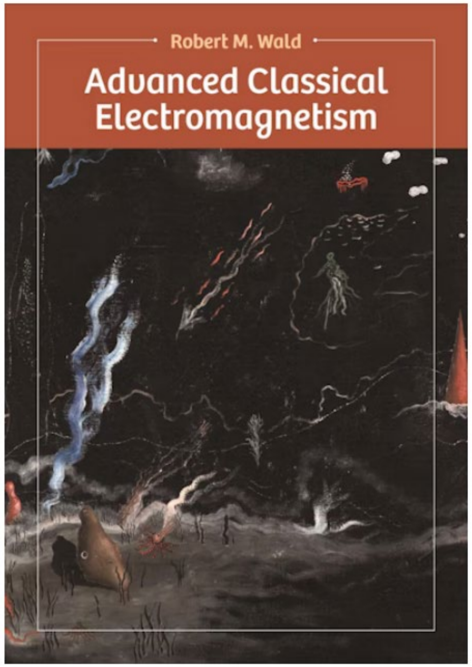
Last modfied: Friday, 12-Jan-2024 12:57:00 EST

Optional supplements

Textbook







Third edition

http://users.wfu.edu/natalie/s24phy712/info

General Information

This course is a one semester survey of Electrodynamics at the graduate level, using the textbook: **Classical Electrodynamics**, 3rd edition, by John David Jackson (John Wiley & Sons, Inc., 1999) -- "**JDJ**". (link to errata for early printings) Note that it is necessary to get the **third** edition in order to synchronize with the class lectures and homework. The more recent textbook: **Modern Electrodynamics**, by Andrew Zangwill (Cambridge University Press, 2013) will be used as a supplement. LINK An even more recent textbook: **Advanced Classical Electromagnetism**, by Robert M. Wald (Princeton University Press, 2022) may be of interest to some of you. LINK

The course will consist of the following components:

- In person meetings in Olin 103 MWF 10-10:50 AM. Zoom connections can be made available if requested, but not on a regular basis. The class sessions will focus on discussion of the material, particularly answering your prepared and spontaneous questions.
- Asynchronous review of annotated lecture notes and corresponding textbook sections. The reading assignment and annotated lecture notes will be available one day before the corresponding synchronous online discussion. For each class meeting, students will be expected to submit (by email) at least one question for class discussion at least 3 hours before the class meeting.
- Homework sets. Typically there will be one homework problem associated with each class meeting.
- There will be two take-home exams, one at mid-term and the other during finals week.
- There will be one project on a chosen topic related to electrodynamics.
- It is highly recommended that each student arrange for weekly one-on-one meetings with the instructor to discuss the course material, homework, and/or projects. These may be face-to-face or online as appropriate.

It is likely that your grade for the course will depend upon the following factors:

Class participation	15%
Problem sets*	35%
Project	15%
Exams	35%

*In general, there will a new assignment after each lecture, so that for optimal learning, it would be best to complete each assignment before the next scheduled lecture. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts.

➔ Email your questions >= 1 hour before each class, schedule weekly one-on-one meetings, and/or attend office hours

Some Ideas for Computational Project

The purpose of the "Computational Project" is to provide an opportunity for you to study a topic of your choice in greater depth. The general guideline for your choice of project is that it should have something to do with electrodynamics, and there should be some degree of computation or analysis with the project. The completed project will include a short write-up and a ~20min presentation to the class. You may design your own project or use one of the following list (which will be updated throughout the term).

- Evaluate the Ewald sum of various ionic crystals using Maple or a programing language. (Template available in Fortran code.)
- Work out the details of the finite difference or finite element methods.
- Work out the details of the hyperfine Hamiltonian as discussed in Chapter 5 of Jackson.
- Work out the details of Jackson problem 7.2 and related problems.
- Work out the details of reflection and refraction from birefringent materials.
- Analyze the Kramers-Kronig transform of some optical data or calculations.
- Determine the classical electrodynamics associated with an infrared or optical laser.
- Analyze the radiation intensity and spectrum from an interesting source such as an atomic or molecular transition, a free electron laser, etc.
- Work out the details of Jackson problem 14.15.

http://users.wfu.edu/natalie/s24phy712/homework/

PHY 712 Electrodynamics

MWF 10-10:50 AM	Olin 103	Webpage: http://www.wfu.edu/~natalie/s24phy712/

Instructor: Natalie Holzwarth	Office:300 OPL	e-mail:natalie@wfu.edu

Course schedule for Spring 2024

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Wed: 01/17/2024	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/19/2024
2	Fri: 01/19/2024	Chap. 1	Electrostatic energy calculations	#2	01/26/2024

PHY 712 – Problem Set #1

Assigned: 01/17/2024 Due: 01/19/2024

Read Chapters I and 1 and Appendix 1 in **Jackson**.

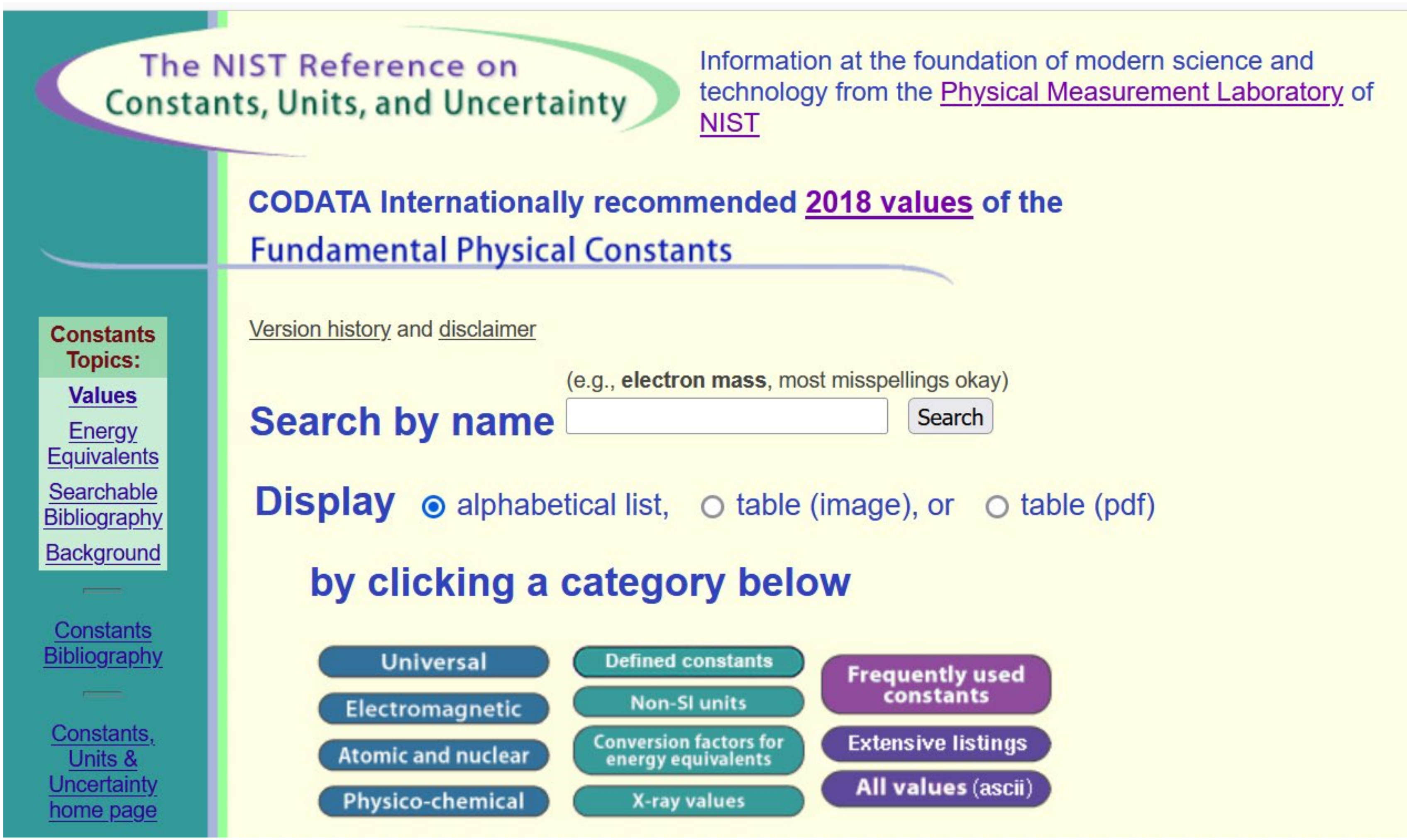
In SI units, the electrostatic interaction energy between two point particles is given by:

$$E_{ES} = \frac{e^2}{4\pi\varepsilon_0}\frac{z_1 z_2}{r},$$

where e denotes the elementary charge unit (in Coulombs), ε_0 denotes the permittivity of vacuum, z_i denotes the charge of particle i in units of e, and r denotes the separation of the two particles in units of m. Suppose that $z_1 = z_2 = 1$ and $r = 1$ Å.

1. Using the standard values of the constants from the NIST website (`https://physics.nist.gov/cuu/Constants/`), determine E_{ES} in SI units (Joules).
2. Using the standard values of the constants from the NIST website (`https://physics.nist.gov/cuu/Constants/`), determine E_{ES} in eV units.

https://physics.nist.gov/cuu/Constants/



Preferences on HW

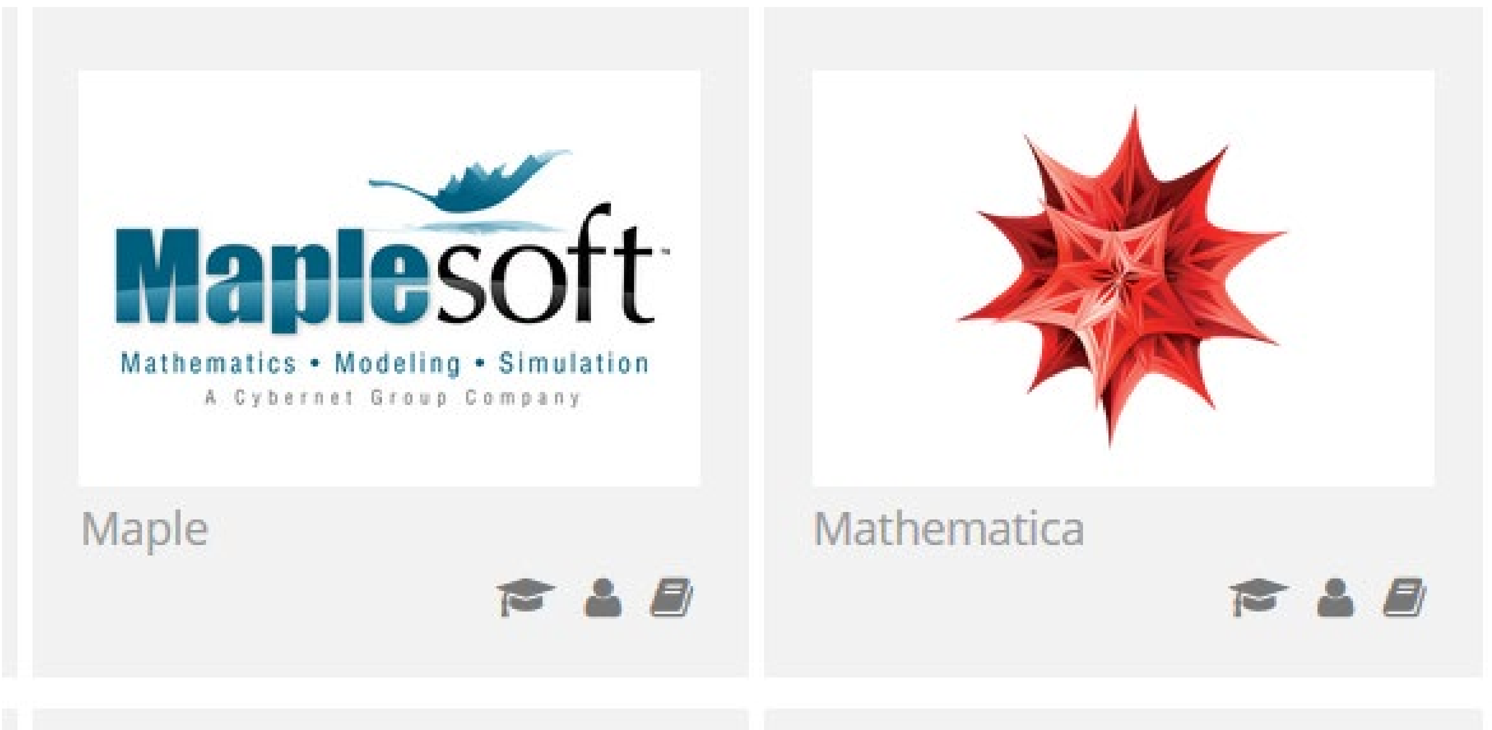
- ❑ Due 3 times a week?
- ❑ Due once a week on Friday?
- ❑ Due once a week on Wednesday?
- ❑ Due once a week on Monday?
- ❑ Other?

Tentative additional information –

Spring break March 9-17
Mid term grades due March 11
Wed May 1 – Last day of class
May 3-10 – Final exams

Remember to check your algebraic manipulation software --

https://software.wfu.edu/audience/students/



Material discussed in Appendix of textbook --

Units - SI vs Gaussian

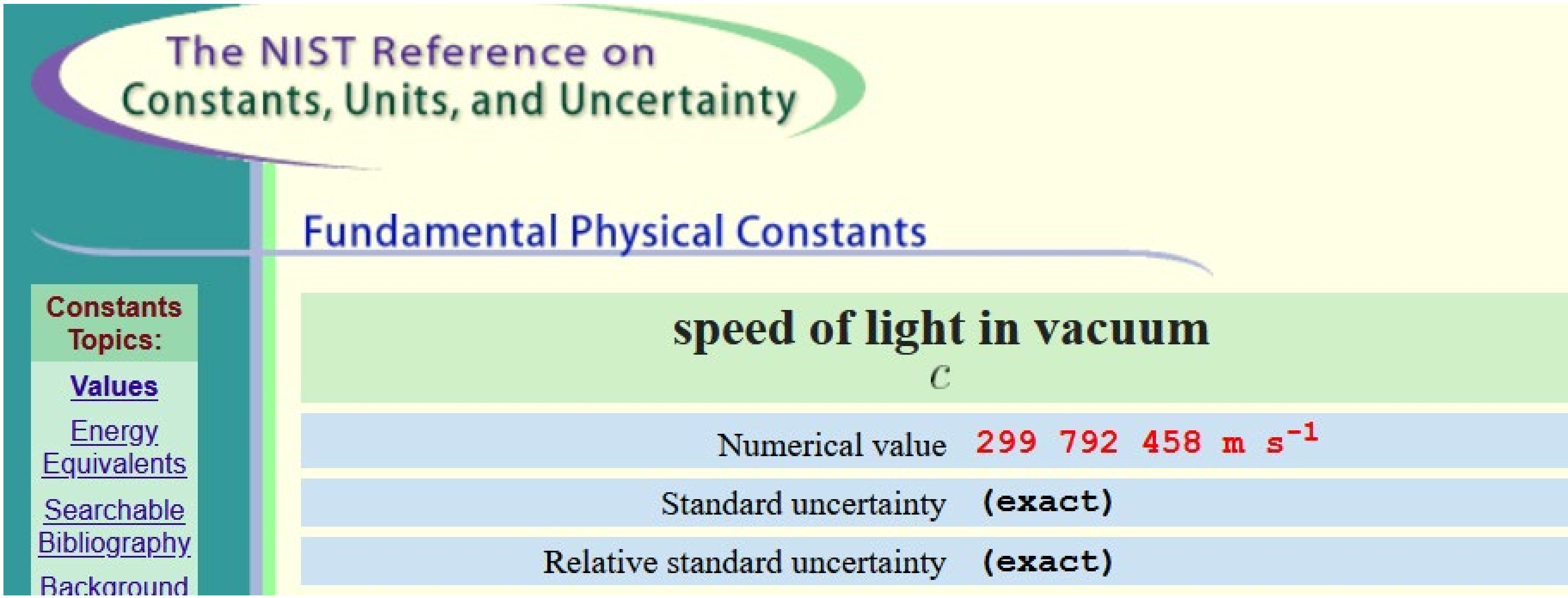
Coulomb's Law

$$F = K_C \frac{q_1 q_2}{r_{12}^2}. \quad (1)$$

Ampere's Law

$$F = K_A \frac{i_1 i_2}{r_{12}^2}\, d\mathbf{s_1} \times d\mathbf{s_2} \times \hat{\mathbf{r}}_{\mathbf{12}}, \quad (2)$$

In the equations above, the current and charge are related by $i_1 = dq_1/dt$ for all unit systems. The two constants K_C and K_A are related so that their ratio K_C/K_A has the units of $(m/s)^2$ and it is *experimentally* known that the ratio has the value $K_C/K_A = c^2$, where c is the speed of light.


The NIST Reference on
Constants, Units, and Uncertainty
Fundamental Physical Constants
Constants Topics:
Values
Energy Equivalents
Searchable Bibliography
Background
speed of light in vacuum
c
Numerical value 299 792 458 m s^{-1}
Standard uncertainty (exact)
Relative standard uncertainty (exact)

Units - SI vs Gaussian – continued

The choices for these constants in the SI and Gaussian units are given below:

	CGS (Gaussian)	SI
K_C	1	$\frac{1}{4\pi\epsilon_0}$
K_A	$\frac{1}{c^2}$	$\frac{\mu_0}{4\pi}$

Here, $\frac{\mu_0}{4\pi} \equiv 10^{-7} N/A^2$ and $\frac{1}{4\pi\epsilon_0} = c^2 \cdot 10^{-7} N/A^2 = 8.98755 \times 10^9 N \cdot m^2/C^2$.

Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m/s^2$	$dyne$	$gm \cdot cm/s^2$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$, $\mathbf{H}$, $\mathbf{P}$, $\mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

As noted by Sam, form many materials in the linear response approximation,

$$\mathbf{D}=\epsilon\mathbf{E} \qquad \text{in vacuum } \epsilon = 1 \quad \text{(for cgs Gaussian);} \quad \epsilon = \epsilon_0 \quad \text{(for SI)}$$

$$\mathbf{B}=\mu\mathbf{H} \qquad \text{in vacuum } \mu = 1 \quad \text{(for cgs Gaussian);} \quad \mu = \mu_0 \quad \text{(for SI)}$$

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

As we will see throughout the course, the E and B fields represent the basic electric and magnetic fields while the other fields include or represent electric and magnetic effects of matter.

Basic equations of electrodynamics

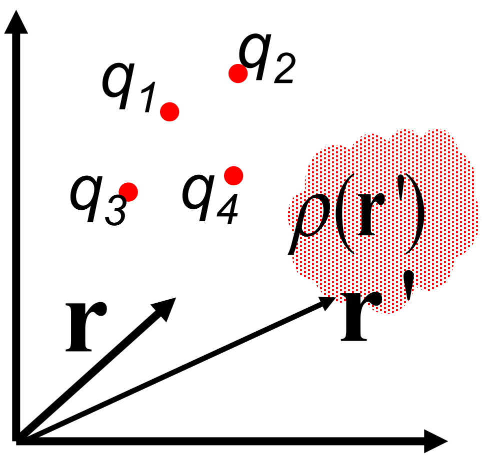
CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$	$u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

Units choice for this course:

SI units for Jackson in Chapters 1-10

Gaussian units for Jackson in Chapters 11-16

Electrostatics



Force on particle i due to particles j :

$$\mathbf{F}_i(\mathbf{r}_i) = q_i \mathbf{E}(\mathbf{r}_i) = \frac{1}{4\pi\epsilon_0} \sum_j q_i q_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

electric field: $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_j q_j \frac{\mathbf{r} - \mathbf{r}_j}{|\mathbf{r} - \mathbf{r}_j|^3}$

continuum field: $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \rho(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$

Electrostatics

Discrete versus continuous charge distributions

In terms of Dirac delta function:

$$\rho(\mathbf{r}) = \sum_i q_i \ \delta(\mathbf{r} - \mathbf{r}_i)$$

Digression: Note that in cartesian coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \delta(x - x_i)\delta(y - y_i)\delta(z - z_i)$$

in spherical polar coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \frac{1}{r^2}\delta(r - r_i)\delta(\cos\theta - \cos\theta_i)\delta(\phi - \phi_i)$$

Differential equations --

Electrostatics

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{E} = 0$$

Electrostatic potential

$$\mathbf{E} = -\nabla\Phi(r).$$

$$\nabla^2\Phi(r) = -\rho(r)/\epsilon_0.$$

Relationship between integral and differential forms of electrostatics --

Differential form

$$\nabla^2\Phi(\mathbf{r}) = -\rho(\mathbf{r}) / \epsilon_0$$

Integral form

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Relationship between integral and differential forms of electrostatics --

Need to show: $$\nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) = -4\pi \delta^3(\mathbf{r}-\mathbf{r}').$$

Noting that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \delta^3(\mathbf{r}-\mathbf{r}') f(\mathbf{r}) = f(\mathbf{r}'),$$

we see that we must show that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \; \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) f(\mathbf{r}) = -4\pi f(\mathbf{r}').$$

We introduce a small radius a such that:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \lim_{a\to 0} \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}}.$$

For a fixed value of a,

$$\nabla^2 \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}} = \frac{-3a^2}{(|\mathbf{r}-\mathbf{r}'|^2 + a^2)^{5/2}}.$$

Some details --

Let $|\mathbf{r}-\mathbf{r}'| \equiv u$ $\qquad \nabla^2 = \frac{\partial^2}{\partial u^2} + \frac{2}{u}\frac{\partial}{\partial u}$

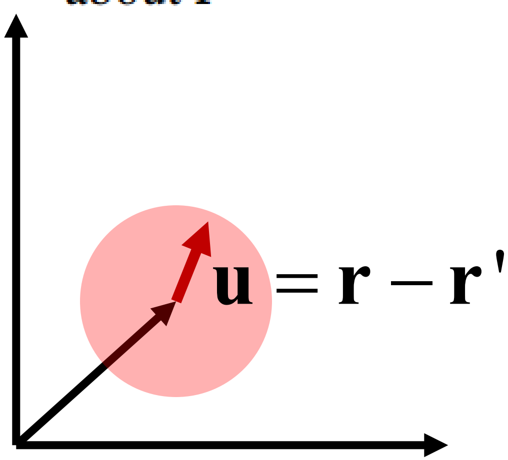
$$\nabla^2 \frac{1}{\sqrt{u^2+a^2}} = \left(\frac{\partial^2}{\partial u^2} + \frac{2}{u}\frac{\partial}{\partial u}\right)\frac{1}{\sqrt{u^2+a^2}} = -\frac{1}{\left(u^2+a^2\right)^{3/2}} + \frac{3u^2}{\left(u^2+a^2\right)^{5/2}} - \frac{2}{\left(u^2+a^2\right)^{3/2}}$$

$$= -\frac{3a^2}{\left(u^2+a^2\right)^{5/2}}$$

If the function $f(\mathbf{r})$ is continuous, we can make a Taylor expansion of it about the point $\mathbf{r} = \mathbf{r}'$, keeping only the first term. The integral over the small sphere about $\mathbf{r}'$ can be carried out analytically, by changing to a coordinate system centered at $\mathbf{r}'$;

so that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \, \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) \approx f(\mathbf{r}') \lim_{a \to 0} \int_{u<R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}}.$$



$$\mathbf{r} = \mathbf{r}' + \mathbf{u} \qquad f(\mathbf{r}) \approx f(\mathbf{r}')$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}} = 4\pi \int_0^R du \, \frac{-3a^2u^2}{(u^2 + a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2 + a^2)^{3/2}}.$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

For $a \ll R$, $4\pi \frac{-R^3}{\left(R^2+a^2\right)^{3/2}} \approx -4\pi$

$$\rightarrow \int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \, \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) f(\mathbf{r}) \approx f(\mathbf{r}')(-4\pi),$$

$$\rightarrow \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$$

Memorable identity

$$\nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -4\pi\delta^3(\mathbf{r} - \mathbf{r}')$$