PHY 712 Electrodynamics
10-10:50 AM in Olin 103

Discussion for Lecture 19:

Complete reading of Chapter 7 (Sec. 7.1-7.5, 7.10)

1. **Comments on reflectivity of plane waves**
2. **Summary of complex response functions for electromagnetic fields**
3. **Comment on spectral properties of electromagnetic waves**

Physics Colloquium

Thursday

4 PM Olin 101

February 29th, 2024

Mother Nature and the unexpected materials science of advanced optical fiber lasers and amplifiers

This Colloquium will overview the glass science of optical fibers and fiber-based amplifiers of interest for scaling to high output power levels. A particular focus will be on all-glass routes to internal thermal management and intrinsically low nonlinearities that may permit marked reductions in laser system size, weight, and power-consumption. Specific topics to be discussed will include low quantum defect core compositions, anti-Stokes fluorescence cooling in silica, and the numerous considerations that must be considered during the processing and fabrication of the preforms and fibers to permit the realization of these phenomena.

Bio: John Ballato is a professor of materials science and engineering at Clemson University where he also holds the Sirrine Endowed Chair of Optical Fiber. The author of over 600 technical articles, Ballato's work has historically focused on light-matter interactions, with a particular focus on novel optical fibers. A Fellow of the AAAS, APS, IEEE, Optica, SPIE and ACerS, he has also received the William Streifer Scientific Achievement Award from the IEEE Photonics Society and the George W. Morey Award from the American Ceramic Society.

Professor
John Ballato
Clemson University

9	Mon: 02/05/2024	Chap. 3 & 4	Spherical geometry and multipole moments	#8	02/12/2024
10	Wed: 02/07/2024	Chap. 4	Dipoles and Dielectrics	#9	02/12/2024
11	Fri: 02/09/2024	Chap. 4	Dipoles and Dielectrics	#10	02/12/2024
12	Mon: 02/12/2024	Chap. 5	Magnetostatics	#11	02/19/2024
13	Wed: 02/14/2024	Chap. 5	Magnetic dipoles and hyperfine interaction	#12	02/19/2024
14	Fri: 02/16/2024	Chap. 5	Magnetic dipoles and dipolar fields	#13	02/19/2024
15	Mon: 02/19/2024	Chap. 6	Maxwell's Equations	#14	02/26/2024
16	Wed: 02/21/2024	Chap. 6	Electromagnetic energy and forces	#15	02/26/2024
17	Fri: 02/23/2024	Chap. 7	Electromagnetic plane waves	#16	02/26/2024
18	Mon: 02/26/2024	Chap. 7	Electromagnetic plane waves	#17	03/01/2024
19	Wed: 02/28/2024	Chap. 7	Optical effects of refractive indices	#18	03/01/2024
20	Fri: 03/01/2024	Chap. 1-7	Review		
21	Mon: 03/04/2024	Chap. 8	Short lectures on waveguides	Exam	
22	Wed: 03/06/2024	Chap. 8	Short lectures on waveguides	Exam	
23	Fri: 03/08/2024	Chap. 8	Short lectures on waveguides	Exam	
	Mon: 03/11/2024	No class	*Spring Break*		
	Wed: 03/13/2024	No class	*Spring Break*		
	Fri: 03/15/2024	No class	*Spring Break*		
24	Mon: 03/18/2024	Chap. 9	Radiation from localized oscillating sources		

PHY 712 -- Assignment #18

Assigned: 2/28/2024 Due: 3/01/2024

Continue reading Chapter 7, particularly Sec. 7.4 in **Jackson** .

1. This problem concerns the phenomenon of total internal reflection. Imagine that plane polarized monochromatic light is relected and refracted at an interface between two media which have real refractive indices $n=2$ and $n'=1.1$. Draw a diagram of the incident, reflected, and refracted beams. What is the range of incident angles i for which total internal reflection occurs?

For Friday, 3/01/2024:

- Please turn in any outstanding HW
- Suggest your preferred review topics

For 3/04/2024-3/08/2024:

- Individual work on take home exam
- Shortened class lectures on Chapter 8 of **Jackson**

Review of Fresnel equations --
Electromagnetic plane waves in isotropic medium with linear and real permeability and permittivity: $\mu\ \varepsilon$.

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right) \qquad n^2 = c^2\mu\varepsilon$$

$$\mathbf{B}(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t) = \sqrt{\mu\varepsilon}\,\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)$$

Note that special care is needed when these quantities are complex.

Poynting vector for plane electromagnetic waves:

$$\langle\mathbf{S}\rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\hat{\mathbf{k}}$$

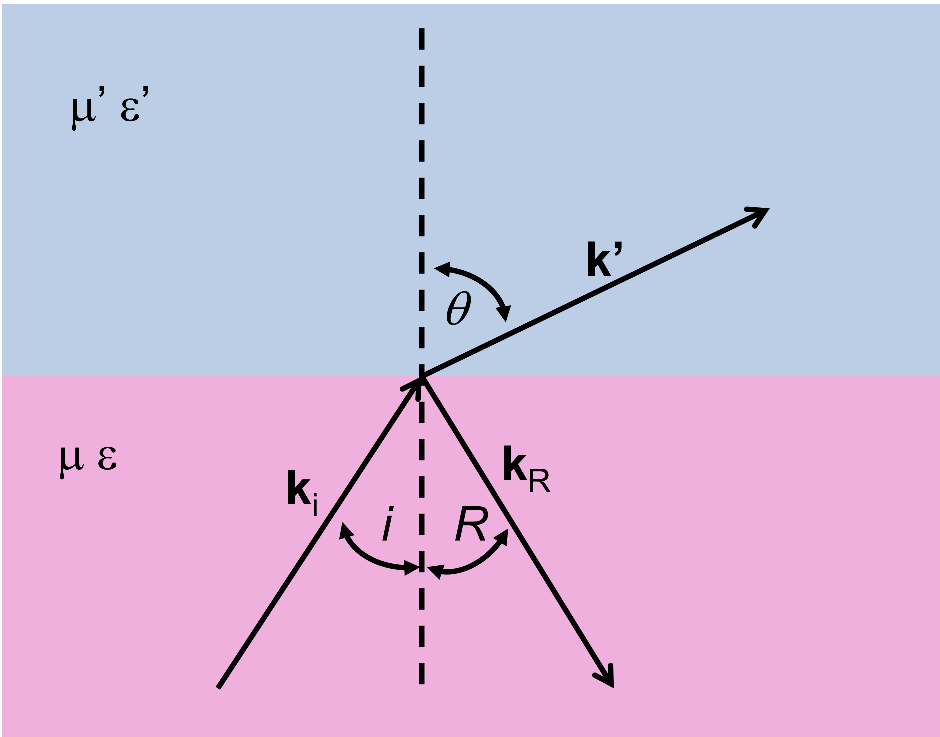
Energy density for plane electromagnetic waves:

$$\langle u\rangle_{avg} = \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$

Some comments on the Fresnel Equations

1. Different behaviors of *s* and *p* polarization
2. Brewster's angle
3. Total internal reflection

Review:
Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



$$n' \equiv \sqrt{\frac{\varepsilon' \mu'}{\varepsilon_0 \mu_0}}$$

$$n \equiv \sqrt{\frac{\varepsilon\ \mu}{\varepsilon_0 \mu_0}}$$

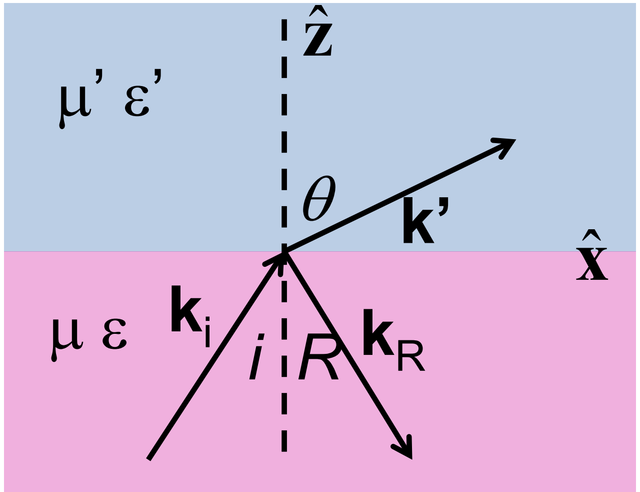
$$i = R$$

$$n \sin i = n' \sin \theta$$

$$\left|\mathbf{k}_i\right| = \left|\mathbf{k}_R\right| = n\frac{\omega}{c}$$

$$\left|\mathbf{k}'\right| = n'\frac{\omega}{c}$$

Review:
Reflection and refraction between two isotropic media



Reflectance, transmittance:

$$\mathrm{R} = \frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E_{0R}}{E_{0i}} \right|^2 \qquad \mathrm{T} = \frac{\mathbf{S}' \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n'}{n} \frac{\mu}{\mu'} \frac{\cos\theta}{\cos i}$$

Note that $\mathrm{R} + \mathrm{T} = 1$

For s-polarization (E perpendicular to plane of incidence)

$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'} n'\cos\theta}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

For p-polarization (E in plane of incidence)

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n'^2\cos i - nn'\cos\theta}{\frac{\mu}{\mu'} n'^2\cos i + nn'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2nn'\cos i}{\frac{\mu}{\mu'} n'^2\cos i + nn'\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

Special case: normal incidence (i=0, θ=0)

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n'-n}{\frac{\mu}{\mu'}n'+n} \qquad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'}n'+n}$$

Reflectance, transmittance:

$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'-n}{\frac{\mu}{\mu'}n'+n}\right|^2$$

$$\mathrm{T} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'} = \left|\frac{2n}{\frac{\mu}{\mu'}n'+n}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}$$

Fresnel equations for reflectivity in general --

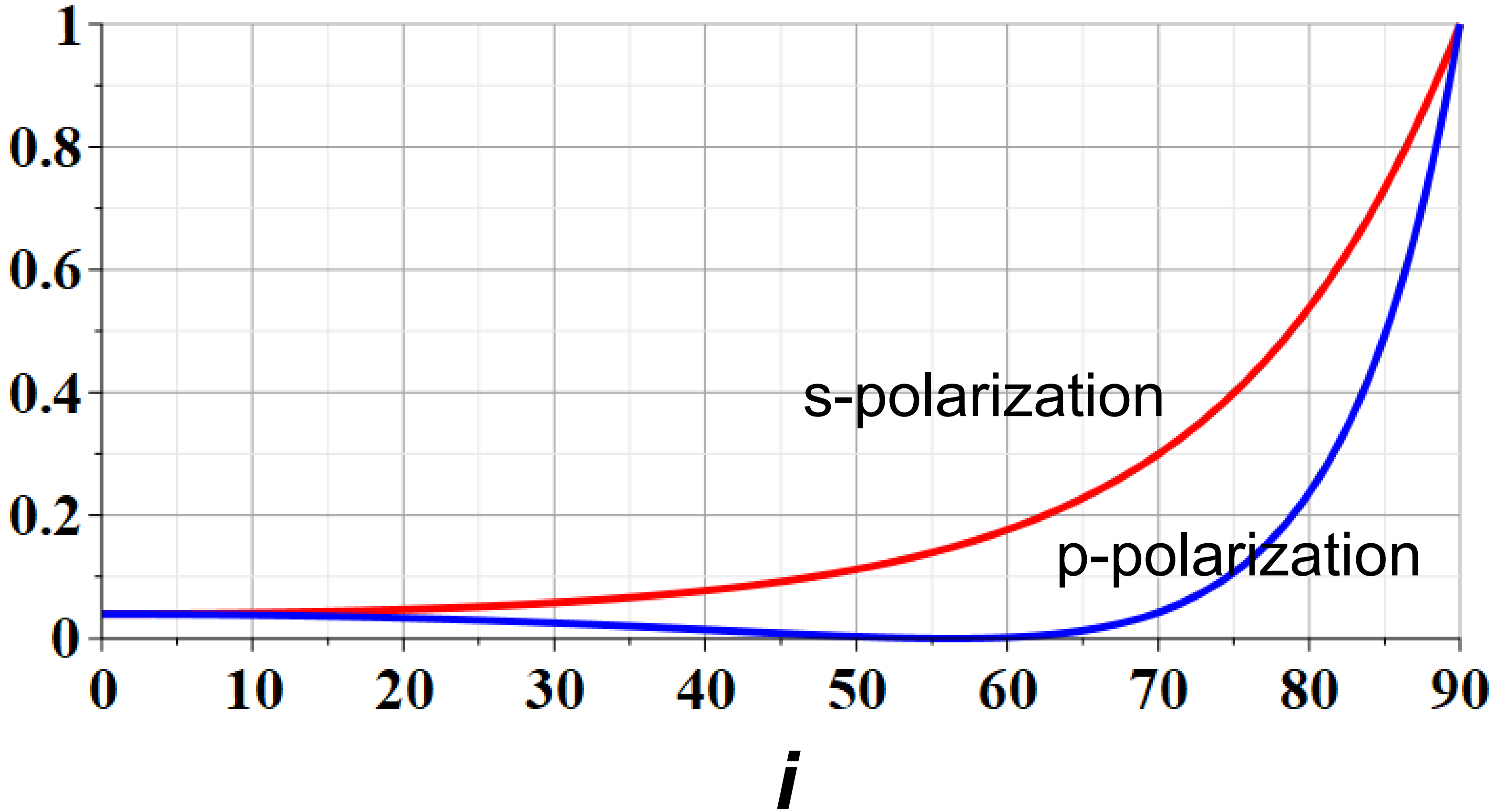
Reflectance for s-polarization

$$R_s = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Reflectance for p-polarization

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'\cos i - \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'\cos i + \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Example for $\mu = \mu'$; $n = 1$ and $n' = 1.5$



Analysis --

Reflectance for s-polarization

$$R_s = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2 \quad \Rightarrow R_s \neq 0 \quad \text{for typical angles } i$$

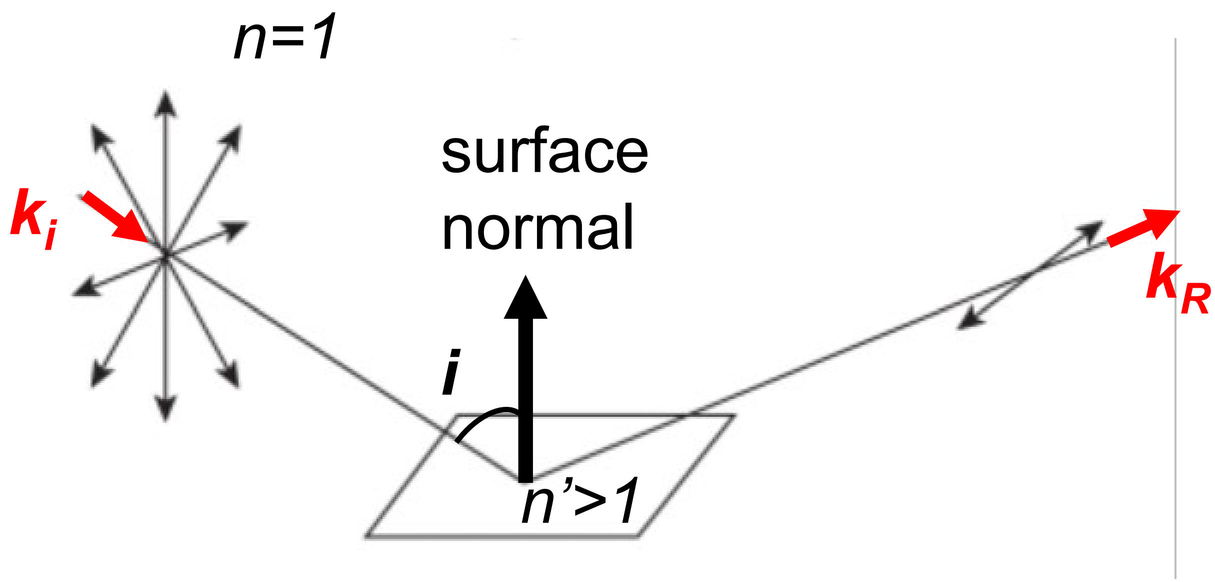
Reflectance for p-polarization

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'\cos i - \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'\cos i + \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

$$\Rightarrow R_p = 0 \quad \text{when}$$

$$\tan i_B = \frac{n'}{n} \quad \text{for } \mu = \mu'$$

$$i_B \equiv \text{Brewster's angle}$$



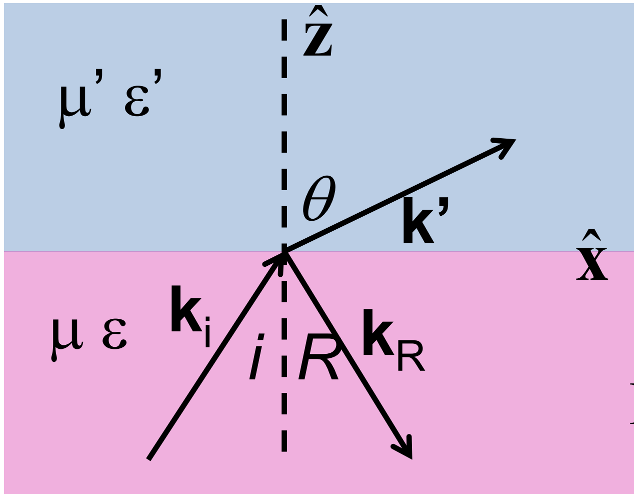
Polarization due to reflection from a refracting surface

Brewster's angle: for $i = i_B$, $\mathrm{R}_p(i_B) = 0$

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'\cos i - \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'\cos i + \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

For $\mu' = \mu$, $\quad i_B = \tan^{-1}\left(\frac{n'}{n}\right)$

Reflection and refraction between two isotropic media -- continued



For each wave:

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right) \qquad n^2 = c^2\mu\varepsilon$$

$$\mathbf{B}(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t) = \sqrt{\mu\varepsilon}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)$$

Matching condition at interface:

$$n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$$

Total internal reflection:

If $n > n'$, for $i > i_0 \equiv \sin^{-1}\left(\frac{n'}{n}\right)$,

refracted field no longer propagates in medium $\mu'\ \varepsilon'$

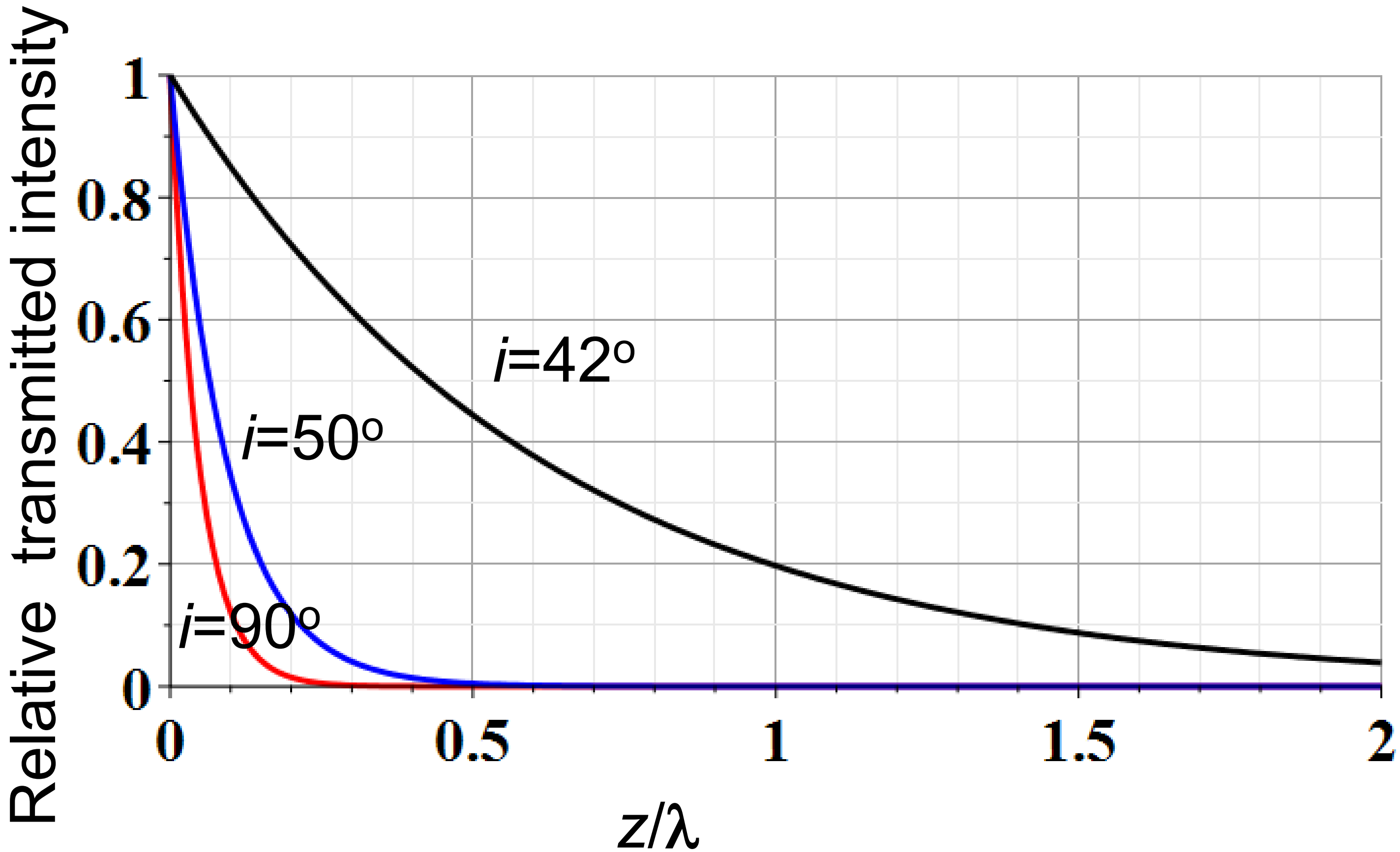
For $i > i_0$

$$n'\cos\theta = i\sqrt{n^2\sin^2 i - n'^2} = i\, n\sqrt{\frac{\sin^2 i}{\sin^2 i_0} - 1}$$

$$\mathbf{E}'(\mathbf{r},t) = e^{-\left(\frac{n\omega}{c}\sqrt{\frac{\sin^2 i}{\sin^2 i_0}-1}\right)z}\,\Re\left(\mathbf{E}'_0\, e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_{\parallel}\cdot\mathbf{r}-ct\right)}\right)$$

Example of total internal reflection

$n'=1$ and $n=1.5$ ➔ $i_0 = \sin^{-1}(1/1.5)=41.81^o$



Transmitted illumination confined within a few wavelengths of the surface.

TIRF (total internal reflection fluorescence)

www.nikon.com/products/microscope-solutions/bioscience.../nikon_note_10_lr.pdf

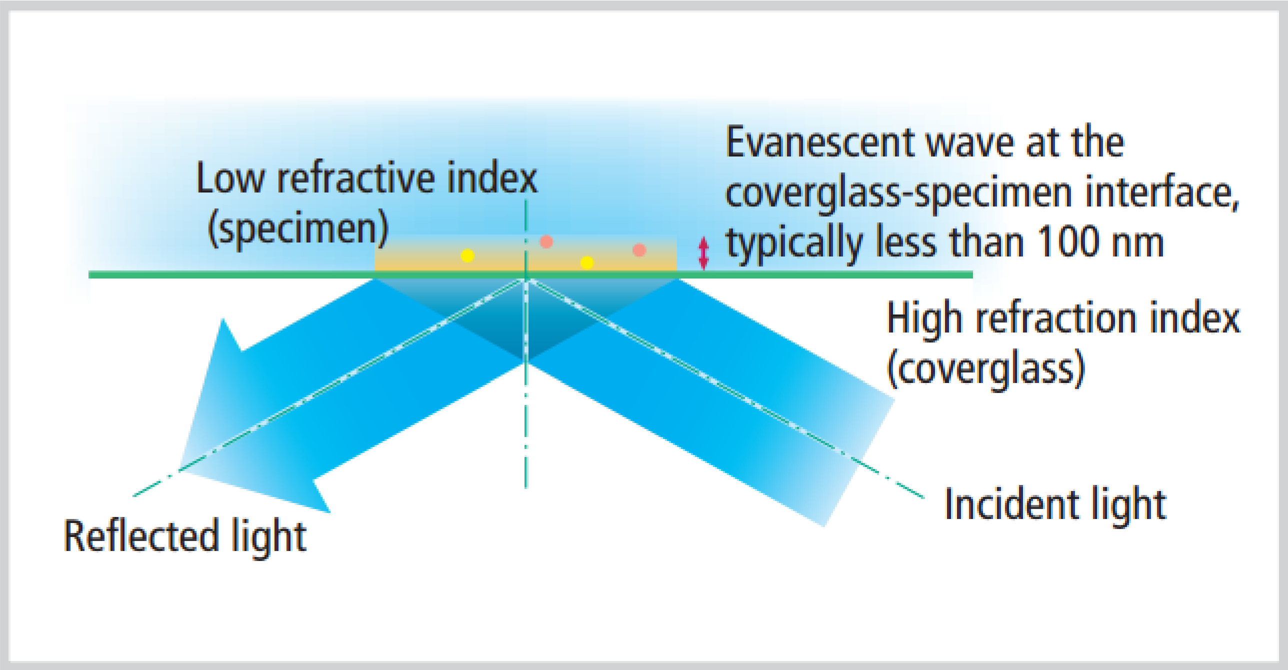


Figure 1: Creation of an evanescent wave at the coverglass-specimen interface

Design of TIRF device using laser and high power lens

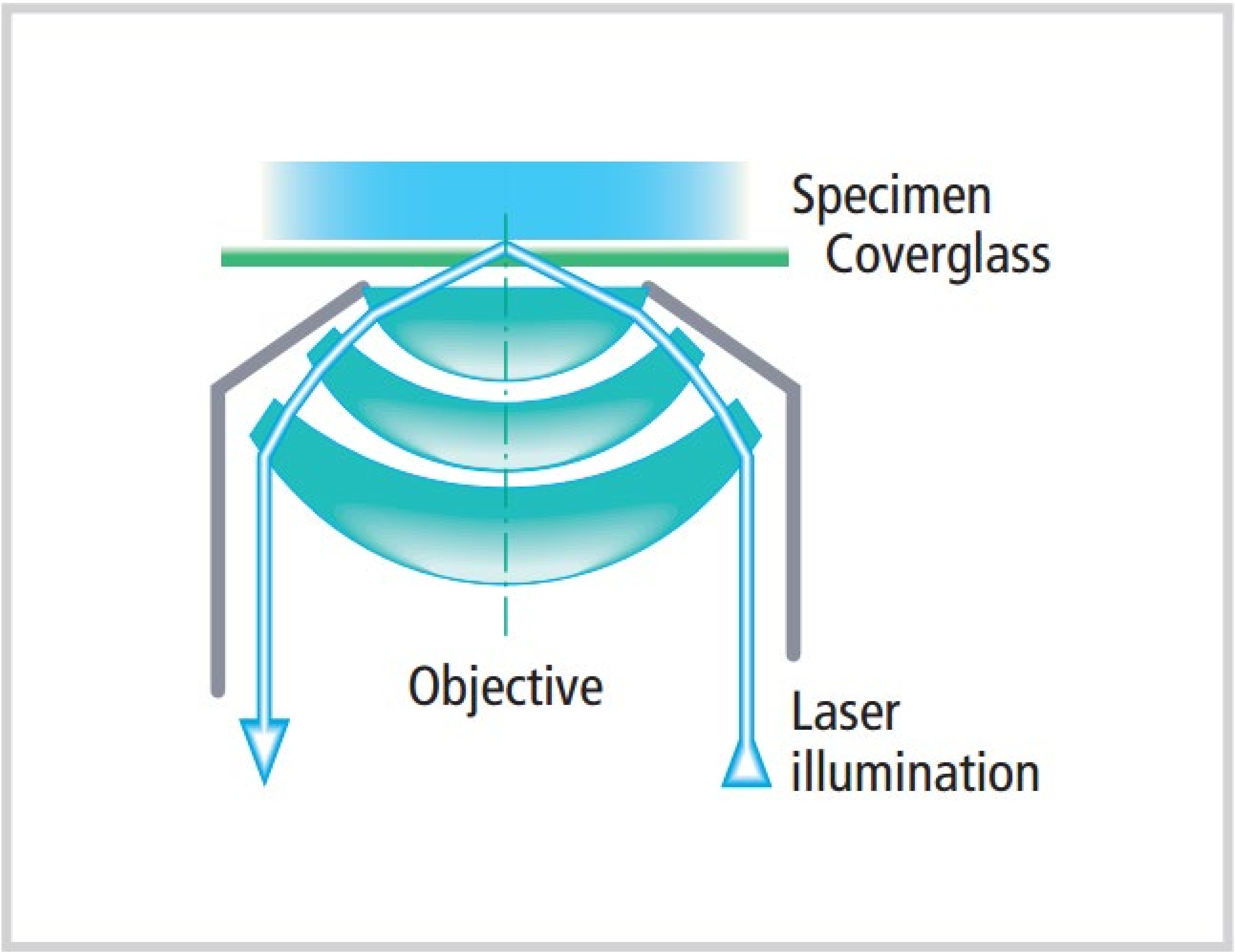
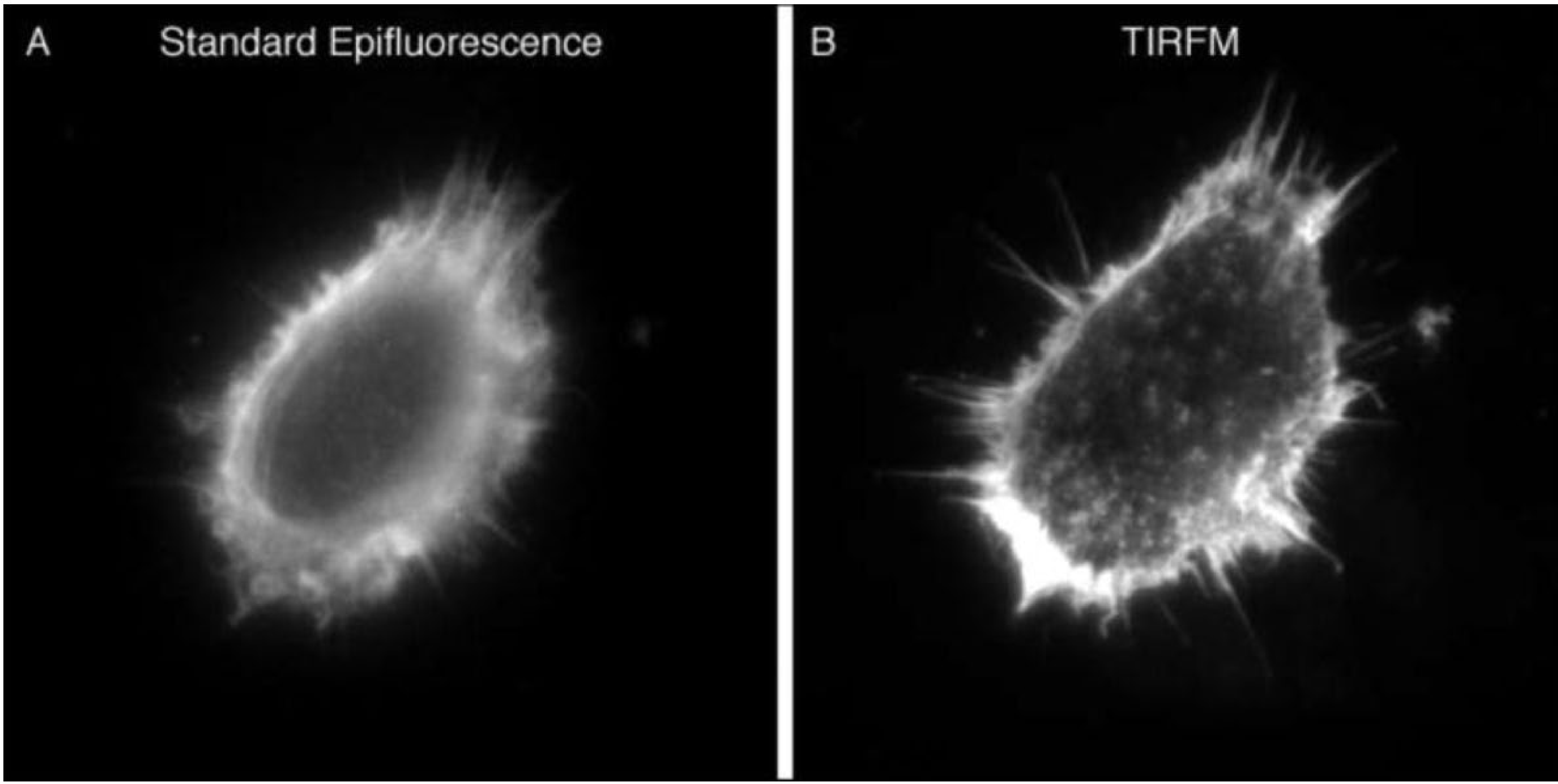


Figure 2: Through-the-lens laser TIRF.

PMC full text: Curr Protoc Cytom. Author manuscript; available in PMC 2015 Aug 18.
Published in final edited form as:
Curr Protoc Cytom. 2009 Oct; 0 12: Unit12.18.
doi: 10.1002/0471142956.cy1218s50

Figure 1



Extension to complex refractive index $n = n_R + i\, n_I$

$$\text{Suppose } \mu = \mu', \; n = \text{real}, \; n' = n'_R + i n'_I$$

Reflectance at normal incidence:

$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'} n' - n}{\frac{\mu}{\mu'} n' + n}\right|^2 = \frac{(n'_R - n)^2 + (n'_I)^2}{(n'_R + n)^2 + (n'_I)^2}$$

Note that for $n'_I >> |n'_R \pm n|$:

$$\mathrm{R} \approx 1$$

The general Fresnel equations can be similarly adapted for complex refractive indices.

Origin of imaginary contributions to permittivity --
Review: Drude model dielectric function:

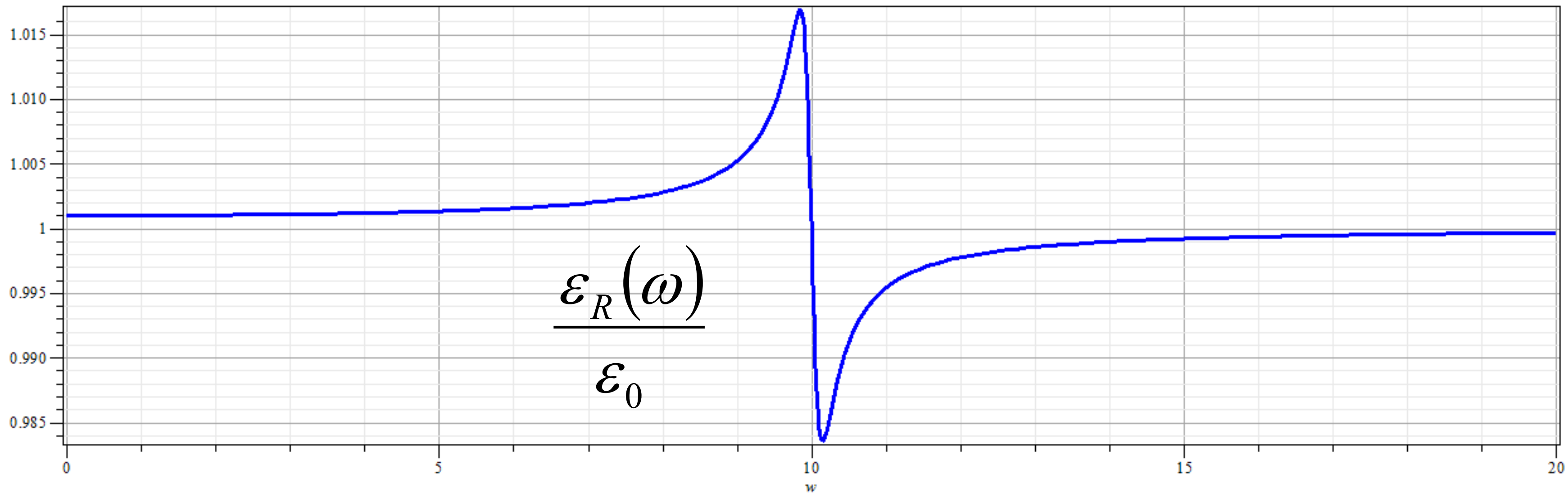
$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^{\ 2}}{\varepsilon_0 m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

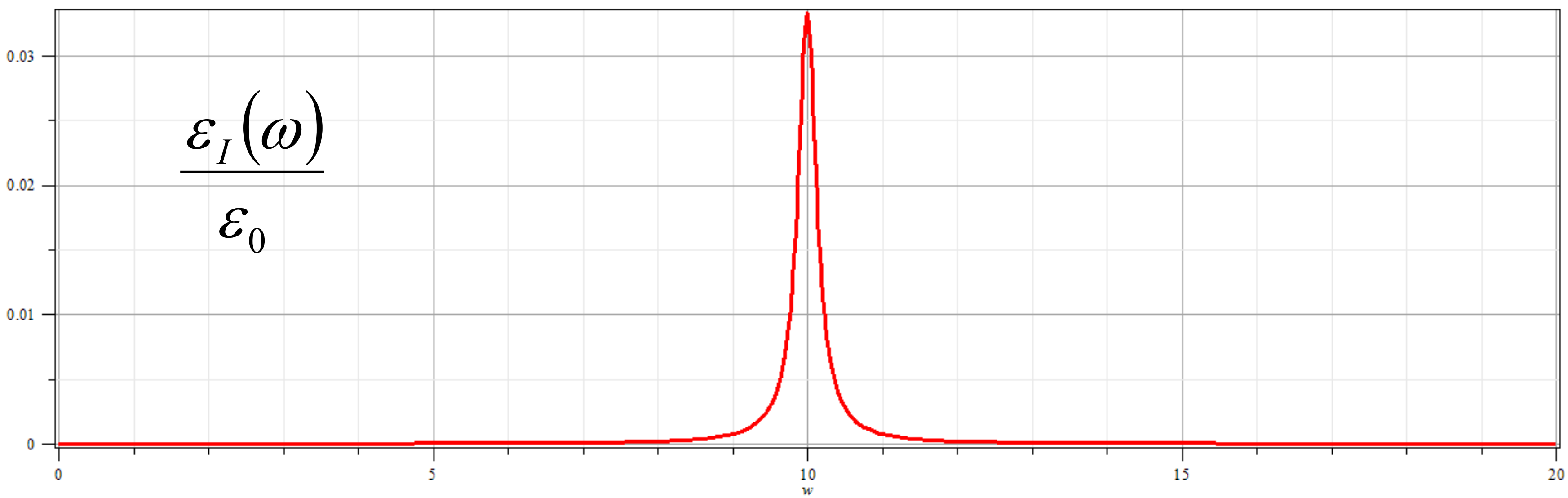
$$= \frac{\varepsilon_R(\omega)}{\varepsilon_0} + i\frac{\varepsilon_I(\omega)}{\varepsilon_0}$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^{\ 2}}{\varepsilon_0 m_i} \frac{\omega_i^2 - \omega^2}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2 \gamma_i^{\ 2}}$$

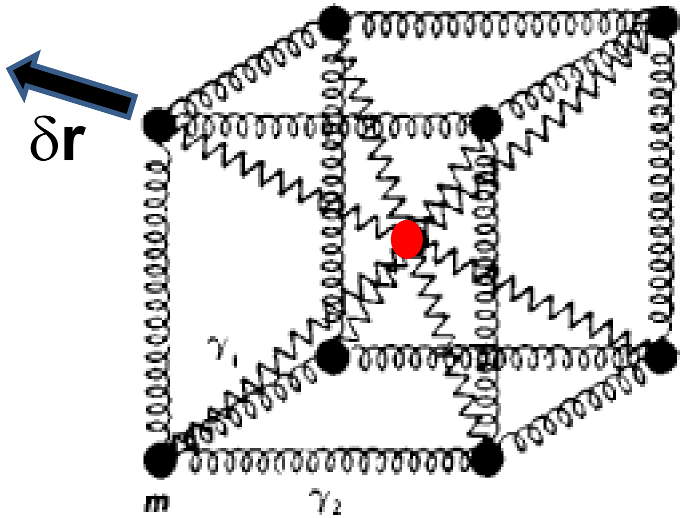
$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = N\sum_i f_i \frac{q_i^{\ 2}}{\varepsilon_0 m_i} \frac{\omega\gamma_i}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2 \gamma_i^{\ 2}}$$

Drude model dielectric function:





Extensions of the Drude model for lattice vibrations



In principle, the ideas of the Drude model apply both to the ionic vibrations which occur at low frequency (~10^{12} Hz) contributing to the so called static permittivity function ε_s and to the electronic vibrations which occur at high frequency (~10^{15} Hz) contributing to the so called high frequency permittivity function ε_∞.

In this model at high frequencies, only the electrons contribute to the polarization: $\epsilon_\infty = \epsilon_0 + \frac{|\mathbf{P}_{electron}|}{|\mathbf{E}|}$

At low frequencies both electrons and ions contribute to the polarization: $\epsilon_s = \epsilon_0 + \frac{|\mathbf{P}_{electron}|}{|\mathbf{E}|} + \frac{|\mathbf{P}_{ion}|}{|\mathbf{E}|}$

$$\Rightarrow \frac{|\mathbf{P}_{ion}|}{|\mathbf{E}|} = \epsilon_s - \epsilon_\infty$$

In terms of the Drude model form:

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

This form applies to classical lattice vibration modes and also to quantum treatments of electronic transitions in which case, the prefactor $f_i \frac{q_i^2}{\varepsilon_0 m_i}$ should be reinterpreted as an "oscillator" strength calculated as a transition matrix element.

$$\frac{\varepsilon_\infty(\omega)}{\varepsilon_0} = 1 + N\sum_{i\,\in\,\text{electrons}} f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$\frac{\varepsilon_s(\omega)}{\varepsilon_0} = 1 + N\sum_{i\,\in\,\text{electrons}} f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i} + N\sum_{i\,\in\,\text{vibrations}} f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$\omega_i = 2\pi\nu_i$$

$$\nu_i \sim 10^{15}\,\text{Hz}$$

$$\omega_i = 2\pi\nu_i$$

$$\nu_i \sim 10^{12}\,\text{Hz}$$

Comment: The Drude model allows us to “derive”:

Kramers-Kronig transform – for dielectric function:

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} - 1 = \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\varepsilon_I(\omega')}{\varepsilon_0} \frac{1}{\omega' - \omega}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = -\frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \left(\frac{\varepsilon_R(\omega')}{\varepsilon_0} - 1 \right) \frac{1}{\omega' - \omega}$$

$$\text{with} \quad \varepsilon_R(-\omega) = \varepsilon_R(\omega); \quad \varepsilon_I(-\omega) = -\varepsilon_I(\omega)$$

Practical applications -- It is often possible/more convenient to calculate the imaginary response and use KK to deduce the real response or visa versa.

Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

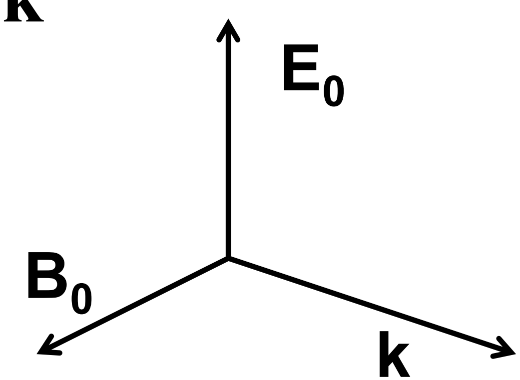
$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$$

Poynting vector and energy density:

$$\langle \mathbf{S}\rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\,\hat{\mathbf{k}}$$

$$\langle u\rangle_{avg} = \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$

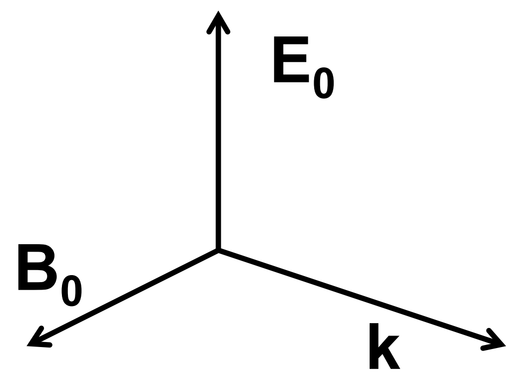


Transverse electric and magnetic waves (TEM)

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}} \times \mathbf{E}_0}{c} e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t} \right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t} \right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}} \cdot \mathbf{E}_0 = 0$$

TEM modes describe electromagnetic waves in lossless media and vacuum



For real
ε, μ, n, k

Effects of complex dielectric; fields near the surface on an ideal conductor

Suppose for an isotropic medium: $\mathbf{D} = \varepsilon_b \mathbf{E} \qquad \mathbf{J} = \sigma \mathbf{E}$

Maxwell's equations in terms of $\mathbf{H}$ and $\mathbf{E}$:

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon_b \frac{\partial \mathbf{E}}{\partial t}$$

$$\left(\nabla^2 - \mu\sigma \frac{\partial}{\partial t} - \mu\varepsilon_b \frac{\partial^2}{\partial t^2} \right) \mathbf{F} = 0 \qquad \mathbf{F} = \mathbf{E}, \mathbf{H}$$

Plane wave form for $\mathbf{E}$:

$$\mathbf{E}(\mathbf{r}, t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t} \right) \qquad \text{where } \mathbf{k} = (n_R + i n_I) \frac{\omega}{c} \hat{\mathbf{k}}$$

$$\Rightarrow \mathbf{E}(\mathbf{r}, t) = e^{-\hat{\mathbf{k}}\cdot\mathbf{r}/\delta} \Re\left(\mathbf{E}_0 e^{i n_R (\omega/c) \hat{\mathbf{k}}\cdot\mathbf{r} - i\omega t} \right)$$

Some details:

Plane wave form for $\mathbf{E}$:

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \text{where } \mathbf{k} = \left(n_R + i n_I\right)\frac{\omega}{c}\hat{\mathbf{k}}$$

$$\left(\nabla^2 - \mu\sigma\frac{\partial}{\partial t} - \mu\varepsilon_b\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = 0$$

$$-\left(n_R + i n_I\right)^2 + i\frac{\mu\sigma c^2}{\omega} + \mu\varepsilon_b c^2 = 0$$

Fields near the surface on an ideal conductor -- continued

For our system:

$$\frac{\omega}{c} n_R = \omega \sqrt{\frac{\mu \varepsilon_b}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_b} \right)^2} + 1 \right)^{1/2}$$

$$\frac{\omega}{c} n_I = \omega \sqrt{\frac{\mu \varepsilon_b}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_b} \right)^2} - 1 \right)^{1/2}$$

$$\text{For } \frac{\sigma}{\omega} >> 1 \qquad \frac{\omega}{c} n_R \approx \frac{\omega}{c} n_I \approx \sqrt{\frac{\mu \sigma \omega}{2}} \equiv \frac{1}{\delta}$$

$$\Rightarrow \mathbf{E}(\mathbf{r},t) = e^{-\hat{\mathbf{k}}\cdot\mathbf{r}/\delta} \Re\left(\mathbf{E}_0 e^{i\hat{\mathbf{k}}\cdot\mathbf{r}/\delta - i\omega t} \right)$$

$$\Rightarrow \mathbf{H}(\mathbf{r},t) = \frac{n}{c\mu} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r},t) = \frac{1+i}{\delta\mu\omega} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r},t)$$

Some representative values of skin depth

Ref: Lorrain and Corson

$$\frac{\omega}{c} n_R \approx \frac{\omega}{c} n_I \approx \sqrt{\frac{\mu\sigma\omega}{2}} \equiv \frac{1}{\delta}$$

	σ (10^7 S/m)	μ/μ_0	δ (0.001m) at 60 Hz	δ (0.001m) at 1 MHz
Al	3.54	1	10.9	84.6
Cu	5.80	1	8.5	66.1
Fe	1.00	100	1.0	10.0
Mumetal	0.16	2000	0.4	3.0
Zn	1.86	1	15.1	117

Relative energies associated with field

Electric energy density: $\varepsilon_b \left|\mathbf{E}\right|^2$

Magnetic energy density: $\mu \left|\mathbf{H}\right|^2$

Ratio inside conducting media:

$$\frac{\varepsilon_b \left|\mathbf{E}\right|^2}{\mu \left|\mathbf{H}\right|^2} = \frac{\varepsilon_b}{\mu \left|\frac{1+i}{\delta\mu\omega}\right|^2} = \frac{\varepsilon_b \mu \omega^2 \delta^2}{2}$$

Here wavelength is defined:

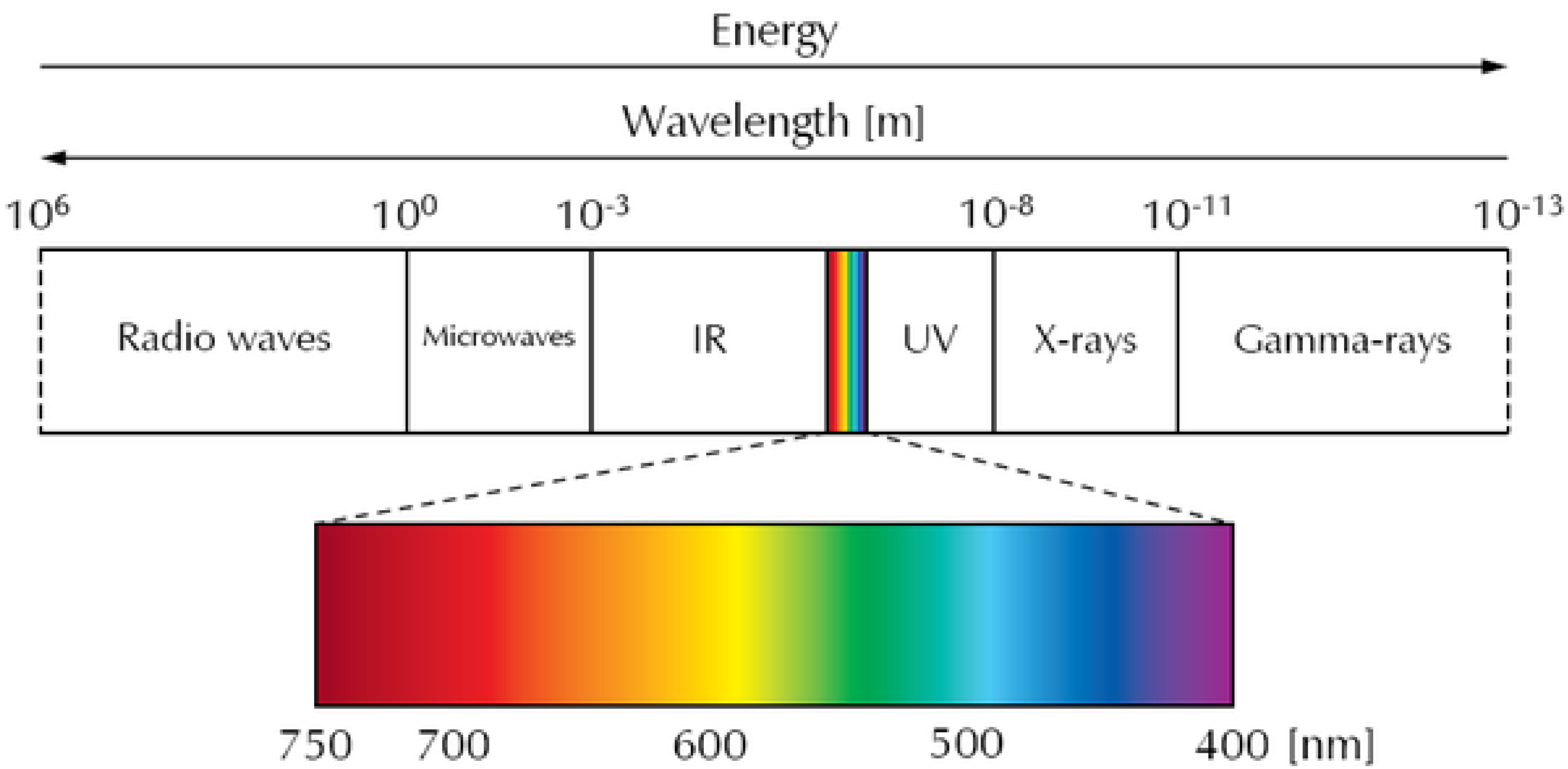
$$\lambda = \frac{2\pi c}{\omega}$$

$$= 2\pi^2 \frac{\varepsilon_b}{\varepsilon_0} \frac{\mu}{\mu_0} \frac{\delta^2}{\lambda^2}$$

For $\frac{\varepsilon_b \left|\mathbf{E}\right|^2}{\mu \left|\mathbf{H}\right|^2} \ll 1 \quad \Rightarrow$ magnetic energy dominates

Note that in free space, $\frac{\varepsilon_0 \left|\mathbf{E}\right|^2}{\mu_0 \left|\mathbf{H}\right|^2} = 1$

Various wavelengths λ --



General relationships

Comment on complex dielectric and refractive index functions

For $\mu = \mu_0$:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\varepsilon_R}{\varepsilon_0} + i\frac{\varepsilon_I}{\varepsilon_0} \equiv a + ib = \left(n_R + in_I\right)^2$$

$$a = n_R^2 - n_I^2$$

$$b = 2n_R n_I$$

$$\Rightarrow n_R^2 = \frac{1}{2}\left(a + \sqrt{a^2 + b^2}\right) \qquad n_I^2 = \frac{1}{2}\left(-a + \sqrt{a^2 + b^2}\right)$$