PHY 712 Electrodynamics
10-10:50 AM in Olin 103

Notes for Lecture 22:
Wave Guides

Chap. 8 (Sec. 8.2++) in Jackson –

1. Cylindrical wave guides
2. Coaxial cables

14	Fri: 02/16/2024	Chap. 5	Magnetic dipoles and dipolar fields	#13	02/19/2024
15	Mon: 02/19/2024	Chap. 6	Maxwell's Equations	#14	02/26/2024
16	Wed: 02/21/2024	Chap. 6	Electromagnetic energy and forces	#15	02/26/2024
17	Fri: 02/23/2024	Chap. 7	Electromagnetic plane waves	#16	02/26/2024
18	Mon: 02/26/2024	Chap. 7	Electromagnetic plane waves	#17	03/01/2024
19	Wed: 02/28/2024	Chap. 7	Optical effects of refractive indices	#18	03/01/2024
20	Fri: 03/01/2024	Chap. 1-7	Review		
21	Mon: 03/04/2024	Chap. 8	Short lecture on waveguides - rectangular	Exam	
22	Wed: 03/06/2024	Chap. 8	Short lecture on waveguides - cylindrical	Exam	
23	Fri: 03/08/2024	Chap. 8	Short lecture on waveguides - resonant cavities	Exam	
	Mon: 03/11/2024	No class	*Spring Break*		
	Wed: 03/13/2024	No class	*Spring Break*		
	Fri: 03/15/2024	No class	*Spring Break*		
24	Mon: 03/18/2024	Chap. 9	Radiation from localized oscillating sources		

For 3/03/2024-3/08/2024:

- Individual work on take home exam
- Shortened class lectures on Chapter 8 of **Jackson**

Physics Colloquium

Thursday

4 PM Olin 101

March 7th, 2024

Audio Forensic Analysis of Gunshot Sounds

Audio forensic analysis is the field of forensic science relating to the acquisition, analysis, and evaluation of sound recordings that may ultimately be presented in court or some official venue. The primary forensic concerns are assessing authenticity, enhancing speech to aid intelligibility, and scientific interpretation and documentation. Audio forensic evidence may include gunshot sounds, and this sort of evidence is increasingly available due to home doorbell cameras, law enforcement body-worn cameras, and bystanders with smartphones. This presentation includes a description of contemporary methods and research about the interpretation of gunshot sounds for forensic purposes. Case study examples are also presented.

Robert C. (Rob) Maher, Ph.D., P.E.
Professor of Electrical and Computer Engineering
Montana State University

Physics Career Event - Prof. Rob Maher, Montana State University

Please join us for a meet and greet and small presentation with Professor Robert Maher from Montana State university. He has a background in digital signal processing, particularly applied to audio engineering and acoustics, including audio forensic analysis. Pizza will be served!

Thursday, March 7 at 12:00pm

Olin 105 1834 Wake Forest Road, Olin Physical Laboratory, Olin Physic

Boundary values of EM fields near the surface of an ideal conductor

Inside the conductor:

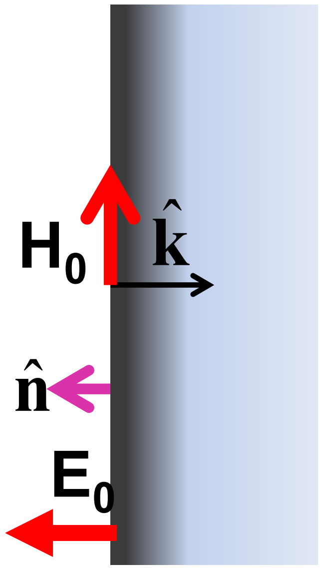
$$\mathbf{H}(\mathbf{r},t) = e^{-\hat{\mathbf{k}}\cdot\mathbf{r}/\delta}\,\Re\left(\mathbf{H}_0 e^{i\hat{\mathbf{k}}\cdot\mathbf{r}/\delta - i\omega t}\right)$$

$$\mathbf{E}(\mathbf{r},t) = \delta\mu\omega\frac{1-i}{2}\hat{\mathbf{k}}\times\mathbf{H}(\mathbf{r},t)$$

At the boundary of an ideal conductor, the **E** and **H** fields decay in the direction normal to the interface.

Ideal conductor boundary conditions:

$$\hat{\mathbf{n}}\times\mathbf{E}\Big|_S = 0 \qquad\qquad \hat{\mathbf{n}}\cdot\mathbf{H}\Big|_S = 0$$



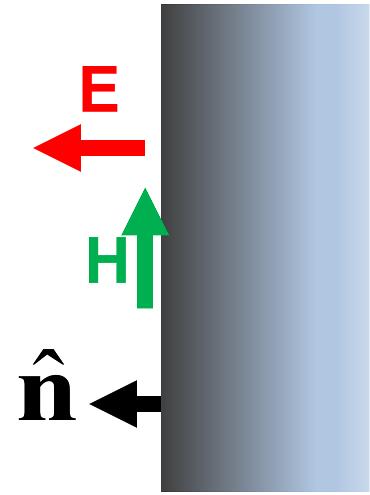
➔Use these properties to manipulate the form of EM waves

Wave guides – dielectric media with one or more metal boundary

Continuity conditions for fields near metal boundaries --

Ideal conductor boundary conditions:

$$\hat{\mathbf{n}} \times \mathbf{E}\Big|_S = 0 \qquad \hat{\mathbf{n}} \cdot \mathbf{H}\Big|_S = 0$$



Waveguide terminology

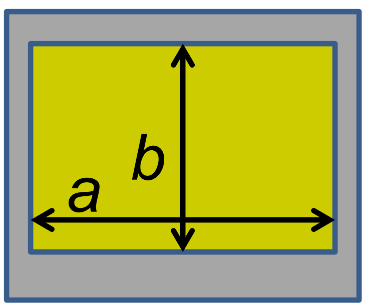
- TEM: transverse electric and magnetic (both E and H fields are perpendicular to wave propagation direction)
- TM: transverse magnetic (H field is perpendicular to wave propagation direction)
- TE: transverse electric (E field is perpendicular to wave propagation direction)

Analysis of rectangular waveguide

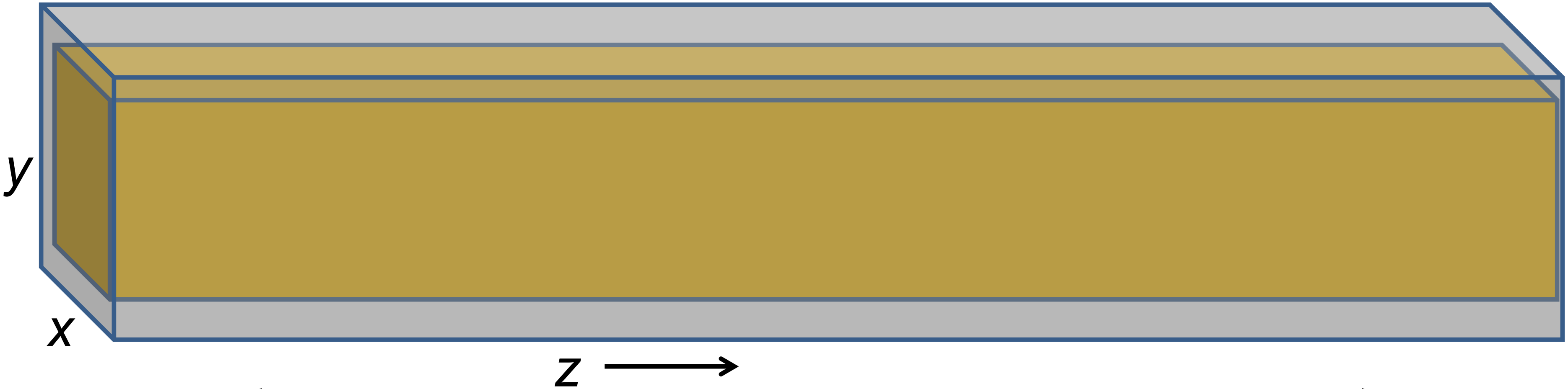
Boundary conditions at surface of waveguide:

$\mathbf{E}_{\text{tangential}}=0, \quad \mathbf{B}_{\text{normal}}=0$

Cross section view



Analysis of rectangular waveguide



$$\mathbf{B} = \Re\left\{\left(B_x(x,y)\hat{\mathbf{x}} + B_y(x,y)\hat{\mathbf{y}} + B_z(x,y)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

$$\mathbf{E} = \Re\left\{\left(E_x(x,y)\hat{\mathbf{x}} + E_y(x,y)\hat{\mathbf{y}} + E_z(x,y)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

Inside the dielectric medium: (assume ε to be real)

$\nabla\cdot\mathbf{E} = 0$ $\qquad$ $\nabla\cdot\mathbf{B} = 0$

$\nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0$ $\qquad$ $\nabla\times\mathbf{B} - \varepsilon\mu\frac{\partial\mathbf{E}}{\partial t} = 0$

Solution of Maxwell's equations within the pipe:

Combining Faraday's Law and Ampere's Law, we find that each field component must satisfy a two-dimensional Helmholz equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-k^2+\mu\varepsilon\omega^2\right)\mathbf{F}(x,y) = 0.$$

F = **E** or **H**
propagation along *z*.

For the rectangular wave guide discussed in Section 8.4 of your text a solution for a TE mode can have:

$$E_z(x,y)\equiv 0 \quad \text{and} \quad B_z(x,y)=B_0\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right),$$

$$\text{with } k^2 \equiv k_{mn}^2 = \mu\varepsilon\omega^2-\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]$$

Maxwell's equations within the pipe in terms of all 6 components:

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+ikB_z=0.$$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+ikE_z=0.$$

For TE mode with $E_z \equiv 0$

$$B_x=-\frac{k}{\omega}E_y$$

$$B_y=\frac{k}{\omega}E_x$$

$$\frac{\partial E_z}{\partial y}-ikE_y=i\omega B_x.$$

$$ikE_x-\frac{\partial E_z}{\partial x}=i\omega B_y.$$

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=i\omega B_z.$$

$$\frac{\partial B_z}{\partial y}-ikB_y=-i\mu\varepsilon\omega E_x.$$

$$ikB_x-\frac{\partial B_z}{\partial x}=-i\mu\varepsilon\omega E_y.$$

$$\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=-i\mu\varepsilon\omega E_z.$$

TE modes for rectangular wave guide continued:

$$E_z(x,y) \equiv 0 \quad \text{and} \quad B_z(x,y) = B_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right),$$

$$E_x = \frac{\omega}{k} B_y = \frac{-i\omega}{k^2 - \mu\varepsilon\omega^2}\frac{\partial B_z}{\partial y} = \frac{-i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}\frac{n\pi}{b} B_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right),$$

$$E_y = -\frac{\omega}{k} B_x = \frac{i\omega}{k^2 - \mu\varepsilon\omega^2}\frac{\partial B_z}{\partial x} = \frac{i\omega}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}\frac{m\pi}{a} B_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right).$$

Check boundary conditions:

$\mathbf{E}_{\text{tangential}} = 0$ because: $E_z(x,y) \equiv 0, \;\; E_x(x,0) = E_x(x,b) = 0$

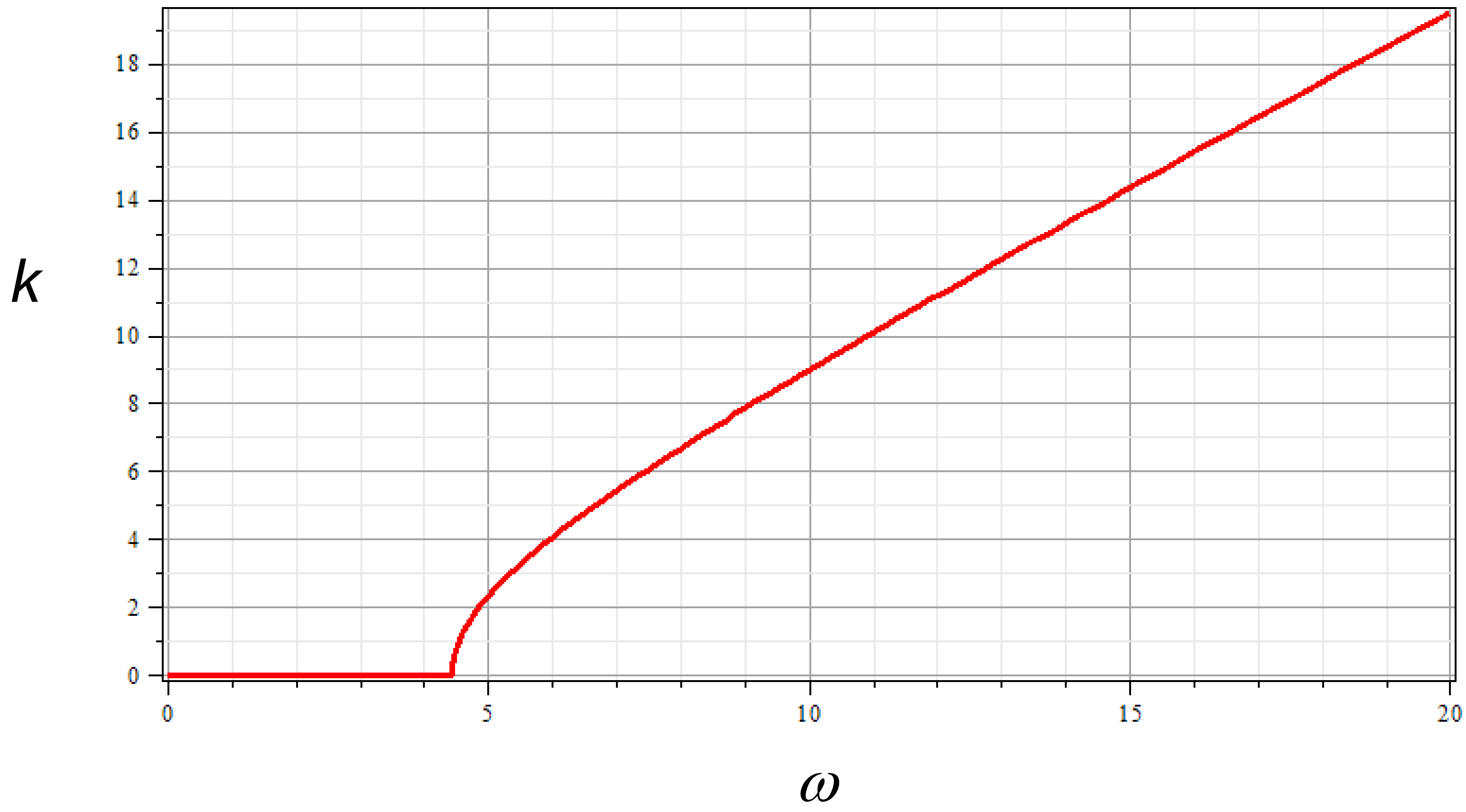
and $E_y(0,y) = E_y(a,y) = 0.$

$\mathbf{B}_{\text{normal}} = 0$ because: $B_y(x,0) = B_y(x,b) = 0$

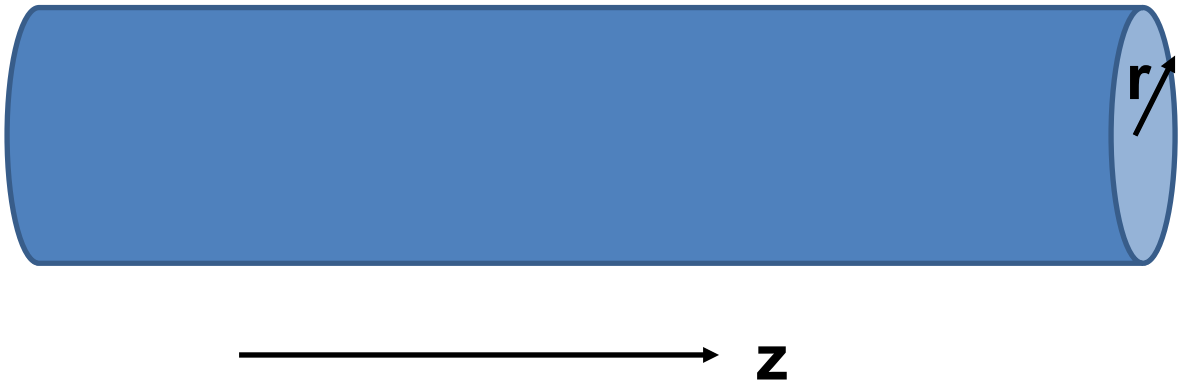
and $B_x(0,y) = B_x(a,y) = 0.$

Solution for m=n=1

$$k^2 \equiv k_{mn}^2 = \mu\varepsilon\omega^2 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$



Now consider a simple cylindrical waveguide



For the rectangular geometry, Maxwell's equations within the pipe:

$$\mathbf{B} = \Re\left\{\left(B_x(x,y)\hat{\mathbf{x}} + B_y(x,y)\hat{\mathbf{y}} + B_z(x,y)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

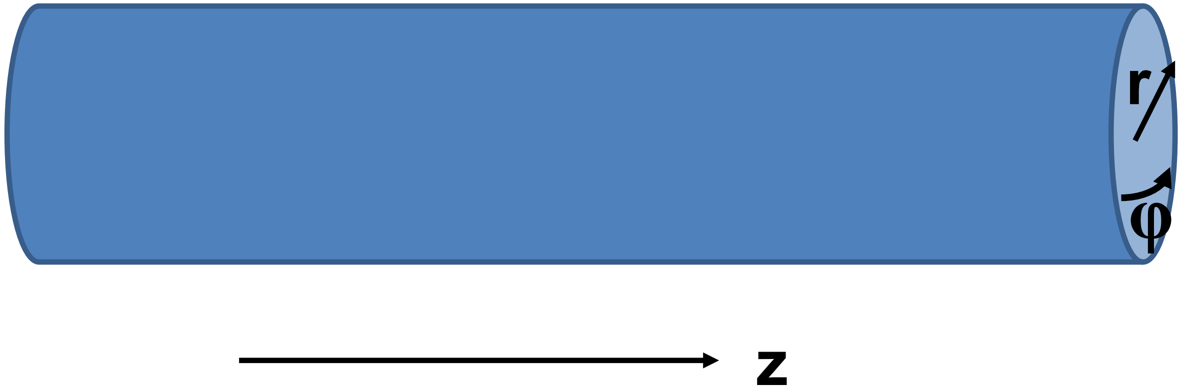
$$\mathbf{E} = \Re\left\{\left(E_x(x,y)\hat{\mathbf{x}} + E_y(x,y)\hat{\mathbf{y}} + E_z(x,y)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

Inside the dielectric medium: (assume ε, μ to be real)

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \times \mathbf{B} - \varepsilon\mu \frac{\partial \mathbf{E}}{\partial t} = 0$$

Simple cylindrical waveguide -- continued



For the cylindrical geometry, Maxwell's equations within the pipe:

$$\mathbf{B} = \Re\left\{\left(\mathbf{B}_T(r,\varphi) + B_z(r,\varphi)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

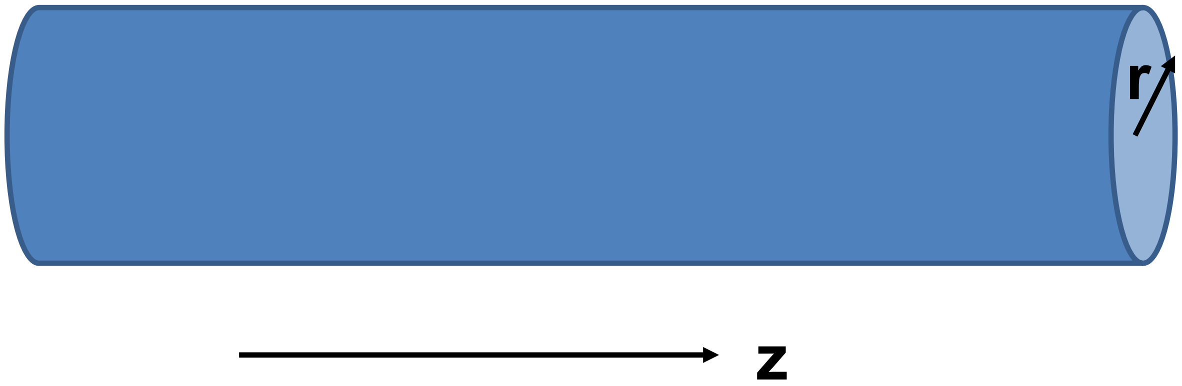
$$\mathbf{E} = \Re\left\{\left(\mathbf{E}_T(r,\varphi) + E_z(r,\varphi)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

$$\mathbf{E}_\mathbf{T} = E_r(r,\varphi)\hat{\mathbf{r}} + E_\varphi(r,\varphi)\hat{\boldsymbol{\varphi}}$$

Using Maxwell's equations within medium:

$$\nabla\cdot\mathbf{E}=0 \quad \nabla\cdot\mathbf{B}=0 \quad \nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0 \qquad \nabla\times\mathbf{B} - \varepsilon\mu\frac{\partial\mathbf{E}}{\partial t}=0$$

Simple cylindrical waveguide



$$\mathbf{B} = \Re\left\{\left(\mathbf{B}_T(\mathbf{r}_T) + B_z(\mathbf{r}_T)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

$$\mathbf{E} = \Re\left\{\left(\mathbf{E}_T(\mathbf{r}_T) + E_z(\mathbf{r}_T)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

Decoupling the equations for $F(\mathbf{r}_T) = B_z(\mathbf{r}_T)$ or $F(\mathbf{r}_T) = E_z(\mathbf{r}_T)$:

Combining equations: $\left(\nabla_T^2 - k^2 + \mu\varepsilon\omega^2\right) F(\mathbf{r}_T) \;=\; 0.$

For the cylindrical case, $F(\mathbf{r}_T) \rightarrow F(r,\varphi)$

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} - k^2 + \mu\varepsilon\omega^2\right)\mathbf{F}(r,\varphi) \;=\; 0.$$

Simple cylindrical waveguide -- continued

Evaluating $F(r,\varphi) = B_z(\mathbf{r}_T)$ for TE mode or $F(r,\varphi) = E_z(\mathbf{r}_T)$ for TM mode:

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} - k^2 + \mu\varepsilon\omega^2\right) F(r,\varphi) \;=\; 0$$

The components take form $F(r,\varphi) = f_m(r)e^{im\varphi}$

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2}{r^2} - k^2 + \mu\varepsilon\omega^2\right) f_m(r) = 0$$

For $\kappa^2 \equiv k^2 - \mu\varepsilon\omega^2$ $\quad \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2}{r^2} - \kappa^2\right) f_m(r) = 0$

$\Rightarrow f_m(r) = J_m(\kappa r)$ Bessel functions

Boundary values will be applied for $r = a$

In some cases, for zeroes of Bessel function $J_m(\kappa a) = 0$

$$\Rightarrow \kappa = \frac{x_{mn}}{a} \quad \text{for } J_m(x_{mn}) = 0$$

After finding $F(r,\varphi)=B_z(\mathbf{r}_T)$ for TE mode or $F(r,\varphi)=E_z(\mathbf{r}_T)$ for TM mode, the analysis continues, finding $\mathbf{B_T}(\mathbf{r}_T)$ and $\mathbf{E_T}(\mathbf{r}_T)$ --

$$\mathbf{B}=\Re\left\{\left(\mathbf{B}_T(\mathbf{r}_T)+B_z\left(\mathbf{r}_T\right)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

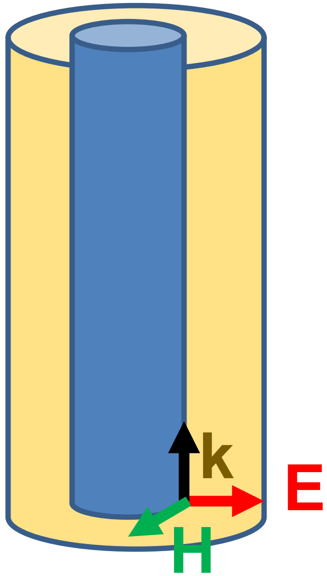
$$\mathbf{E}=\Re\left\{\left(\mathbf{E}_T(\mathbf{r}_T)+E_z\left(\mathbf{r}_T\right)\hat{\mathbf{z}}\right)e^{ikz-i\omega t}\right\}$$

Note that in the examples with the “simple” waveguide, having a single outer metallic boundary, TE or TM modes can be produced.

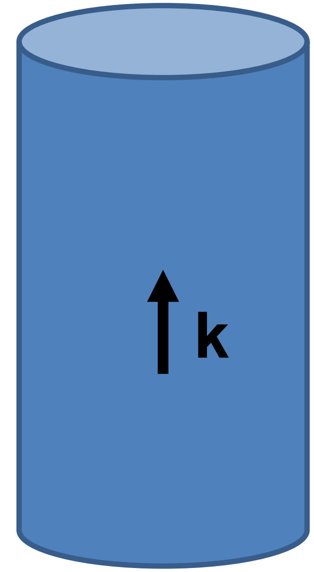
For a more complicated design, it is possible to have TEM modes --

Wave guides – dielectric media with one or more metal boundary

Coaxial cable
TEM modes



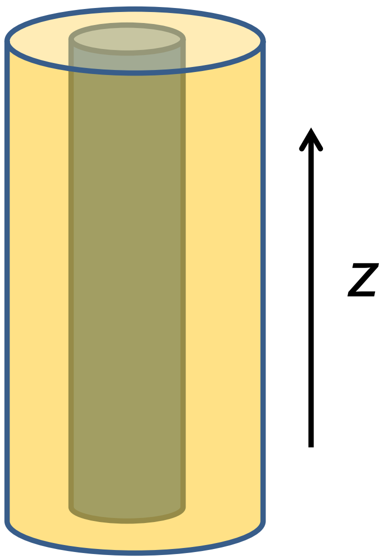
Simple optical pipe
TE or TM modes



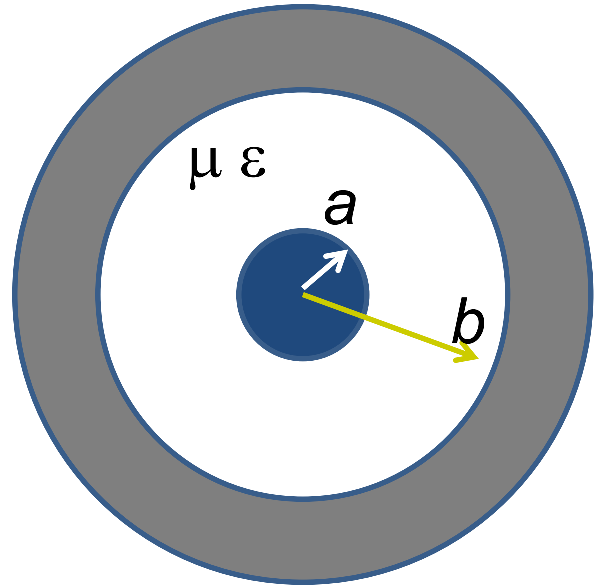
Waveguide terminology

- TEM: transverse electric and magnetic (both E and H fields are perpendicular to wave propagation direction)
- TM: transverse magnetic (H field is perpendicular to wave propagation direction)
- TE: transverse electric (E field is perpendicular to wave propagation direction)

Wave guides



Top view:



Inside medium, μ ε assumed to be real

Coaxial cable
TEM modes

(following problem 8.2 in Jackson's text)

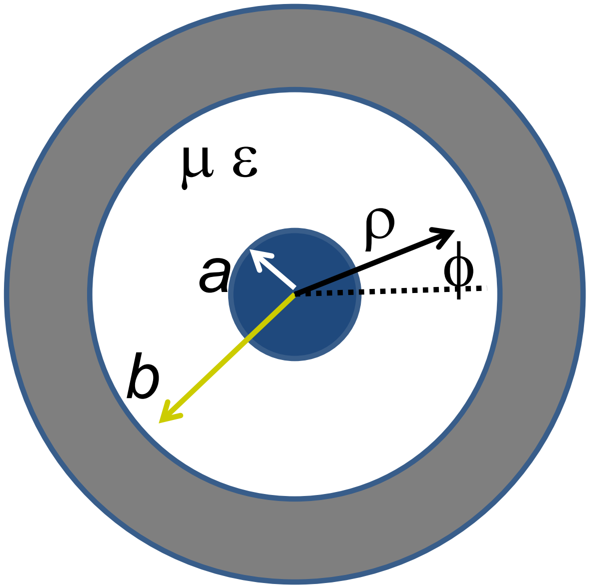
Maxwell's equations inside medium: for $a \leq \rho \leq b$

$$\nabla \times \mathbf{E} = i\omega\mathbf{B} \qquad \nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{B} = -i\omega\mu\varepsilon\mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0$$

Electromagnetic waves in a coaxial cable -- continued

Top view:



Example solution for $a \le \rho \le b$

$$\mathbf{E} = \hat{\boldsymbol{\rho}}\Re\left(\frac{E_0 a}{\rho} e^{ikz-i\omega t}\right)$$

$$\mathbf{B} = \hat{\boldsymbol{\varphi}}\Re\left(\frac{B_0 a}{\rho} e^{ikz-i\omega t}\right)$$

$$\hat{\boldsymbol{\rho}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}}$$

$$\hat{\boldsymbol{\varphi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

Find:

$$k = \omega\sqrt{\mu\varepsilon}$$

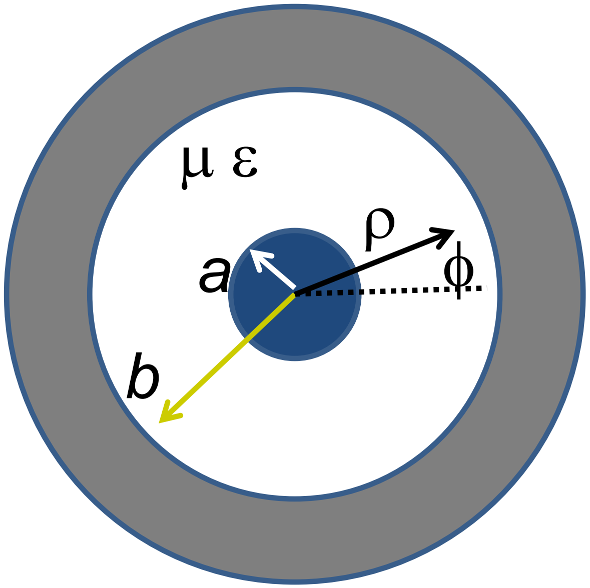
$$E_0 = \frac{B_0}{\sqrt{\mu\varepsilon}}$$

Poynting vector within cable medium (with μ, ε):

$$\langle \mathbf{S} \rangle_{avg} = \frac{1}{2\mu}\Re\left(\mathbf{E}\times\mathbf{B}^*\right) = \frac{|B_0|^2}{2\mu\sqrt{\mu\varepsilon}}\left(\frac{a}{\rho}\right)^2 \hat{\mathbf{z}}$$

Electromagnetic waves in a coaxial cable -- continued

Top view:



Time averaged power in cable material:

$$\int_0^{2\pi} d\phi \int_a^b \rho d\rho \left(\langle \mathbf{S} \rangle_{avg} \cdot \hat{\mathbf{z}} \right) = \frac{|B_0|^2 \pi a^2}{\mu \sqrt{\mu \varepsilon}} \ln\left(\frac{b}{a}\right)$$