PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Notes for Lecture 25:

Continue reading Chap. 9

A. **Electromagnetic waves due to specific sources**

B. **Dipole radiation examples**

C. **Radiation from antennas**

Physics Career Meet and Greet - Dr. Ryan Melvin, University of Alabama, Birmingham, and WFU Alum

Please join us for a career event with Dr. Ryan Melvin, Professor of Anesthesiology and Perioperative Medicine, Heersink School of Medicine, University of Alabamama Birmingham. Pizza will be served.

Thursday, March 21 at 12:00pm to 1:30am

Olin Physical Laboratory, Lounge

2090 Eure Dr., Winston-Salem, NC 27106

PHYSICS COLLOQUIUM

THURSDAY

4 PM Olin 101

MARCH 21ST, 2024

Adventures in Time Series Analysis

This seminar, tailored for physicists, embarks on an explorative journey into time series analysis, demonstrating its impact across finance, healthcare, and artificial intelligence. We begin in the financial world, where time series models are key in forecasting. Here, physicists' familiarity with exponential smoothing and Brownian motion provides a unique lens to understand these financial tools.

Shifting to healthcare, we encounter the physics of blood flow in cerebral autoregulation (the body's dynamic ability to maintain proper oxygenation for the brain). Here we examine how time series analysis is crucial for optimized blood pressure management during cardiovascular surgery and intensive care stays.

Next, treating text as sequential data, we examine generative AI. We'll explore how time series analysis is employed in the realm of AI, specifically in natural language processing, demonstrating its relevance beyond traditional numerical data. The seminar will conclude with a demo of several custom generative AI tools currently used in clinical research and administration.

Designed for physicists, this session highlights the interdisciplinary applications of time series analysis, revealing its ubiquitous role in diverse fields from economic predictions to medical technology and AI innovations.

Prof. Ryan Melvin
Heersink School of Medicine
Department of Anesthesiology and Perioperative Medicine
UAB

4 pm - Olin 101

24	Mon: 03/18/2024	Chap. 9	Digression on Math methods and Radiation from localized oscillating sources	#19	03/25/2024
25	Wed: 03/20/2024	Chap. 9	Radiation from localized oscillating sources	#20	03/25/2024
26	Fri: 03/22/2024	Chap. 9 & 10	Radiation and scattering		
27	Mon: 03/25/2024	Chap. 11	Special Theory of Relativity		
28	Wed: 03/27/2024	Chap. 11	Special Theory of Relativity		
29	Fri: 03/29/2024	Chap. 11	Special Theory of Relativity		
30	Mon: 04/01/2024	Chap. 14	Radiation from moving charges		
31	Wed: 04/03/2024	Chap. 14	Radiation from accelerating charged particles		
32	Fri: 04/05/2024	Chap. 14	Synchrotron radiation and Compton scattering		
33	Mon: 04/08/2024	Chap. 15	Radiation from collisions of charged particles		
34	Wed: 04/10/2024	Chap. 13	Cherenkov radiation		
35	Fri: 04/12/2024		Special topic: E & M aspects of superconductivity		
36	Mon: 04/15/2024		Special topic: Quantum Effects in E & M		
37	Wed: 04/17/2024		Special topic: Quantum Effects in E & M		
38	Fri: 04/19/2024		Special topic: Quantum Effects in E & M		
	Mon: 04/22/2024		Presentations I		
	Wed: 04/24/2024		Presentations II		
	Fri: 04/26/2024		Presentations III		
39	Mon: 04/29/2024		Review		
40	Wed: 05/01/2024		Review		

PHY 712 -- Assignment #20

Assigned: 3/18/2024 Due: 3/25/2024

Continue reading Chapter 9 (Sec. 9.1-9.2) in **Jackson** .

1. Problem 9.10 in **Jackson** lists the harmonic frequency dependent charge and current densities of a radiating H atom. Instead of answering **Jackson's** questions, calculate the exact scalar $\Phi(\mathbf{r},\omega_0)$ and vector potential $\mathbf{A}(\mathbf{r},\omega_0)$ fields for $r \gg a_0$ and compare your results with the scalar and vector potential fields calculated within the dipole approximation.

In this problem, you are given the following harmonically oscillating source:

$$\rho(r,\theta,\phi,t) = \frac{2e}{\sqrt{6}a_0^4} r\, e^{-3r/(2a_0)} Y_{00}(\theta,\phi) Y_{10}(\theta,\phi) e^{-i\omega_0 t}$$

$$\mathbf{J}(r,\theta,\phi,t) = -iv_0 \left(\frac{1}{2}\hat{\mathbf{r}} + \frac{a_0}{z}\hat{\mathbf{z}} \right) \rho(r,\theta,\phi,t)$$

Electromagnetic waves from time harmonic sources

Charge density: $\rho(\mathbf{r},t) = \Re\left(\tilde{\rho}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Current density: $\mathbf{J}(\mathbf{r},t) = \Re\left(\tilde{\mathbf{J}}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Note that the continuity condition applies:

$$\frac{\partial \rho(\mathbf{r},t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r},t) = 0 \quad \Rightarrow -i\omega\tilde{\rho}(\mathbf{r},\omega) + \nabla \cdot \tilde{\mathbf{J}}(\mathbf{r},\omega) = 0$$

General source: $f(\mathbf{r},t) = \Re\left(\tilde{f}(\mathbf{r},\omega)e^{-i\omega t}\right)$

For $\tilde{f}(\mathbf{r},\omega) = \frac{1}{4\pi\varepsilon_0}\tilde{\rho}(\mathbf{r},\omega)$

or $\tilde{f}(\mathbf{r},\omega) = \frac{\mu_0}{4\pi}\tilde{J}_i(\mathbf{r},\omega)$

Electromagnetic waves from time harmonic sources – continued:

$$\Psi(\mathbf{r},t) = \Psi_{f=0}(\mathbf{r},t) +$$

$$\int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|} \delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'| \right)\right) f(\mathbf{r}',t')$$

$$\tilde{\Psi}(\mathbf{r},\omega)e^{-i\omega t} = \tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} +$$

$$\int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|} \delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'| \right)\right) \tilde{f}(\mathbf{r}',\omega)e^{-i\omega t'}$$

$$= \tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{f}(\mathbf{r}',\omega)e^{-i\omega t}$$

Why is this result so different from the Liénard-Wiechert case?

Important results from last time – EM waves from time harmonic sources – open isotropic boundaries

For scalar potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega),$$

where $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\Phi}_0(\mathbf{r},\omega) = 0$

For vector potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \frac{\mu_0}{4\pi}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega),$$

where $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\mathbf{A}}_0(\mathbf{r},\omega) = 0$

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr) = j_l(kr) + in_l(kr)$

$$\widetilde{\Phi}(\mathbf{r},\omega) = \widetilde{\Phi}_0(\mathbf{r},\omega) + \sum_{lm}\widetilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\widetilde{\phi}_{lm}(r,\omega) = \frac{ik}{\varepsilon_0}\int d^3r'\,\widetilde{\rho}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

$$\widetilde{\mathbf{A}}(\mathbf{r},\omega) = \widetilde{\mathbf{A}}_0(\mathbf{r},\omega) + \sum_{lm}\widetilde{\mathbf{a}}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\widetilde{\mathbf{a}}_{lm}(r,\omega) = ik\mu_0\int d^3r'\,\widetilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

Example radiation source -- exact results for r>>R:

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\,(ik\mu_0)\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0 k}{\varepsilon_0\omega R}\cos\theta\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

Evaluation for $r>>R$:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\;\frac{2R^3}{\left(1+k^2R^2\right)^2}$$

$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{J_0 k}{\varepsilon_0\omega}\cos\theta\,\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right)\;\frac{2R^3}{\left(1+k^2R^2\right)^2}$$

Example radiation source – continued --

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

Note that the continuity of charge and current must be satisfied. For the Fourier amplitudes, the relations are as below:

Recall continuity condition: $-i\omega\,\tilde{\rho}(\mathbf{r},\omega)+\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)=0$

$$-i\omega\mathbf{r}\,\tilde{\rho}(\mathbf{r},\omega)+\mathbf{r}\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)$$

Jackson's clever trick for dipole moment --

$$\mathbf{p}(\omega)=\int d^3r\;\mathbf{r}\;\tilde{\rho}(\mathbf{r},\omega)=\frac{1}{i\omega}\int d^3r\;\mathbf{r}\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)$$

$$=-\frac{1}{i\omega}\int d^3r\;\tilde{\mathbf{J}}(\mathbf{r},\omega)$$

*Provided that $\left(\mathbf{r}\mathbf{J}(\mathbf{r},\omega)\right)_{r\to\infty}=0$

Example of radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega) = \hat{\mathbf{z}} J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega) = \frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

Dipole moment: $$\mathbf{p}(\omega) = \hat{\mathbf{z}}\frac{J_0}{-i\omega R}\int d^3r\; r\cos(\theta)\left(\cos(\theta)e^{-r/R}\right)$$

$$= \hat{\mathbf{z}}\frac{J_0}{-i\omega R}\frac{4\pi}{3}\int_0^\infty dr\; r^3 e^{-r/R} = \hat{\mathbf{z}} J_0 \frac{8\pi R^3}{-i\omega}$$

$$= \frac{1}{-i\omega}\int d^3r\; \tilde{\mathbf{J}}(\mathbf{r},\omega)$$

From the analysis valid for *kr>>1* and *kR<<1*:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = -\frac{i\mu_0\omega}{4\pi}\mathbf{p}(\omega)\frac{e^{ikr}}{r} = \hat{\mathbf{z}} J_0 \mu_0 2R^3 \frac{e^{ikr}}{r}$$

$$\tilde{\Phi}(\mathbf{r},\omega) = -\frac{ik}{4\pi\varepsilon_0}\mathbf{p}(\omega)\cdot\hat{\mathbf{r}}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r} = \frac{J_0 2R^3}{\varepsilon_0 c}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}\cos\theta$$

More details --

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\,(ik\mu_0)\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0 k}{\varepsilon_0\omega R}\cos\theta\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

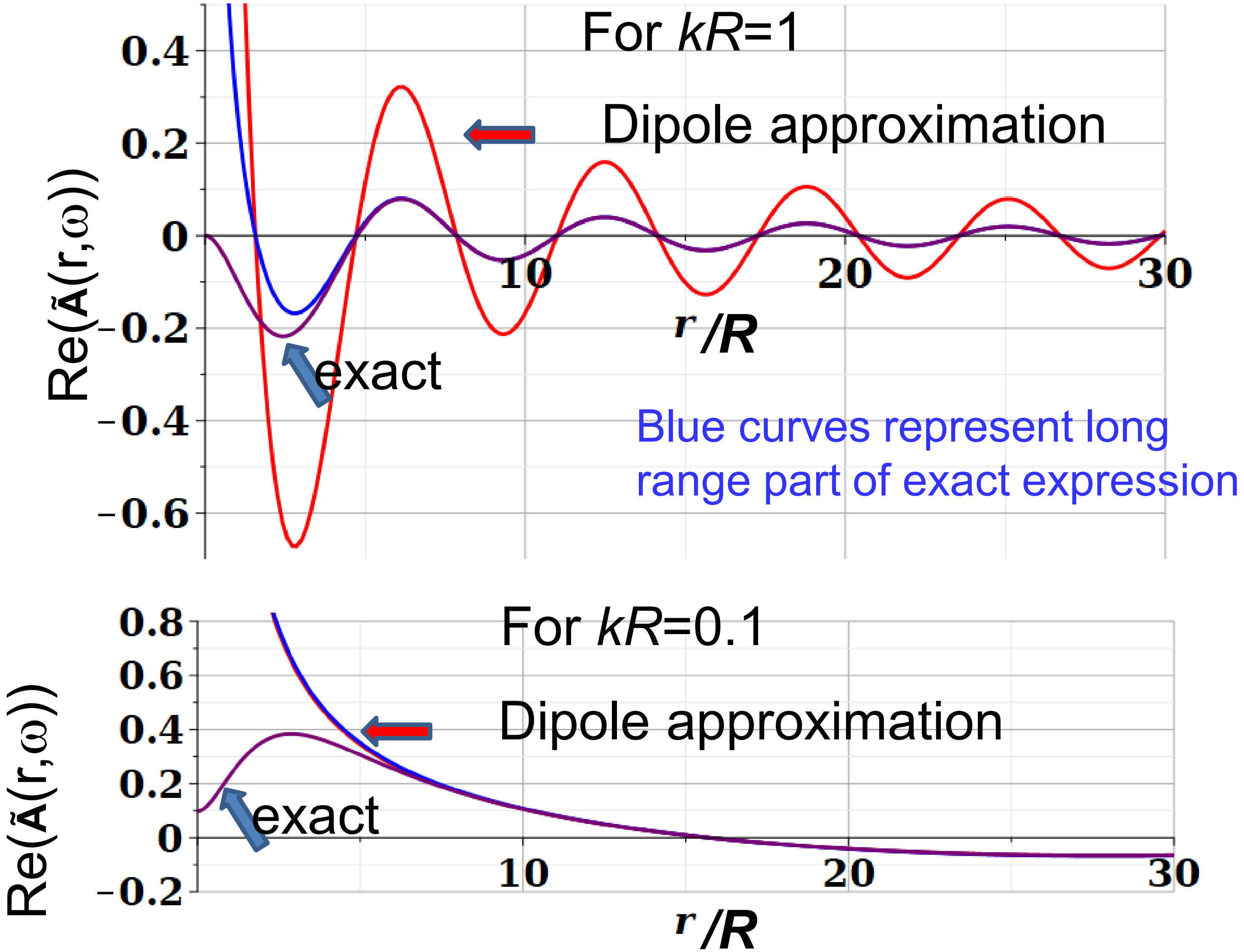
$$\tilde{\mathbf{A}}(r,\omega)=\hat{\mathbf{z}}J_0\mu_0\left(\frac{e^{ikr}}{kr}\int_0^r r'dr'e^{-r'/R}\sin(kr')+\frac{\sin(kr)}{kr}\int_r^\infty r'dr'e^{-r'/R+ikr'}\right)$$

$$\underset{r\gg R}{\approx}\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\frac{2R^3}{\left(k^2R^2+1\right)^2}$$

Correct when this term is negligible.

$$\tilde{\mathbf{A}}_{\text{dipole approx}}(r,\omega)=\hat{\mathbf{z}}J_0\mu_0\left(\frac{e^{ikr}}{kr}\int_0^\infty r'dr'e^{-r'/R}(kr')\right)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}2R^3$$

Example continued



Continued review of dipole results --

Power in the dipole approximation; Section 9.2 of Jackson

Here we use our notation with $\mathbf{n} \rightarrow \hat{\mathbf{r}}$ and $Z_0 \equiv \sqrt{\frac{\mu_0}{\epsilon_0}}$

$$\frac{dP}{d\Omega} = \frac{r^2}{2} \Re \left| \left(\hat{\mathbf{r}} \cdot \left(\mathbf{E} \times \mathbf{H}^* \right) \right) \right|^2$$

Using the expressions for the dipole fields far from the source:

$$\mathbf{H} = \frac{ck^2}{4\pi} \left(\hat{\mathbf{r}} \times \mathbf{p} \right) \frac{e^{ikr}}{r} \qquad \mathbf{E} = Z_0 \mathbf{H} \times \hat{\mathbf{r}}$$

The power can be written $\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2} k^4 \left| \left(\left(\hat{\mathbf{r}} \times \mathbf{p} \right) \times \hat{\mathbf{r}} \right) \right|^2$

Defining the angle θ by $\mathbf{p} \cdot \hat{\mathbf{r}} = |\mathbf{p}| \cos\theta$,

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2} k^4 |\mathbf{p}|^2 \sin^2\theta$$ integrating over solid angles $P = \frac{c^2 Z_0}{12\pi} k^4 |\mathbf{p}|^2$

Review:

Electromagnetic waves from time harmonic sources – continued:

Dipole radiation case:

Define dipole moment at frequency ω:

$$\mathbf{p}(\omega) \equiv \int d^3r \; \mathbf{r}\tilde{\rho}(\mathbf{r},\omega) = -\frac{1}{i\omega}\int d^3r \; \tilde{\mathbf{J}}(\mathbf{r},\omega)$$

For fields outside extent of source and $kr' << 1$ within the source:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = -\frac{i\mu_0\omega}{4\pi}\mathbf{p}(\omega)\frac{e^{ikr}}{r}$$

$$\tilde{\Phi}(\mathbf{r},\omega) = -\frac{ik}{4\pi\varepsilon_0}\mathbf{p}(\omega)\cdot\hat{\mathbf{r}}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}$$

Review:

Electromagnetic waves from time harmonic sources – continued:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = -\nabla\tilde{\Phi}(\mathbf{r},\omega) + i\omega\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\left(k^2\left(\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\times\hat{\mathbf{r}}\right) + \left(\frac{3\hat{\mathbf{r}}\left(\hat{\mathbf{r}}\cdot\mathbf{p}(\omega)\right)-\mathbf{p}(\omega)}{r^2}\right)(1-ikr)\right)$$

$$\tilde{\mathbf{B}}(\mathbf{r},\omega) = \nabla\times\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$= \frac{1}{4\pi\varepsilon_0 c}\frac{e^{ikr}}{r}k^2\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\left(1-\frac{1}{ikr}\right)$$

Power radiated for $kr >> 1$:

$$\frac{dP}{d\Omega} = r^2\hat{\mathbf{r}}\cdot\langle\mathbf{S}\rangle_{avg} = \frac{r^2}{2\mu_0}\hat{\mathbf{r}}\cdot\Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)\right)$$

$$= \frac{c^2k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left|\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)\times\hat{\mathbf{r}}\right|^2$$

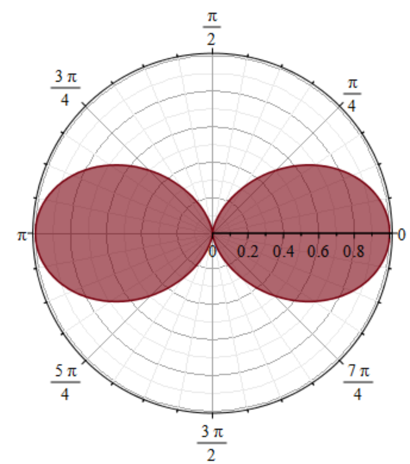
Properties of dipole radiation field for *kr* >>1:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = \frac{1}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\left(k^2\left((\hat{\mathbf{r}}\times\mathbf{p}(\omega))\times\hat{\mathbf{r}}\right)\right)$$

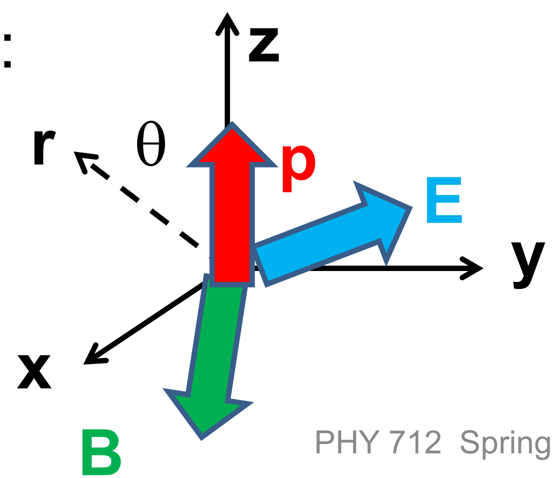
$$\tilde{\mathbf{B}}(\mathbf{r},\omega) = \frac{1}{4\pi\varepsilon_0 c^2}\frac{e^{ikr}}{r}k^2\left(\hat{\mathbf{r}}\times\mathbf{p}(\omega)\right)$$

Power radiated for $kr >> 1$:

$$\frac{dP}{d\Omega} = r^2\hat{\mathbf{r}}\cdot\langle\mathbf{S}\rangle_{avg} = \frac{c^2k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left|(\hat{\mathbf{r}}\times\mathbf{p}(\omega))\times\hat{\mathbf{r}}\right|^2$$



Example:



Note that vectors **r, E, B** are mutually orthogonal

Alternative approach

Fields from time harmonic source:

$$\tilde{\Phi}(\mathbf{r},\omega) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \frac{\mu_0}{4\pi}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

For $r >> r'$: $\quad |\mathbf{r}-\mathbf{r}'| \approx r - \hat{\mathbf{r}}\cdot\mathbf{r}' + \ldots$

$$\tilde{\Phi}(\mathbf{r},\omega) \approx \frac{1}{4\pi\epsilon_0}\frac{e^{ikr}}{r}\int d^3r' \, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'} \, \tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r' \, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'} \, \tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

For our example:

$$\tilde{\mathbf{J}}(\mathbf{r},\omega) = \hat{\mathbf{z}} J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega) = \frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

For $r >> r'$: $\qquad |\mathbf{r}-\mathbf{r}'| \approx r - \hat{\mathbf{r}}\cdot\mathbf{r}' + \ldots$

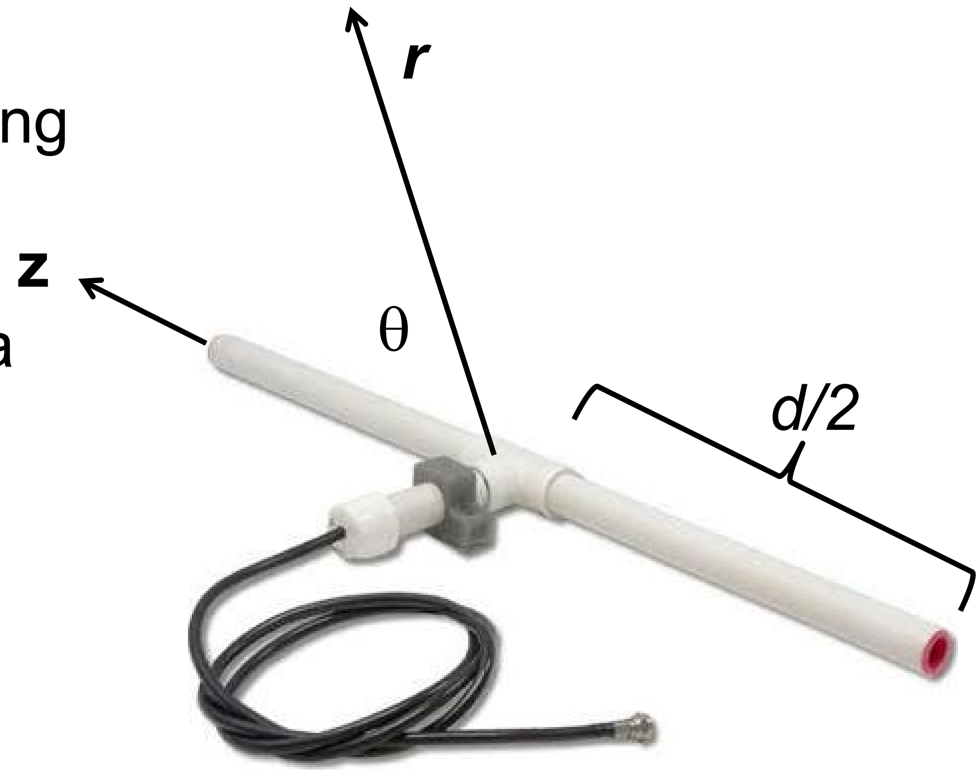
$$\tilde{\Phi}(\mathbf{r},\omega) \approx \frac{1}{4\pi\epsilon_0}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\;\tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\;\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

➔Results equivalent to Bessel function expansion in the limit $kr \rightarrow \infty$.

Other radiation sources using ``alternative approach"

Linear center-fed antenna



$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx$$

$$\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\;\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{J}}(\mathbf{r}',\omega) = I_0 \sin\left(\frac{kd}{2} - k\,|\,z\,|\right)\delta(x)\delta(y)\hat{\mathbf{z}}$$

Lecture 25 concluded here.

Alternative approach – linear center-fed antenna continued

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx \hat{\mathbf{z}} \frac{\mu_0 I_0}{4\pi} \frac{e^{ikr}}{r} \int_{-d/2}^{d/2} dz' \, e^{-ik\cos(\theta) z'} \sin\left(\frac{kd}{2} - k|z'|\right)$$

$$= \hat{\mathbf{z}} \frac{\mu_0 I_0}{2\pi} \frac{e^{ikr}}{kr} \left(\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin^2\theta} \right)$$

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$

Alternative approach – linear center-fed antenna continued

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^{\,2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{8\pi^2}\left|\frac{\cos\left(\frac{kd}{2}\cos\theta\right)-\cos\left(\frac{kd}{2}\right)}{\sin\theta}\right|^2$$

for $kd = \pi$:

$$\frac{dP}{d\Omega} = I_0^{\,2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{8\pi^2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

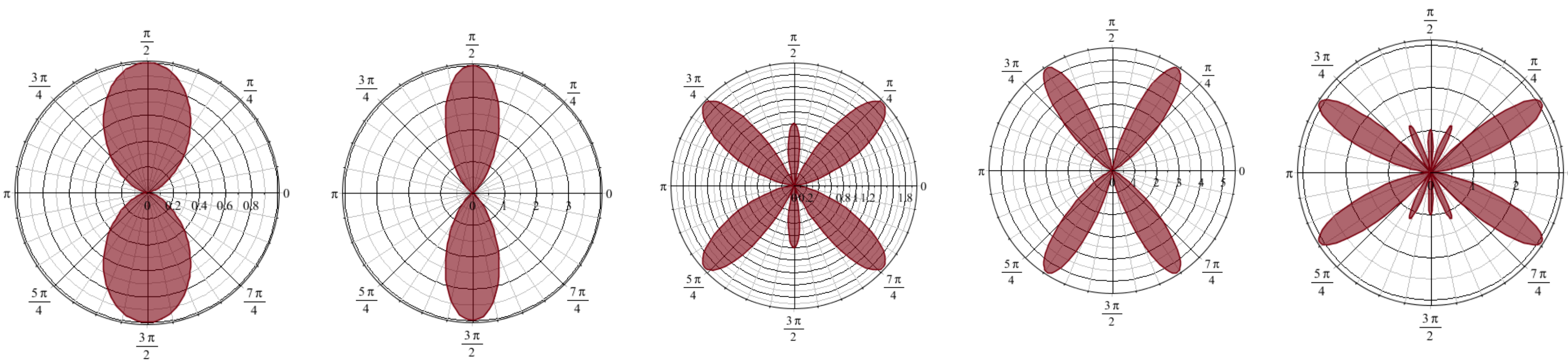
for $kd = 2\pi$:

$$\frac{dP}{d\Omega} = I_0^{\,2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{4}{8\pi^2}\frac{\cos^4\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

Alternative approach – linear center-fed antenna continued

Time averaged power:

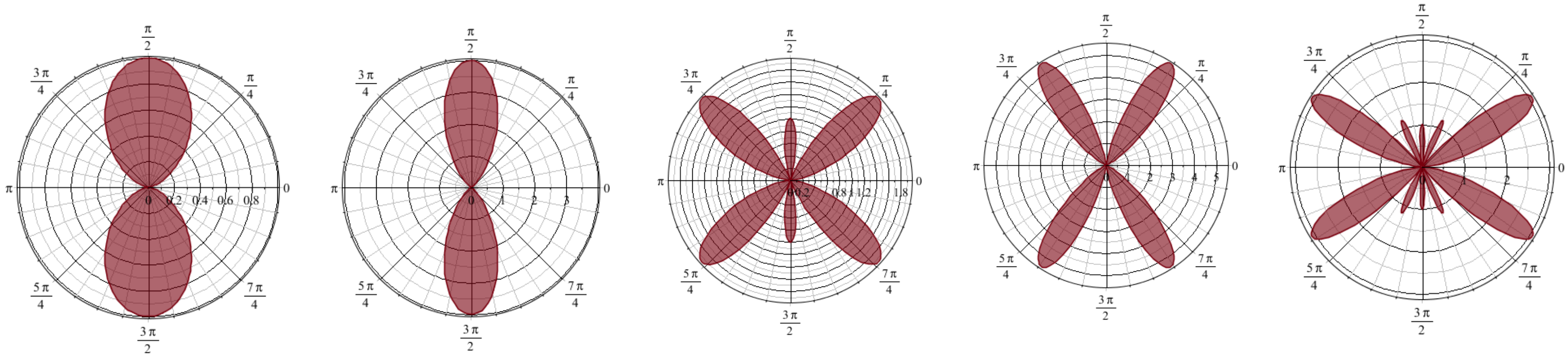
$$\frac{dP}{d\Omega} = I_0{}^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$



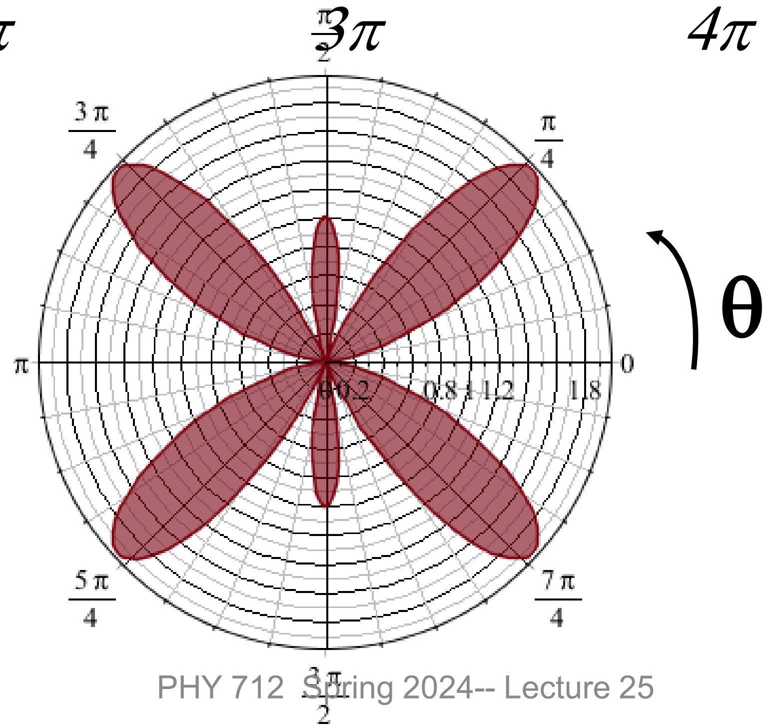
kd=π $\qquad$ 2π $\qquad$ 3π $\qquad$ 4π $\qquad$ 5π

Interesting patterns for special values of *kd*

Some details --



kd=π   *2π*   *3π*   *4π*   *5π*



Polar plot:
Angle indicates values of theta

Radius indicates value scaled to 1.

Next time – we will consider the effects of multiple antennas (antenna arrays including interference effects) and radiation due to light scattering (from Chapter 10 of your textbook).