PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Notes for Lecture 37:

Quantum effects in electrodynamics

Connections to experiment

a. Coherent states
b. Squeezed states
c. More complicated states

			localized oscillating sources		
25	Wed: 03/20/2024	Chap. 9	Radiation from localized oscillating sources	#20	03/25/2024
26	Fri: 03/22/2024	Chap. 9 & 10	Radiation and scattering	#21	03/25/2024
27	Mon: 03/25/2024	Chap. 11	Special Theory of Relativity	#22	04/01/2024
28	Wed: 03/27/2024	Chap. 11	Special Theory of Relativity	#23	04/01/2024
29	Fri: 03/29/2024	Chap. 11	Special Theory of Relativity		
30	Mon: 04/01/2024	Chap. 14	Radiation from moving charges	#24	04/08/2024
31	Wed: 04/03/2024	Chap. 14	Radiation from accelerating charged particles	#25	04/08/2024
32	Fri: 04/05/2024	Chap. 14	Synchrotron radiation and Compton scattering	#26	04/08/2024
	Mon: 04/08/2024	No class	Eclipse related absences		
33	Wed: 04/10/2024	Chap. 13 & 15	Other radiation -- Cherenkov & bremsstrahlung	#27	04/22/2024
34	Fri: 04/12/2024		Special topic: E & M aspects of superconductivity		
	Mon: 04/15/2024		Presentations I		
	Wed: 04/17/2024		Presentations II		
	Fri: 04/19/2024		Presentations III		
35	Mon: 04/22/2024		Special topic: Quantum Effects in E & M		
36	Wed: 04/24/2024		Special topic: Quantum Effects in E & M		
37	Fri: 04/26/2024		Special topic: Quantum Effects in E & M		
38	Mon: 04/29/2024		Review		
39	Wed: 05/01/2024		Review		

Final take-home exam – due May 10, 2024

- Will need focus involving analysis and evaluations
- Will cover topics discussed throughout course

References –

- Consultation with Professor Kandada
- Rodney Loudon, “The quantum theory of light” (1983)
- Leonard Mandel and Emil Wolf, “Optical Coherence and Quantum Optics” (2013)
- Yanhua Shih, “An Introduction to Quantum Optics” (2021)   (some typos, but generally informative)
- Paul R Berman and Vladimir S. Malinovsky, “Principles of Laser Spectroscopy and Quantum Optics” (2011)
- Christopher C. Gerry and Peter L. Knight, “Introductory Quantum Optics” (2^{nd} Edition) (2024)

Review of equations related to quantized EM fields --

Recall that we can write the EM Hamiltonian for a single mode $\omega_{\mathbf{k}} \equiv \omega$ --

$$H_{rad} = \frac{1}{2}\hbar\omega\left(a^\dagger a + a a^\dagger\right) = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) \quad \text{where } \left[a, a^\dagger\right] = 1$$

Field eigenstates $\quad a^\dagger a|n\rangle = n|n\rangle$

For further analysis, it is convenient to define "Quadrature operators" which are unitless and Hermitian:

$$\hat{Q} \equiv \left(a^\dagger + a\right) \quad \text{and} \quad \hat{P} \equiv i\left(a^\dagger - a\right) \quad \Rightarrow \left[\hat{Q}, \hat{P}\right] = 2i$$

Note that: $\quad H = \frac{\hbar\omega}{2}\left(\hat{Q}^2 + \hat{P}^2\right)$

Note that some texts define Q and P with a prefactor of ½.

From the Heisenberg uncertainty ideas applied to the standard deviations:

$$\Delta\hat{Q}\Delta\hat{P} \geq 1$$

Also note that $\langle n|\hat{Q}|n\rangle = 0 = \langle n|\hat{P}|n\rangle$

For the coherent state:

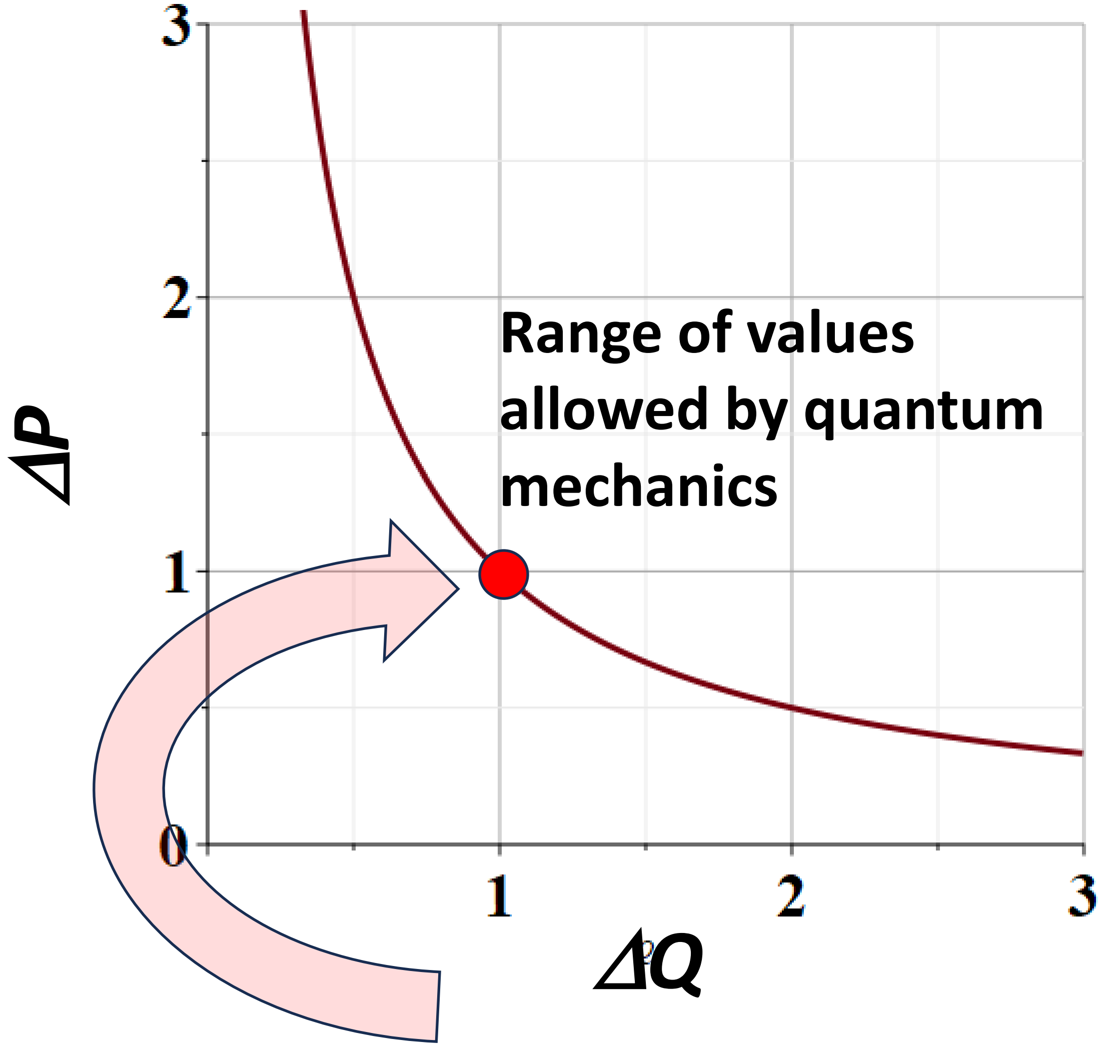
$$\left|\lambda\right\rangle = e^{-|\lambda|^2} \sum_{n=0}^{\infty} \frac{\lambda^n}{\sqrt{n!}} \left|n\right\rangle$$

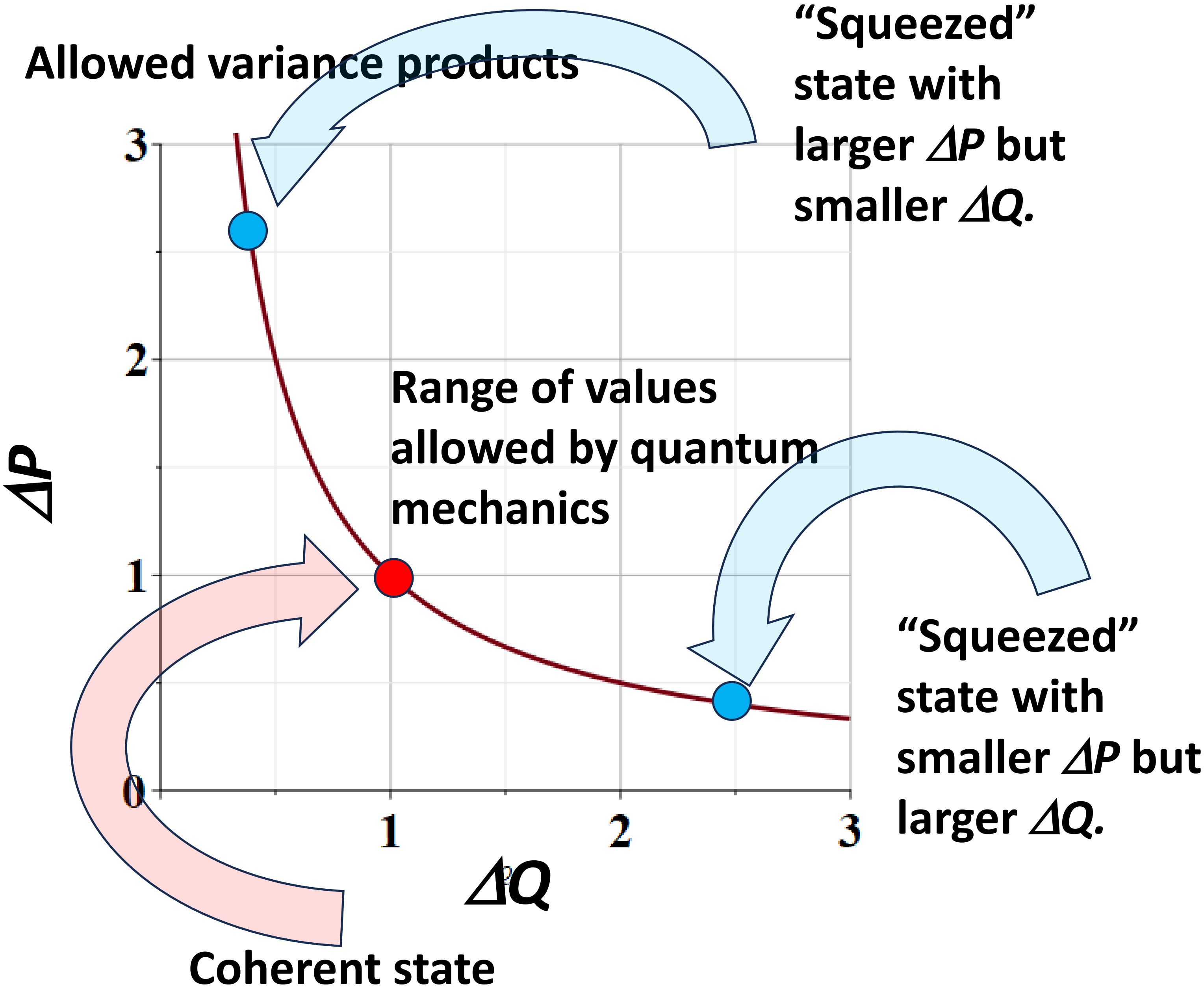
$$\Delta\hat{Q}_\lambda = \sqrt{\left\langle\lambda\left|\hat{Q}^2\right|\lambda\right\rangle - \left|\left\langle\lambda\left|\hat{Q}\right|\lambda\right\rangle\right|^2} = 1 = \Delta\hat{P}_\lambda$$

$$\Rightarrow \Delta\hat{Q}_\lambda \Delta\hat{P}_\lambda = 1$$

In this sense, the coherent state represents the minimum uncertainty process.

Allowed variance products




Allowed variance products
"Squeezed" state with larger ΔP but smaller ΔQ.
Range of values allowed by quantum mechanics
"Squeezed" state with smaller ΔP but larger ΔQ.
Coherent state
ΔP
ΔQ
0
1
2
3
1
2
3

In terms of the eigenstates of the EM Hamiltonian:

$$H_{rad}\left|n\right\rangle = \hbar\omega\left(n+\frac{1}{2}\right)\left|n\right\rangle$$

$$\Delta\hat{Q}_n = \sqrt{\left\langle n\left|\hat{Q}^2\right|n\right\rangle - \left|\left\langle n\left|\hat{Q}\right|n\right\rangle\right|^2} = \sqrt{2n+1} = \Delta\hat{P}_n$$

$$\Rightarrow \Delta\hat{Q}_n\Delta\hat{P}_n = 2n+1 \geq 1$$

In terms of coherent states: --

For the coherent state:

$$\left|\lambda\right\rangle = e^{-|\lambda|^2} \sum_{n=0}^{\infty} \frac{\lambda^n}{\sqrt{n!}} \left|n\right\rangle$$

$$\Delta\hat{Q}_\lambda = \sqrt{\left\langle\lambda\left|\hat{Q}^2\right|\lambda\right\rangle - \left|\left\langle\lambda\left|\hat{Q}\right|\lambda\right\rangle\right|^2} = 1 = \Delta\hat{P}_\lambda$$

$$\Rightarrow \Delta\hat{Q}_\lambda \Delta\hat{P}_\lambda = 1$$

In this sense, the coherent state represents the minimum uncertainty process.

How can we transform the quadrature functions to reduce the variances of ΔQ or ΔP?

Following Mandel and Wolf, we introduce the squeeze operator

$$\hat{S}(z) \equiv \exp\left(\frac{1}{2}\left(z^* \hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right)$$

$$= 1 + \frac{1}{2}\left(z^* \hat{a}^2 - z\hat{a}^{\dagger 2}\right) + \frac{1}{2}\left(\frac{1}{2}\left(z^* \hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right)^2 + \frac{1}{3!}\left(\frac{1}{2}\left(z^* \hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right)^3 \ldots\ldots$$

Note that $\hat{S}(z)$ is a unitary operator $\quad \hat{S}(z)\left(\hat{S}(z)\right)^\dagger = 1$

Let $z = re^{i\theta}$

Squeeze operator with $z = re^{i\theta}$

$$\hat{S}(z) \equiv \exp\left(\frac{1}{2}\left(z^*\hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right) \qquad z = re^{i\theta}$$

$$\hat{A}(z) \equiv \hat{S}(z)\hat{a}\left(\hat{S}(z)\right)^\dagger \qquad \text{and} \qquad \hat{A}^\dagger(z) \equiv \hat{S}(z)\hat{a}^\dagger\left(\hat{S}(z)\right)^\dagger$$

$$\hat{A}(z) = \hat{a} + z\hat{a}^\dagger + \frac{|z|^2\,\hat{a}}{2!} + \frac{z|z|^2\,\hat{a}^\dagger}{3!} + ... \qquad \text{(not totally trivial...)}$$

$$\Rightarrow \hat{A}(z) = \hat{a}\,\cosh r + \hat{a}^\dagger e^{i\theta}\sinh r$$

$$\hat{A}^\dagger(z) = \hat{a}\;e^{-i\theta}\sinh r + \hat{a}^\dagger\cosh r$$

Inverting these relations --

$$\hat{a} = \hat{A}(z)\cosh r - \hat{A}^\dagger(z)e^{i\theta}\sinh r$$

$$\hat{a}^\dagger = \hat{A}^\dagger(z)\cosh r - \hat{A}(z)e^{-i\theta}\sinh r$$

Now recall the "Quadrature operators"

$\hat{Q} \equiv \left(a^\dagger + a \right)$ and $\hat{P} \equiv i\left(a^\dagger - a \right)$ $\Rightarrow \left[\hat{Q}, \hat{P} \right] = 2i$

From the Heisenberg uncertainty ideas -- $\Delta\hat{Q}\Delta\hat{P} \geq 1$

More generally, we can use the altered operators --

$\hat{Q}_\beta \equiv \left(a^\dagger e^{i\beta} + ae^{-i\beta} \right)$ and $\hat{P}_\beta \equiv i\left(a^\dagger e^{i\beta} - ae^{-i\beta} \right)$

Note that $[\hat{Q}_\beta,\ \hat{P}_\beta] = 2i$ which implies $\Delta\hat{Q}_\beta \Delta\hat{P}_\beta \geq 1$

We are seeking a "squeezed" states for which $\Delta\hat{Q}_\beta < 1$

Consider a "squeezed" coherent state: $\left| z, \lambda \right\rangle \equiv S(z)\left| \lambda \right\rangle$

Evaluating the variance $\Delta\hat{Q}_\beta$ for this squeezed coherent state --

Evaluating the variance --

$$\left\langle z,\lambda\left|\hat{Q}_\beta\right|z,\lambda\right\rangle=\left\langle z,\lambda\left|\hat{a}^\dagger e^{i\beta}+\hat{a}e^{-i\beta}\right|z,\lambda\right\rangle$$

$$\hat{a}=\hat{A}(z)\cosh r-\hat{A}^\dagger(z)e^{i\theta}\sinh r$$

$$\hat{a}^\dagger=\hat{A}^\dagger(z)\cosh r-\hat{A}(z)e^{-i\theta}\sinh r$$

When the dust clears -- (Details in Mandel and Wolf and other references)

$$\left\langle z,\lambda\left|\hat{Q}_\beta\right|z,\lambda\right\rangle=\left(\lambda^*\cosh r-\lambda e^{-i\theta}\sinh r\right)e^{i\beta}+\left(\lambda\cosh r-\lambda^* e^{i\theta}\sinh r\right)e^{-i\beta}$$

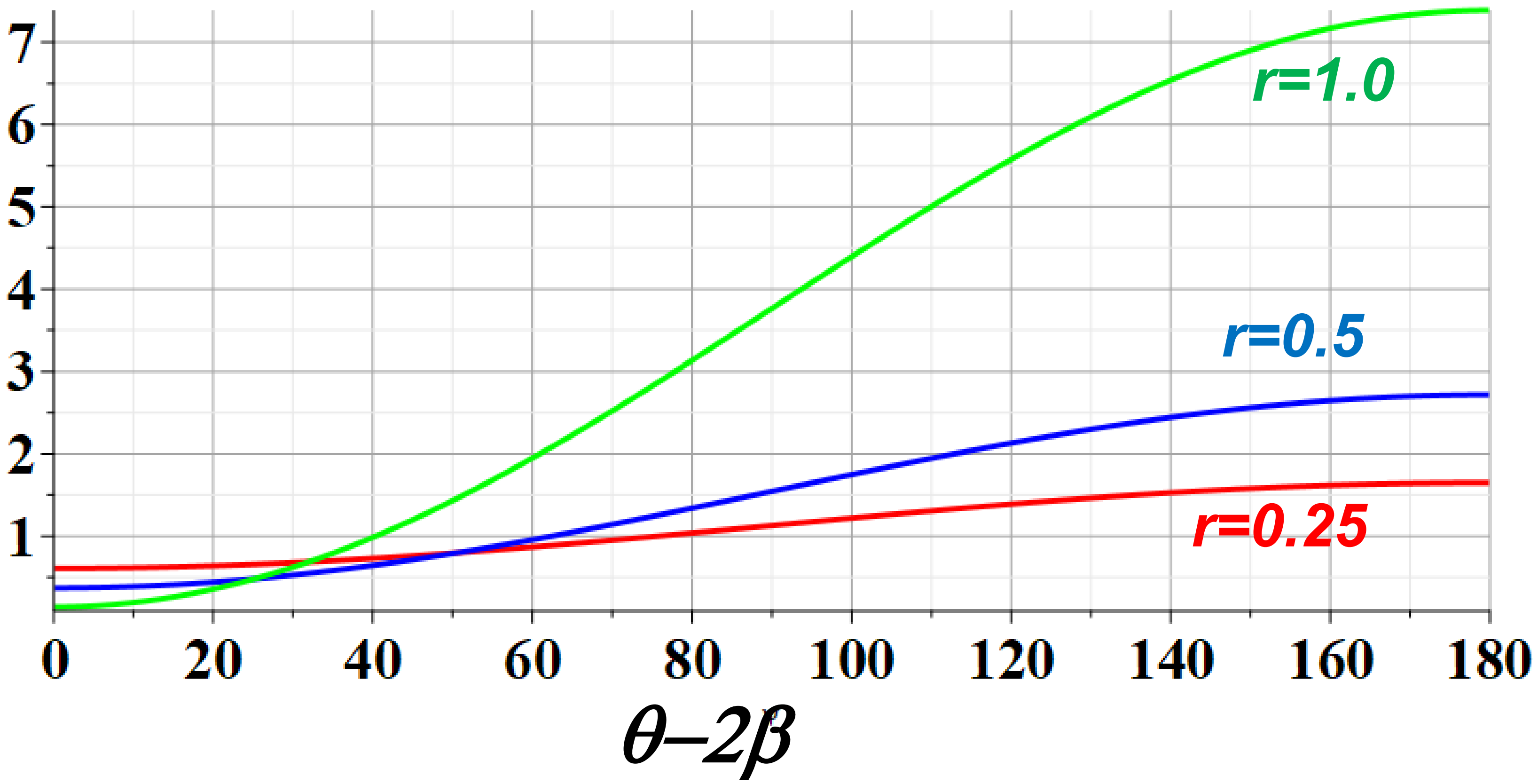
After more dust --

$$\left\langle z,\lambda\left|\left(\Delta\hat{Q}_\beta\right)^2\right|z,\lambda\right\rangle=\left\langle z,\lambda\left|\left(\hat{Q}_\beta\right)^2\right|z,\lambda\right\rangle-\left|\left\langle z,\lambda\left|\hat{Q}_\beta\right|z,\lambda\right\rangle\right|^2$$

$$=\cosh(2r)-\sinh(2r)\cos\left(\theta-2\beta\right)$$

$$\langle z,\lambda|\left(\Delta\hat{Q}_\beta\right)^2|z,\lambda\rangle = \langle z,\lambda|\left(\hat{Q}_\beta\right)^2|z,\lambda\rangle - \left|\langle z,\lambda|\hat{Q}_\beta|z,\lambda\rangle\right|^2$$

$$= \cosh(2r) - \sinh(2r)\cos\left(\theta - 2\beta\right)$$



Searching for the best squeeze parameters

$$\left\langle z,\lambda\left|\left(\Delta\hat{Q}_\beta\right)^2\right|z,\lambda\right\rangle = \left\langle z,\lambda\left|\left(\hat{Q}_\beta\right)^2\right|z,\lambda\right\rangle - \left|\left\langle z,\lambda\left|\hat{Q}_\beta\right|z,\lambda\right\rangle\right|^2$$

$$= \cosh(2r) - \sinh(2r)\cos\left(\theta - 2\beta\right)$$

For each r, the smallest result is obtained when $\beta=\theta/2$

$$\left\langle z,\lambda\left|\left(\Delta\hat{Q}_\beta\right)^2\right|z,\lambda\right\rangle = \cosh(2r) - \sinh(2r) = \exp(-2r) \leq 1$$

It can also be shown that for the same choice of parameters

$$\left\langle z,\lambda\left|\left(\Delta\hat{P}_\beta\right)^2\right|z,\lambda\right\rangle = \cosh(2r) + \sinh(2r) = \exp(2r) \geq 1$$

➔Despite the constraints of the uncertainty principle, it is possible to improve the measurement of one of the two non-commuting processes.

Experimental evidence

VOLUME 57, NUMBER 20 PHYSICAL REVIEW LETTERS 17 NOVEMBER 1986

Generation of Squeezed States by Parametric Down Conversion

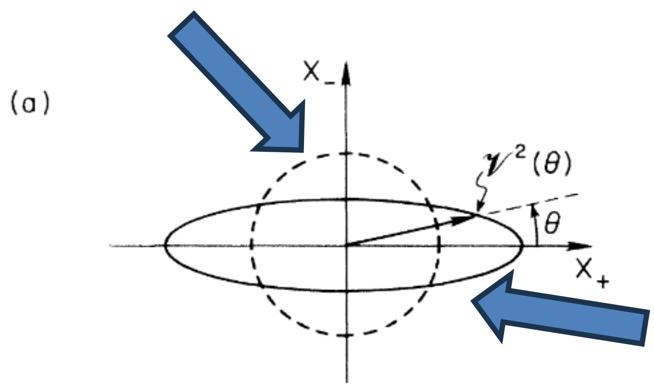
Ling-An Wu, H. J. Kimble, J. L. Hall,[a] and Huifa Wu

Department of Physics, University of Texas at Austin, Austin, Texas 78712

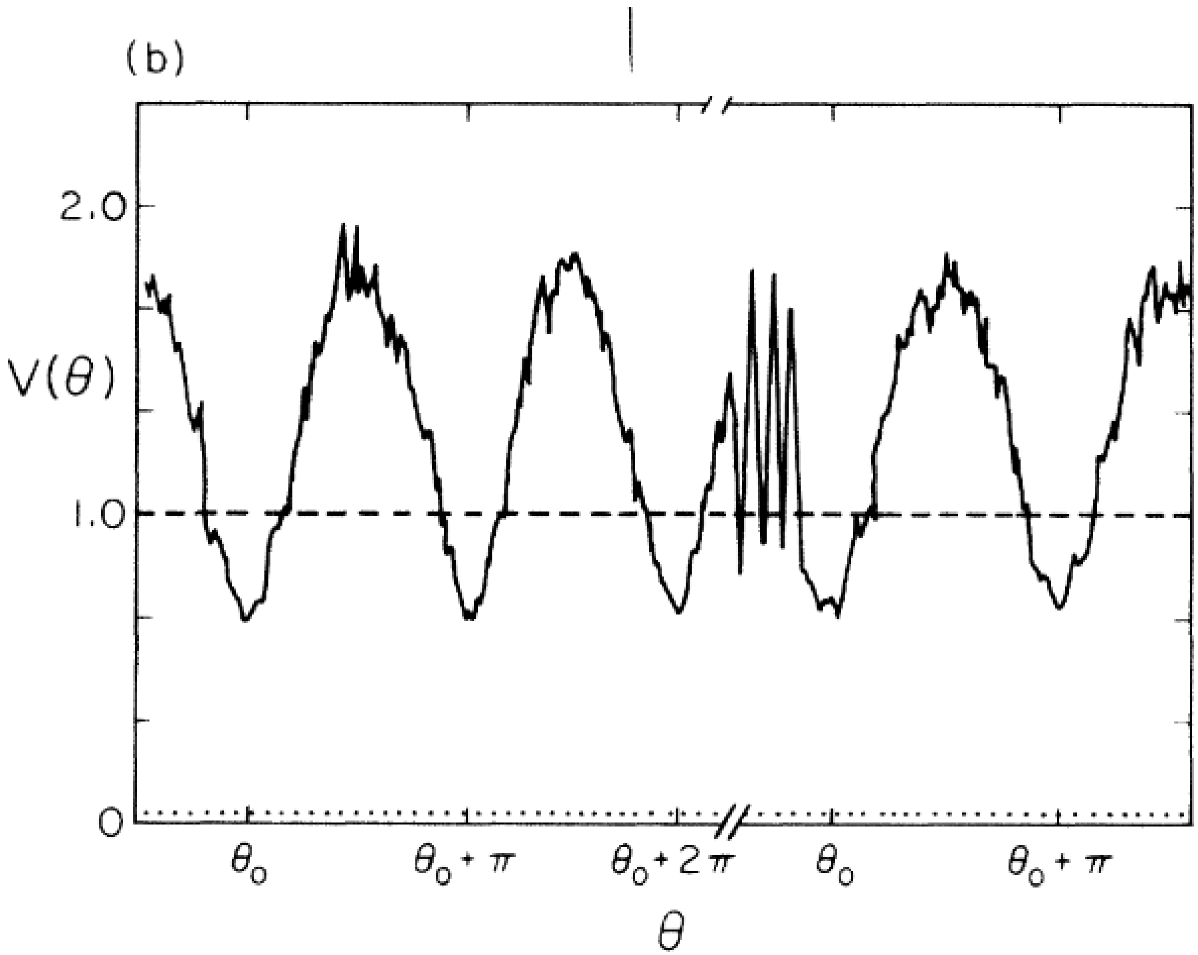
(Received 11 September 1986)

Squeezed states of the electromagnetic field are generated by degenerate parametric down conversion in an optical cavity. Noise reductions greater than 50% relative to the vacuum noise level are observed in a balanced homodyne detector. A quantitative comparison with theory suggests that the observed squeezing results from a field that in the absence of linear attenuation would be squeezed by greater then tenfold.

Coherent state



Squeezed state


(b)
2.0
V(θ)
1.0
0
θ₀
θ₀ + π
θ₀ + 2π
θ₀
θ₀ + π
θ

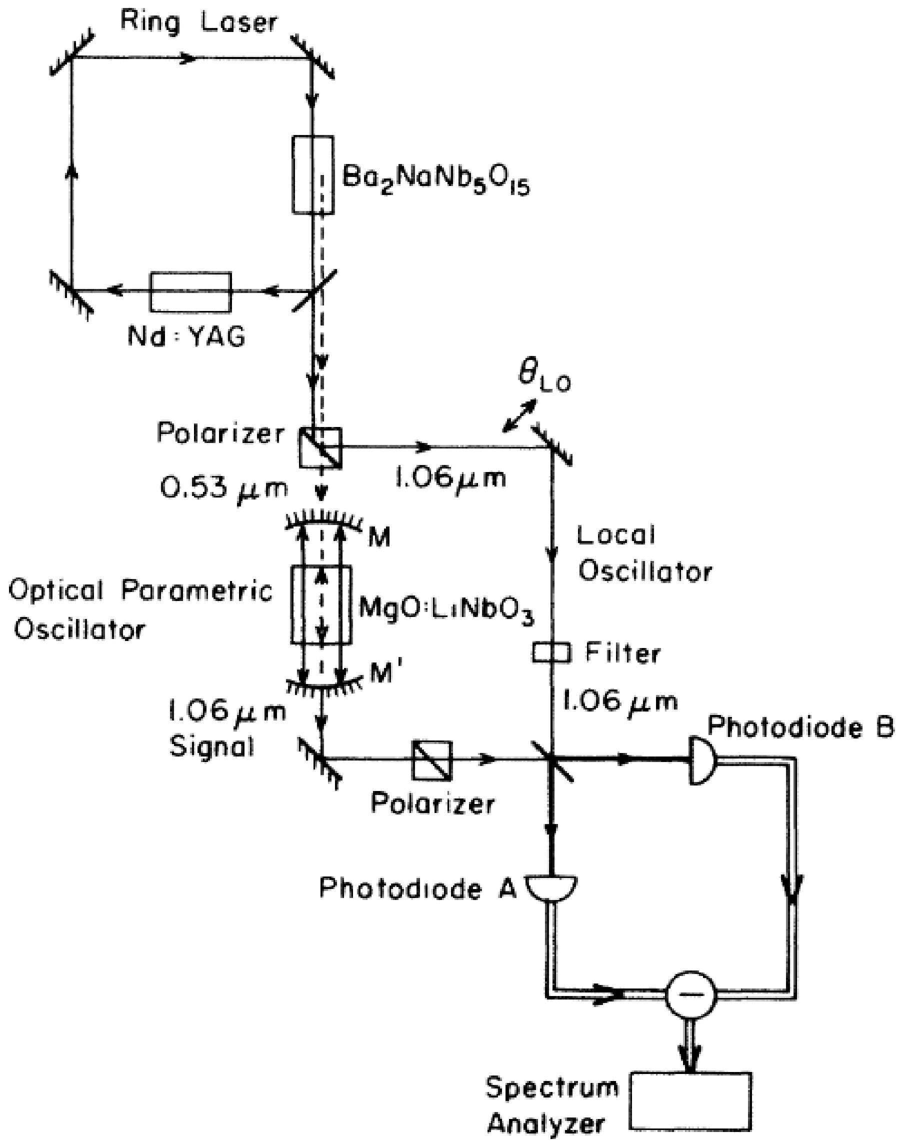


FIG. 2. Diagram of the principal elements of the apparatus for squeezed-state generation by degenerate parametric down conversion.