



PHY 712 Electrodynamics

10-10:50 AM MWF in Olin 103

Class notes for Lecture 6:

Reading: Chapter 1 - 3 in JDJ

Introduction to numerical methods

- 1. Finite difference**
- 2. Finite element**

Course schedule for Spring 2024

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Wed: 01/17/2024	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/19/2024
2	Fri: 01/19/2024	Chap. 1	Electrostatic energy calculations	#2	01/29/2024
3	Mon: 01/22/2024	Chap. 1	Electrostatic energy calculations	#3	01/29/2024
4	Wed: 01/24/2024	Chap. 1 & 2	Electrostatic potentials and fields	#4	01/29/2024
5	Fri: 01/26/2024	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions	#5	01/29/2024
6	Mon: 01/29/2024	Chap. 1 - 3	Brief introduction to numerical methods	#6	02/05/2024
7	Wed: 01/31/2024	Chap. 2 & 3	Image charge constructions		

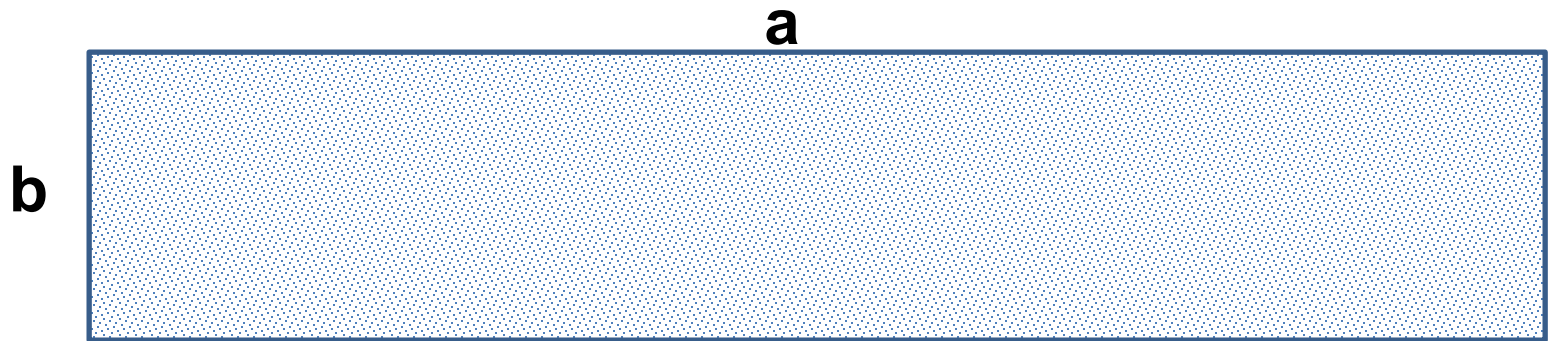
PHY 712 -- Assignment #6

Assigned: 1/29/2024 Due: 2/05/2024

Continue reading Chaps. 1-3 in **Jackson**.

1. Consider a square of length a which has a uniform charge density ρ_0 . In Lecture 5 we found an "exact" expression for the electrostatic potential of this system $\Phi(x,y)$ for the case that the potential vanishes on the 4 boundary lines $\Phi(x,0)=\Phi(x,a)=\Phi(0,y)=\Phi(a,y)$.
 - a. For the $5 \times 5 \times 5$ grid discussed in Lecture 6 with $h=a/4$, evaluate the unique values of $\Phi(x,y)$ on the grid points by summing the series in the exact expressions.
 - b. Using the finite difference method, find the approximate unique values of $\Phi(x,y)$ on the grid points and compare with the exact values.
 - c. Using the finite element method, find the approximate unique values of $\Phi(x,y)$ on the grid points and compare with the exact values.

Recall example from Lecture 5:



$$\rho(x, y) = \rho_0 \quad \text{for } 0 \leq x \leq a \text{ and } 0 \leq y \leq b$$

With boundary values $\Phi(0, y) = 0, \Phi(a, y) = 0, \Phi(x, 0) = 0, \Phi(x, b) = 0$

Solution: $\Phi(x, y) =$

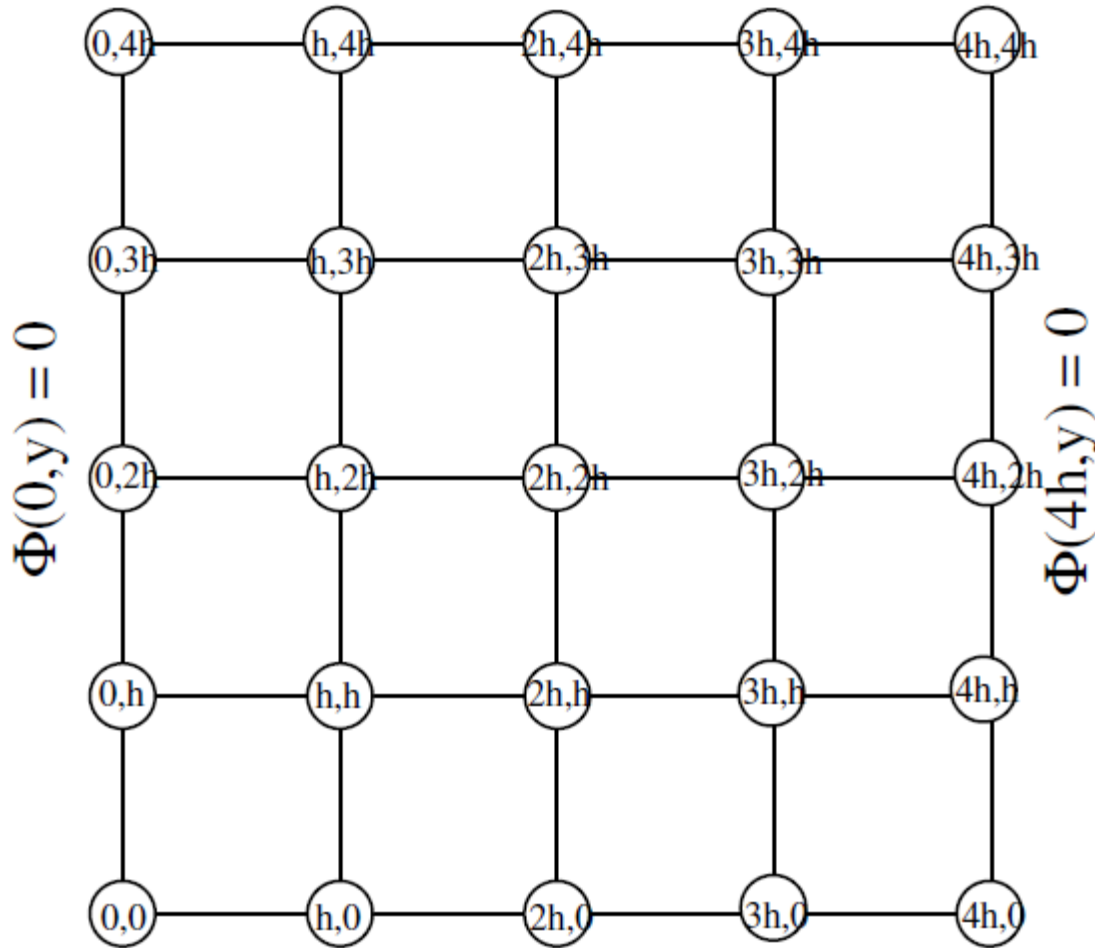
$$\frac{16\rho_0 a}{4\pi^2 \epsilon_0} \sum_{n \text{ (odd)}} \frac{\sin\left(\frac{n\pi x}{a}\right)}{n^2 \sinh\left(\frac{n\pi b}{a}\right)} \left(\sinh\left(\frac{n\pi(b-y)}{a}\right) \int_0^y dy' \sinh\left(\frac{n\pi y'}{a}\right) + \sinh\left(\frac{n\pi y}{a}\right) \int_y^b dy' \sinh\left(\frac{n\pi(b-y')}{a}\right) \right)$$

$$= \frac{16\rho_0 a^2}{4\pi^3 \epsilon_0} \sum_{n \text{ (odd)}} \frac{\sin\left(\frac{n\pi x}{a}\right)}{n^3 \sinh\left(\frac{n\pi b}{a}\right)} \left(\sinh\left(\frac{n\pi b}{a}\right) - \sinh\left(\frac{n\pi y}{a}\right) - \sinh\left(\frac{n\pi(b-y)}{a}\right) \right)$$

=

Some details for fine grid --

$$\Phi(x, 4h) = 0$$



9 interior grid points

→ reduced to 6 evaluation points using symmetry

$$\Phi(x, 0) = 0$$