

Math 732: Knot Theory
Asst. 1, due F, 1/21

Problems to think about, but not submit

1. Adams 1.5, 1.6, 1.7, 1.8, 1.9
2. Adams 1.33 (forming a knot with your arms, body, and yardstick)
3. Adams 1.36
4. Cromwell, 1.8 (p. 30) – let me know if you find knots where you cannot achieve the stick number with the straws.

Problems to submit

You must submit 3 of the following; clearly indicate which 3 you want me to grade. You are welcome to submit any others that you want me to provide feedback on. Everyone must do the first problem.

1. (required) For two knots J and K , show $s(J\#K) \leq s(J) + s(K) - 1$.
2. Prove that there are no nontrivial knots with 2 crossings.
(This is more complicated than it seems; start by drawing 2 pairs of crossings, without the arcs connected. Then connect each of the eight endpoints by an arc to another of the eight endpoints – forming 2 pairs. Show that no matter how you do this, you obtain either an unknot or a link.)
3. Cromwell, 1.3 (p. 30)
4. Cromwell, 1.6 (p. 30)
5. ★ Adams 1.35
6. ★ Adams 1.38 (relating stick number and crossing number)
7. Explore the stick number for composite knots. We proved a theorem in class on 1/18 about $s(J\#K) \leq s(J) + s(K) - 2$. Find a stick diagram with $s(J) + s(K) - 2$ sticks for composite knots $3_1\#4_1$ and $4_1\#4_1$ (or two others of your choice). Can you find one with fewer sticks? (Ensure that your diagrams are physically attainable.)