Math 732: Knot Theory
Asst. 5, due F., 2/18

Problems to submit

You must submit 5 of the following; clearly indicate which ones you want me to grade. You are welcome to submit any others that you want me to provide feedback on.

1. (required) Show that the unknotting number $u(K) \leq \frac{1}{2}Cr(K)$.

2. Adams 1.14 (show a sequence of isotopies)

3. Adams 3.1 – find the unknotting number of the Figure-8 knot. (Prove that this works by giving a projection with the changed crossing indicated; then show a sequence of isotopies that transform the resulting diagram into the unknot.)

4. (required) Find the unknotting number for each of your knots. (same instructions as above)
   If $u(K) > 1$, can you argue why it cannot be 1? (proving this can be difficult)
   One of the 7-crossing knots has a difficult argument as to why $u(K) = 2$; another 7-crossing knot has $u(K) = 3$; all others are either 1 or 2.

5. Show that if M^3 is an irreducible manifold (not homeomorphic to S^3), then it is prime.

6. Show that $S^2 \times S^1$ is not irreducible. (It is the only example of a prime 3-manifold which is not irreducible; you do not have to show it is prime.)

7. Prove Fary's Theorem; it is sketched in Cromwell, section 2.11 (unless you did this on hw4)

8. (required) Can two knots in $\mathbb{R}^3$ be transversal? If so, describe their intersection; if not, explain why.