Math 732: Knot Theory
Asst. 6, due Th., 2/24

Problems to think about, but not submit

- Show that each of your knots has an alternating reduced diagram with the expected number n of crossings. By the theorem of Kauffman, Murasagi, & Thistlethwaite, this proves that $Cr = n$.
- Show every link has some diagram with writhe $w(D) = 0$.

Problems to submit

You must submit 4 of the following; clearly indicate which ones you want me to grade. You are welcome to submit any others that you want me to provide feedback on.

1. (required) Calculate the writhe for each of your knots (i.e., use an alternating reduced diagram). Draw a diagram with $w = 0$ for each one.

2. (required) (a) Show that for a knot diagram D, mirroring reverses the sign of its writhe: $w(D^*) = -w(D)$. Explain why this implies for alternating knots, $w(K^*) = -w(K)$.
(b) Show $w(-D) = w(D)$.
(c) Using parts a,b, conclude by stating a necessary condition using writhe for an alternating knot to be amphichiral (i.e., having full or $\pm$ amphichiral symmetry).

3. Find an alternating knot with $w(K) = 0$ that is chiral. (I believe there are examples but haven't looked for one.)

4. Show that if a curve γ is planar, then its geometric writhe $Wr(\gamma) = 0$.

5. Either find the first non-alternating torus knot (i.e., the one with the lowest crossing number), or argue why all torus knots are alternating.

6. Argue precisely why there are only a finite number of links with crossing number n.