

# History of Knot Tabulation

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# Brief History of (Prime) Knot Tabulation

Gauss knew and thought about knots – 1833 integral formula for *linking number*. Before him, Vandermonde (1771) wrote a seminal paper on topology & discussed knots.

## Atomic model [Kelvin, late 1800's]

Atoms are knotted vortices in the *ether*.

This theory, albeit vastly incorrect, led to the first serious work in knot theory.

- Tait (1876), a colleague of Kelvin – knots to 7 crossings
- Kirkman (1885, British) – knot projections
- Little (1885, Nebraska) – knots to 10 crossings
- by 1900, Tait, Kirkman, Little had produced all  $\leq 10$  crossing knots and all 11 crossing alternating knots

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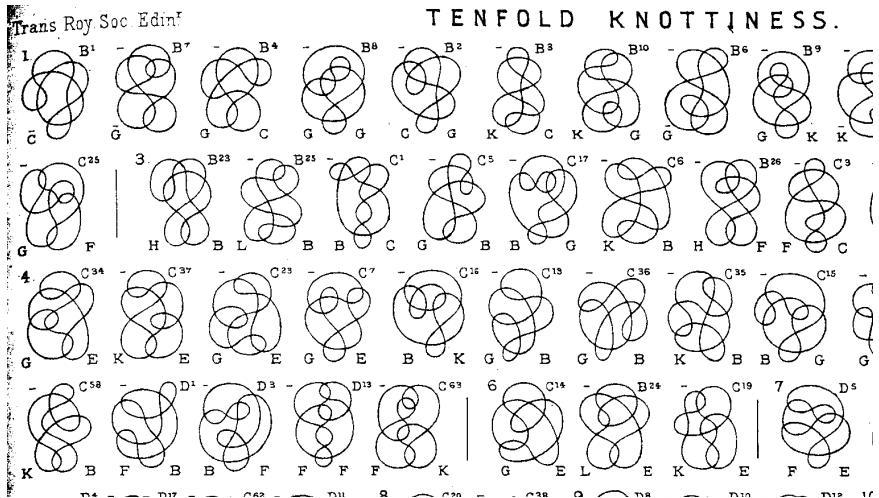
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## Tait (1898): Knots to 10 crossings



# Showing knots are distinct

## Issue

These knots were not known to be distinct.

- Brunn (1892) – linking number from a diagram
- Wirtinger (1905) – presentation of a knot group  $\pi_1(S^3 \setminus K)$
- Tietze (1908) – nontrivial knots exist
- Dehn (1914) – trefoil is chiral
- Alexander-Briggs (1927) – all 9 crossing knots distinct, except 3 pairs
- Reidemeister (1932) – these 3 pairs

Many of these included significant advances in algebraic topology. e.g., Alexander polynomial

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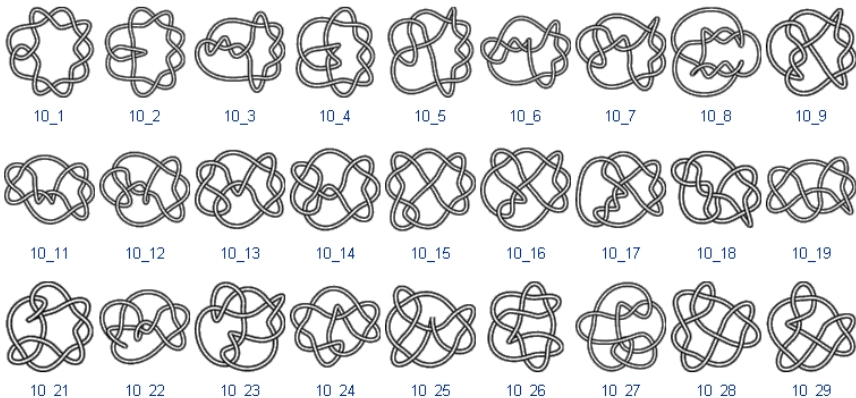
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# Brief History of Knot Tabulation II

Rolfsen (1976): Publishes standard textbook, knots to 10 crossings. 1 error. Below: 27 of the **165** 10-crossing knots



# Brief History of Knot Tabulation III

- 1 Conway (1964)  
Knots to 11 crossings, links to 10 crossings; errors.
- 2 Rolfsen (1976)      Knots to 10 crossings. 1 error.
- 3 Caudron (1978) – knots to 11 crossings correctly.
- 4 Doll/Hoste (1991)      Oriented links to 10 crossings.
- 5 Cerf (1998) Oriented alt. links to 10 crossings.
- 6 Hoste/Thistlethwaite/Weeks (1998)  
1,701,936 knots to 16 crossings; determined chirality
- 7 Flint/Rankin (2007)  
98,517,495,461 *alternating* links to 23 crossings.

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