Imaging Through Atmospheric Turbulence in Remote Sensing
PSF Estimation and Deblurring for Hyperspectral Imaging

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WFU
Sanya China, Jan. 2015

Starry Night - Vincent van Gogh

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Outline

- Overview of Optical Imaging in Remote Sensing
- Our Recent Projects
- (Imaging Through) Turbulence: da Vinci, Galileo, Komolgorov, Von Karman
- Hyperspectral Imaging (HSI)
- Estimating HSI PSFs for Atmospheric Turbulence
- Joint Sparse Deblurring and Feature Extraction
- **If time:** Compressive Snapshot Spectral - Polarimetric Imaging
References Related to Presentation


First 3 available at: http://users.wfu.edu/plemmons/
2015 has been proclaimed the International Year of Light and Light-Related Technologies by UNESCO.

Optical sensors form images of objects or scenes by detecting their solar or laser reflectance.

- **Hyperspectral** (2D spatial & 1D wavelength) and **LiDAR** (2D spatial & 1D range) Imaging
  - Hyperspectral imaging (HSI) collects information across the visible and IR spectrum for interrogating scenes.
  - LiDAR measures distance by illuminating targets with a scanning laser and analyzing the reflected light. Laser wavelengths vary to suit the target: $10 \, \mu m$ to UV. Very short.
  - Fused HSI and LiDAR for enhanced analysis. Combined laser ranging & object material identification.

- **Spectro-Polarimetric imaging** (3D hyperspectral & 1D polarization). Data is a 4D tensor. Polarization identifies object shape, metallic surfaces.
NGA Project

- National Geospatial-Intelligence Agency (NGA) Funded Project “Compressive Sensing and Fusion of LiDAR and Hyperspectral Data using Tensor Factorizations”. Boeing and Wake Forest 2011-2013

Randomized matrix and tensor factorization for HSI compression. Low-rank matrix approximations, e.g., approx. tSVD of HSI tensor, then fusion of components with LiDAR data.

Clustering and classification of fused data, and information-theoretic results, detecting objects in shadows, etc.

New project: Information-theoretic Fusion and Analysis of HSI, LiDAR and Polarimetric Data.
AFOSR Project

- **Air Force (AFOSR) Funded Project**

**Space Object Identification with Spectral Imagers**

Analysis of reflectance.

Hubble Space Telescope

**Figure:** Reflectance of an object pixel results from additive reflectance of its constitutive elements.
Image contains sketch by Leonardo da Vinci, along with a remarkably modern description: “the smallest eddies are almost numberless, and large things are rotated only by large eddies and not by small ones.” Called phenomena “turbolenza”, leading to modern word turbulence.
Later, Galileo, 1564-1642, knew effects of atmospheric turbulence on telescopes. Modern mathematical models developed by:

(a) Th. von Karman, 1881-1963
(b) Andrey Kolmogorov, 1903-1987
Earth’s atmosphere is turbulent and variations in the index of refraction cause the plane optical wavefront from distant objects to be distorted. Caused by variation of air temperature in eddies and air currents. Arriving light wavefront is crumpled.

In astronomy, or space situational awareness, resolution of all ground-based telescopes is severely limited by effects of atmospheric turbulence. At best sites, the resolution of a 2.5m telescope is degraded by at least a factor of five.

Can cause blurring in ground-level (horizontal) imaging. For horizontal image beam paths, ground based turbulence typically has a highly non-Kolmogorov power spectral density (PSD), and both phase and amplitude perturbations (scintillations) must be included in PSF model.
Imaging Through Atmospheric Turbulence - Estimating a PSF for Deconvolution

- Ground-level turbulence - video imaging illustration 1km distance.

(a) Image taken in early morning  
(b) Afternoon image through turbulence.

Active research topic: e.g., Zhu and Milanfar, 2012-13. Lou, Kang, Soatto, Bertozzi, 2013. Others ...
Methods generally based on selecting “lucky frames”, patches, image registration, fusion, segmentation etc.
• Astronomical imaging, looking up

AMOS sensors can collect simultaneously from visible to LWIR.

Methods based on wavefront sensing (Shack-Hartman), gradient & phase estimation leading to a PSF, or turbulence modeling.
• Spectral imagers capture a 3D datacube (tensor) containing:
  - 2D spatial information: x-y
  - 1D spectral information at each spatial location: $\lambda$

• Pixel intensity varies with wavelength bands - provides a spectral trace of intensity values.

• Generates a spatial map of spectral variation.

• Challenging to remove atmospheric turbulence blur from HSI.
Figure 2: $\lambda$ generally ranges between 400 and 2500 nanometers.
Structure of HSI Data

[Diagram showing the structure of HSI data with spatial dimensions (x, y) and spectral dimension (z), with a spectral signature of a pixel.]
Extract Spectral Signatures (Traces) from HSI to Identify Materials

Figure 3: Illustration High Resolution - Each pixel represents a vector
Some Applications of HSI

- Detect and identify militarily important objects at a distance.

http://www.globalsecurity.org/intell/library/imint/hyper.htm
Some Applications of HSI

- Identification of plant species.
Some Applications of HSI

- Detect and identify militarily important objects at a distance.
- Identification of plant species.
- Food processing.
- Mineral resource assessments.
- Medicine.
- Enable space object identification & analysis from the ground.

http://photonics.com/Article.aspx?AID=51023
Some Applications of HSI

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www.usgs.gov/blogs/features/usgs_science_pick/hyperspectral-hypercoverage/
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Enable space object identification & analysis from the ground.

German physicist Ernst Abbe realized the resolution of an imaging instrument is constrained by the wavelength of light used, and the aperture of its optics.

Resolution (detail an image holds) of a given telescope is proportional to the size of its aperture, and inversely proportional to the wavelength of the light being observed: \( Res \approx \frac{d}{\lambda} \). - opposite to the effect of atmospheric turbulence. Resolution also affected by blur & noise.

Our purpose: estimate wavelength dependent PSFs for HSI.
Hyperspectral Imaging (HSI) - Atmospheric PSFs

The image acquired at wavelength $\lambda$ can be represented as

$$g_{\lambda}(x, y) = h_{\lambda}(x, y; \phi) \ast f(x, y, \lambda) + \epsilon_{\lambda}(x, y),$$

where the blurring kernel, with diffractive scaling included, is

$$h_{\lambda}(x, y; \phi) = \left(\frac{\lambda_0}{\lambda}\right)^2 h_0 \left(\frac{\lambda_0}{\lambda} x, \frac{\lambda_0}{\lambda} y; \phi\right),$$

with $\lambda_0 = \text{reference baseline wavelength}$, and

$$h_0(x, y; \phi) = \left|\mathcal{F}^{-1}\left(pe^{i\phi}\right)\right|^2,$$

and where the wavefront phase $\phi = \frac{2\pi}{\lambda} \times \text{OPD}$.

• **OPD** is the optical path difference function, i.e. the optical phase shift in passage through turbulent atmosphere. **OPD** is essentially the same for each HSI wavelength, but the wavefront phase is not.
Tracking the PSFs as a Function of Wavelength

- Model phase function using the von Karman phase spectrum, 
  \[ P(\kappa_x, \kappa_y) = \sqrt{0.023} \left( \frac{D}{r_0} \right)^{5/3} \left( \kappa_0^2 + \kappa_x^2 + \kappa_y^2 \right)^{-11/6}, \]
  where \( r_0 \): Fried parameter, \( D \): telescope aperture diameter, \( \kappa_0 \): low-freq cutoff.

- \( \phi = \sqrt{P(\kappa_x, \kappa_y)} \cdot W(\kappa_x, \kappa_y) \), where \( W \) is a zero-mean, unit-variance, white Gaussian noise array. Set \( D/r_0 = 10 \).

- PSF at wavelength \( \lambda \) given by:
  \[ h_\lambda = \left| \mathcal{F}^{-1} \left( p e^{i \phi_\lambda} \right) \right|^2. \]
Some Sources of HSI Blur in Imaging Through the Atmosphere

- **Telescope optics.** Telescope PSF does not vary with position in field of view, but **varies with wavelength.** This is imaging camera system **diffraction blur** - worse at longer wavelengths.

- **Atmospheric turbulence (AT) effects.** AT effects are less at longer wavelengths. If adaptive optics is used, the residual PSF can vary with the spatial position. **Limits use of a PSF obtained from wavefront sensor gradient measurements**, Jefferies, Nagy, et al., 2014.

- **We estimate PSFs by modeling with spatially varying elliptical Moffat functions and parameter identification by HSI of a “guide star” point source.**
  
  Original Physics paper:
  
Movies Illustrating PSF Change with Wavelength
Multi Unit Spectroscopic Explorer (MUSE)

- Second generation Very Large Telescope (VLT) panoramic integral-field spectrograph.
- Operates in the visible wavelength range with \( \approx 4,000 \) wavelengths.
- Assisted by a ground layer adaptive optics system using four laser guide stars.
- Primary application: the study of galaxy formation and evolution.
- Additional applications: monitoring of outer planets atmosphere, young stellar objects environment, supermassive black holes and active nuclei in nearby galaxies or massive spectroscopic survey of stellar fields.
The image formation process for an isolated star or point source

\[ g_\lambda = h_\lambda s_\lambda + \eta_\lambda \]

- \( g_\lambda \) is a vector representing the vectorized form of an observed, blurred, and noisy image of an isolated star corresponding to wavelength \( \lambda \).
The image formation process for an isolated star or point source

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- \( g_\lambda \) is a vector representing the vectorized form of an observed, blurred, and noisy image of an isolated star corresponding to wavelength \( \lambda \).
- \( h_\lambda \) is a vector representing the vectorized form of an exact original image of the isolated star corresponding to wavelength \( \lambda \).
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- \( h_\lambda \) is a vector representing the vectorized form of an exact original image of the isolated star corresponding to wavelength \( \lambda \).
- \( s_\lambda \) is a scalar representing the unknown intensity of the star spectrum at wavelength \( \lambda \).
PSF Star (point source) HSI Image Formation Model

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- \( s_\lambda \) is a scalar representing the unknown intensity of the star spectrum at wavelength \( \lambda \).

By assuming a parametrized formula for the PSF, the image formation model becomes

\[ g_\lambda = h(\phi_\lambda)s_\lambda + \eta_\lambda \]

- \( \phi_\lambda \) is a vector of unknown parameters corresponding to wavelength \( \lambda \).
The circular Moffat function is defined by a positive scale factor $\alpha$ and a shape parameter $\beta$. In this case, $\phi = \begin{bmatrix} \alpha & \beta \end{bmatrix}^T$, and the PSF has the form:

$$h(\phi)_{i_x,j_y} = h(\alpha, \beta)_{i_x,j_y} = \left(1 + \frac{i_x^2 + j_y^2}{\alpha^2}\right)^{-\beta}$$  \hspace{1cm} (1)
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$$h(\phi)_{i_x,j_y} = h(\alpha, \beta)_{i_x,j_y} = \left(1 + \frac{i_x^2 + j_y^2}{\alpha^2}\right)^{-\beta} \tag{1}$$

- The flux: $\int \int h(\alpha, \beta) \, dx \, dy = \frac{\pi \alpha^2}{\beta - 1} \Rightarrow 1 < \beta < \infty$
The circular Moffat function is defined by a positive scale factor \( \alpha \) and a shape parameter \( \beta \). In this case, \( \phi = [\alpha \ \beta]^T \), and the PSF has the form:

\[
h(\phi)_{ix,jy} = h(\alpha, \beta)_{ix,jy} = \left(1 + \frac{i_x^2 + j_y^2}{\alpha^2}\right)^{-\beta}
\]

1. The flux: \( \int \int h(\alpha, \beta) \, dx \, dy = \frac{\pi \alpha^2}{\beta - 1} \Rightarrow 1 < \beta < \infty \)

2. Multiply the PSF by the inverse of the flux to insure that \( \sum_{ix,jy} h(\phi)_{ix,jy} = 1 \)
Wavelength Varying Circular Moffat

Modeling the variation of the PSF with respect to $\lambda$: 

A linear variation of $\alpha(\lambda) = \alpha_0 + \alpha_1 \lambda$

A constant value for $\beta(\lambda) = \beta_0$

Using this model the parameter vector becomes $\phi = [\alpha_0 \alpha_1 \beta]^T$

and the normalized wavelength varying PSF takes the form:

$h(\phi)_{i \times j \times \lambda} = h(\alpha_0, \alpha_1, \beta)_{i \times j \times \lambda} - \frac{1}{\pi} (\alpha_0 + \alpha_1 \lambda)^2 (1 + i_2 x + j_2 y (\alpha_0 + \alpha_1 \lambda)^2) - \beta$

Simple model involving only 3 parameters.
Wavelength Varying Circular Moffat

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- A linear variation of $\alpha(\lambda) = \alpha_0 + \alpha_1 \lambda$
- A constant value for $\beta(\lambda) = \beta_0$

D. Serre, E. Villeneuve, H. Carfantan, L. Jolissaint, V. Mazet, S. Bourguignon, A. Jarno
Modeling the spatial PSF at the VLT focal plane for MUSE WFM data analysis purpose
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Using this model the parameter vector becomes $\phi = [\alpha_0 \quad \alpha_1 \quad \beta]^T$
and the normalized wavelength varying PSF takes the form:

$$h(\phi)_{i_x, j_y, \lambda} = h(\alpha_0, \alpha_1, \beta)_{i_x, j_y, \lambda} = \frac{\beta - 1}{\pi(\alpha_0 + \alpha_1 \lambda)} \left(1 + \frac{i_x^2 + j_y^2}{(\alpha_0 + \alpha_1 \lambda)^2}\right)^{-\beta}$$

- Simple model involving only 3 parameters.
Wavelength Varying Circular Moffats

λ = 450nm

λ = 550nm

λ = 650nm

λ = 750nm

λ = 850nm

λ = 950nm
For **spatially variant blurs**, we need to use an elliptical Moffat function

\[
h(\phi)_{i_x,j_y} = h(\alpha, \beta, \gamma, \Theta)_{i_x,j_y} = \left[1 + \frac{1}{\alpha^2} \left(i_r^2 + \frac{j_r^2}{\gamma^2}\right)\right]^{-\beta} \tag{2}
\]

- \(\gamma\) is the ellipticity parameter.
For **spatially variant blurs**, we need to use an elliptical Moffat function

\[
h(\phi)_{i_x,j_y} = h(\alpha, \beta, \gamma, \Theta)_{i_x,j_y} = \left[ 1 + \frac{1}{\alpha^2} \left( i_r^2 + \frac{j_r^2}{\gamma^2} \right) \right]^{-\beta}
\]

- \(\gamma\) is the ellipticity parameter.
- \(\Theta\) is the rotation angle.
For **spatially variant blurs**, we need to use an elliptical Moffat function

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h(\phi)_{i_x,j_y} = h(\alpha, \beta, \gamma, \Theta)_{i_x,j_y} = \left[ 1 + \frac{1}{\alpha^2} \left( i_r^2 + \frac{j_r^2}{\gamma^2} \right) \right]^{-\beta}
\] (2)

- \(\gamma\) is the ellipticity parameter.
- \(\Theta\) is the rotation angle.
- \[
\begin{bmatrix} i_r \\ j_r \end{bmatrix} = \begin{bmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{bmatrix} \begin{bmatrix} i_x \\ j_y \end{bmatrix}.
\]
The variation of the elliptical Moffat PSF with respect to $\lambda$ and the polar coordinates $(\rho, \theta)$ in the field of view:

- $\gamma(\lambda, \rho) = 1 + (\gamma_0 + \gamma_1 \lambda)\rho$
- $\beta$ is kept as a constant
- $\alpha(\lambda, \rho) = \alpha_0 + \alpha_1 \rho + \alpha_2 \lambda + \alpha_3 \lambda^2$
- $\Theta = \frac{\pi}{2} - \theta$

D. Serre, E. Villeneuve, H. Carfantan, L. Jolissaint, V. Mazet, S. Bourguignon, A. Jarno
Modeling the spatial PSF at the VLT focal plane for MUSE WFM data analysis purpose
Example: Elliptical Moffat - 7 Parameters for each \( \lambda \)

For \( \lambda = 465 \text{nm} \):

\[
\alpha_0 = 3.75, \quad \alpha_1 = -2.99 \cdot 10^{-3}, \quad \alpha_2 = -4.31 \cdot 10^{-3}, \quad \alpha_3 = 1.98 \cdot 10^{-6}
\]

\[
\beta = 1.74, \quad \gamma_0 = 6.86 \cdot 10^{-4}, \quad \gamma_1 = 2.17 \cdot 10^{-6}
\]
Elliptical Moffat: Field of View

We use 20 wavelengths with 25 orientations in the first quadrant of the field of view for each wavelength.
Elliptical Moffat: Field of View

We use 20 wavelengths with 25 orientations in the first quadrant of the field of view for each wavelength.
Optimization Problem (simplified)

The set of observed isolated star images at $N_w$ wavelengths can be written as:

$$g_{\lambda_i} = h_{\lambda_i} s_{\lambda_i} + \eta_{\lambda_i}, \quad i = 1, 2, \cdots, N_w$$

- $g_{\lambda_i} \in \mathbb{R}^{N_p}$, $h_{\lambda_i} \in \mathbb{R}^{N_p}$, $\eta_{\lambda_i} \in \mathbb{R}^{N_p}$.
- $s_{\lambda_i}$ is a scalar representing the reflectance of the star at a particular wavelength $\lambda_i$.

Since there is a one-to-one correspondence between $\lambda_i$ and the index $i$, without loss of generality, we will use the notation:

$$g_i = g_{\lambda_i}, \quad h_i = h_{\lambda_i}, \quad s_i = s_{\lambda_i}, \quad \eta_i = \eta_{\lambda_i}.$$
Optimization Problem

By stacking all observations, we obtain the overall image formation model as:

\[ g = H(\phi)s + \eta \]

where

\[ s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{N_w} \end{bmatrix} \]

and in the simpler case of a circular Moffat PSF,

\[
\begin{align*}
g &= \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{N_w} \end{bmatrix}, \\
H(\phi) &= \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ 0 & h_2 & & \\
& \vdots & & \\
0 & \cdots & \cdots & h_{N_w} \end{bmatrix}, \\
\phi &= \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \beta \end{bmatrix}.
\end{align*}
\]
Optimization Problem

In the case of elliptical Moffat PSF, with \( N_w \) wavelengths and \( N_o \) orientations (i.e., \( N_o \) polar coordinates \((\rho_\ell, \theta_\ell), \ell = 1, 2, \ldots, N_o\)) in the field of view, we have

\[
g = \begin{bmatrix}
g_1^{(1)} \\
\vdots \\
g_1^{(N_o)} \\
g_2^{(1)} \\
\vdots \\
g_2^{(N_o)} \\
g_Nw^{(1)} \\
\vdots \\
g_Nw^{(N_o)}
g_Nw^{(N_o)}
\end{bmatrix}, \quad H(\phi) = \begin{bmatrix}
h_1^{(1)} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
h_1^{(N_o)} & 0 & \cdots & 0 \\
0 & h_2^{(1)} & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
0 & \vdots & \ddots & \vdots \\
0 & \cdots & h_{Nw}^{(N_o)} & \cdots \\
0 & \cdots & \cdots & h_{Nw}^{(N_o)}
\end{bmatrix}, \quad \phi = \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\beta \\
\gamma_0 \\
\gamma_1
\end{bmatrix}.
\]
Optimization Problem

We formulate the PSF parameter estimation and star spectrum reconstruction problem in a nonlinear least squares framework

$$\min_{\phi, s} \left( f(\phi, s) = \|g - H(\phi)s\|_2^2 \right)$$

Note that $f(\phi, s)$ is linear in $s$ and nonlinear in $\phi$. $\phi \in \mathbb{R}^p$, $s \in \mathbb{R}^{N_w}$ and $p < N_w$.

Variable projection method:
Implicitly eliminate linear term $s$.
Optimize over nonlinear term $\phi$ using Gauss-Newton.

$\phi$ used to specify the spatially varying elliptical PSF parameters at each wavelength.
Optimization Problem

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Optimization Problem

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Note that
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Variable projection method:
- Implicitly eliminate linear term \( s \).
- Optimize over nonlinear term \( \phi \) using Gauss-Newton.
- \( \phi \) used to specify the spatially varying elliptical PSF parameters at each wavelength.
HSI Unmixing/Deblurring: Single PSF Case (Zhao et.al., BNP ’15 extended to multiple PSFs)

Solve the deblurring and sparse hyperspectral unmixing of the form

\[
\min_{X \geq 0} \frac{1}{2} \| HXM - G \|_F^2 + \mu_1 \| X \|_1 + \mu_2 TV(X)
\]
Solve the deblurring and sparse hyperspectral unmixing of the form

\[
\min_{X \geq 0} \frac{1}{2} ||HXM - G||_F^2 + \mu_1 ||X||_1 + \mu_2 TV(X)
\]

- \(H \in \mathbb{R}^{N_p \times N_p}\) is a block diagonal blurring matrix constructed using elliptical Moffat function parameters from our estimate \(\phi\).
Solve the deblurring and sparse hyperspectral unmixing of the form

$$\min_{X \geq 0} \frac{1}{2} \|HXM - G\|_F^2 + \mu_1 \|X\|_1 + \mu_2 TV(X)$$

- $H \in \mathbb{R}^{Np \times Np}$ is a block diagonal blurring matrix constructed using elliptical Moffat function parameters from our estimate $\phi$.
- ADMM applied to compute $X$.

X-L. Zhao, F. Wang, T-Z. Huang, M. K. Ng, and R. J. Plemmons
Deblurring and Sparse Unmixing for Hyperspectral Images

# Test Results: Simulated Data for HST Satellite - Materials from NASA Database

<table>
<thead>
<tr>
<th>Material</th>
<th>Color</th>
<th>Constituent Endmembers (%)</th>
<th>Frac. Abun. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material 1</td>
<td>light gray</td>
<td>Em. 1 (100)</td>
<td>11</td>
</tr>
<tr>
<td>Material 2</td>
<td>green</td>
<td>Em. 2 (70), Em. 9 (30)</td>
<td>18</td>
</tr>
<tr>
<td>Material 3</td>
<td>red</td>
<td>Em. 3 (100)</td>
<td>4</td>
</tr>
<tr>
<td>Material 4</td>
<td>dark gray</td>
<td>Em. 4 (60), Em. 10 (40)</td>
<td>19</td>
</tr>
<tr>
<td>Material 5</td>
<td>brown</td>
<td>Em. 5 (100)</td>
<td>7</td>
</tr>
<tr>
<td>Material 6</td>
<td>gold</td>
<td>Em. 6 (40), Em. 11 (30), Em. 12 (30)</td>
<td>32</td>
</tr>
<tr>
<td>Material 7</td>
<td>blue</td>
<td>Em. 7 (100)</td>
<td>3</td>
</tr>
<tr>
<td>Material 8</td>
<td>white</td>
<td>Em. 8 (100)</td>
<td>6</td>
</tr>
</tbody>
</table>
Spectral signatures of eight materials, such as aluminum, solar cell, copper tubing, etc.
Simulated data at different wavelengths

\[ \lambda = 400\text{nm} \quad \lambda = 1107.1\text{nm} \quad \lambda = 1814.3\text{nm} \]
Simulated data at different wavelengths

\[ \lambda = 400\text{nm} \quad \lambda = 1107.1\text{nm} \quad \lambda = 1814.3\text{nm} \]
Simulated data at different wavelengths

$\lambda = 400\text{nm}$  $\lambda = 1107.1\text{nm}$  $\lambda = 1814.3\text{nm}$
Relative Errors, Relative Residual Norms

![Graph showing relative errors and relative residual norms over iterations for multiple PSFs and one PSF.](image)

- Relative errors for multiple PSFs and one PSF are plotted against iterations.
- The graph shows a decrease in relative errors as the number of iterations increases.

Outer ADMM iterations: 200
CG iterations: 1000
PCG iterations: 20

\[ \mu_1 = 0.01, \mu_2 = 5 \times 10^{-4}, \beta = 0.01 \]

Noise = 30dB
Relative Errors, Relative Residual Norms

Relative errors

Relative residual norms

Outer ADMM iterations: 200  CG iterations: 1000  PCG iterations: 20

\( \mu_1 = 0.01, \mu_2 = 5 \times 10^{-4}, \beta = 0.01, \text{Noise} = 30\text{dB} \)
Reconstructed Fractional Abundances

True

Single PSF

Multiple PSF
Reconstructed Fractional Abundances

True

Single PSF

Multiple PSF
Reconstructed Fractional Abundances

<table>
<thead>
<tr>
<th>True</th>
<th>Single PSF</th>
<th>Multiple PSF</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="True Image" /></td>
<td><img src="image2.png" alt="Single PSF Image" /></td>
<td><img src="image3.png" alt="Multiple PSF Image" /></td>
</tr>
<tr>
<td><img src="image4.png" alt="True Image" /></td>
<td><img src="image5.png" alt="Single PSF Image" /></td>
<td><img src="image6.png" alt="Multiple PSF Image" /></td>
</tr>
<tr>
<td><img src="image7.png" alt="True Image" /></td>
<td><img src="image8.png" alt="Single PSF Image" /></td>
<td><img src="image9.png" alt="Multiple PSF Image" /></td>
</tr>
</tbody>
</table>
Reconstructed Fractional Abundances

True  

Single PSF

Multiple PSF
Reconstructed Fractional Abundances

True  Single PSF  Multiple PSF
Reconstructed Fractional Abundances

True

Single PSF

Multiple PSF
Reconstructed Fractional Abundances

True

Single PSF

Multiple PSF
Reconstructed Fractional Abundances

True | Single PSF | Multiple PSF

![Image of reconstructed abundances]
Reconstructed Material Spectral Signatures

The true material spectral signatures: - The computed spectral signatures: -.
• Spectro-Polarimetric Compressive Sensing from Snapshot HSI-Polarization Imaging through Turbulence

• Using data from a polarimetric coded aperture snapshot spectral imager (CASSPI) camera (a 4D tensor problem) in AFOSR project joint with: D. Brady (ECE, Duke), S. Prasad (Physics, UNM), and Sebastian Berisha (Radiology, UPenn).

• Data reconstruction and joint deblurring and feature extraction using ADMM.

Preprint later: http://users.wfu.edu/plemmons/
Coded-Aperture Snapshot Spectral Imagers: DD-CASSI, SD-CASSI, CASSPI. Dave Brady et al., 2007 - present.

\[ g(x, y) = \int_{\lambda} C_{\lambda}(x, y)f(x, y, \lambda)d\lambda + \epsilon_{\lambda}(x, y). \]

\( C_{\lambda}(x, y) \) is wavelength dependent system function.

- Spatial Light Modulator (SLM)-based CASSPI - snapshot spectro-polarimetric imager forward model.

\[ g(x, y) = \sum_{\mu} \int_{\lambda} C_{\mu}(x, y, \lambda)[h(x, y, \lambda) * f_{\mu}(x, y, \lambda)]d\lambda + \epsilon_{\lambda,\mu}(x, y). \]

\( \mu = \) polarimetric variable.
Segmentation Model

Segmentation model:

\[ f(x, y, \lambda) = \sum_{i=1}^{m} s_i(\lambda)u_i(x, y). \]

Resulting system model:

\[ g = \int_{\lambda} C_{\lambda}(x, y)[h_{\lambda}^{\phi}(x, y) \ast \sum_{i=1}^{m} s_i(\lambda)u_i(x, y)]d\lambda + \epsilon(x, y). \]

Knowns: coded, blurred image \( g \), and CASSI system operator \( C \).

Unknowns: phase function \( \phi \) (in terms of the OPD), spectral signatures \( s_i \), and support functions, \( u_i \).

Approach: we take a two-step approach, that is, we estimate the optical path difference function (OPD) first using a HSI of a guide star, and then classify features of object.
• Assume the OPD is known as the estimated $\hat{\phi}$ from Berisha, Nagy, Ple. 2014 so each PSF $h_\lambda$ is available. We estimate support functions $u_i$ and spectral signatures $s_i$.

$$J(u, s) = \frac{1}{2} \left\| \int_\lambda C_\lambda(x, y) \left[ h_\lambda(x, y) * \sum_{i=1}^m s_i(\lambda) u_i(x, y) \right] - g(x, y) \right\|^2_2$$

$$+ \alpha \sum_{i=1}^m \int_{\mathbb{R}^2} \sqrt{\nabla^2 x_i + \nabla^2 y_i} + \frac{\beta}{2} \sum_{i=1}^m \int_\lambda \| s_i(\lambda) \|^2_2,$$

where we use total variation regularization for $u_i$ and Tikhonov for spectral signatures $s_i$.

• Will solve inverse problem using ADMM.
System modulates a 4D tensor array image onto a 2D detector (matrix). Project in progress. To reconstruct, and jointly deblur & extract features.
In retrospect: Did van Gogh envision atmospheric turbulence in his most widely acclaimed painting – Starry Night?

Vincent van Gogh
Optical Imaging in Remote Sensing – 2015 is UNESCO “Year of Light”

Overview of Our Projects - NGA & AFOSR

(Imaging Through) Turbulence - Galileo, Von Karman, Komolgorov, ...

Hyperspectral Imaging (HSI) through Atmospheric Turbulence

Estimating HSI PSFs for Atmospheric Turbulence - HSI image of guide star, Moffat fct. modeling

Joint Deblurring and Feature Extraction - ADMM optimization approach

Overview of Compressive Snapshot HSI - Enables HSI video

Spectro-Polarimetric Imaging - Adds polarization channels for Stokes images used in identifying object shape and surface properties
Thank You!