Computations in Optical Remote Sensing
Structured Low-Rank Approximations in Spectral & LiDAR Imaging

Bob Plemmons
Wake Forest
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Fused Hyperspectral (HSI) and Light Detection and Ranging (LiDAR) Data
Optical Remote Sensing, e.g., LiDAR instead of RaDAR

Optical sensors form images of objects or scenes by detecting their solar or laser reflectance.

- **Hyperspectral** (2D spatial & ID wavelength) and **LiDAR** (2D spatial & ID range) Imaging
  - Hyperspectral imaging (HSI) collects information across the visible and IR spectrum for interrogating scenes.
  - **LiDAR** measures distance by illuminating targets with a scanning laser and analyzing the reflected light. Laser wavelengths vary to suit the target: 10 \( \mu m \) to UV. Very short.
  - A Basic technology - combined (fused) HSI and LiDAR for enhanced analysis. **Combined laser ranging & object material identification**.

- **Spectro-Polarimetric imaging** (3D hyperspectral & ID polarization). Data is a **4D tensor**. Polarization identifies object shape, metallic surfaces.
References Related to Presentation and Co-Authors
Overview of Optical Remote Sensing
Low-Rank Matrix Decomposition - Exposing Structure
Our Motivation: Remote Sensing with Hyperspectral & LiDAR Imaging

(1) Nonnegative Matrix Factorization
(2) Randomized Low-Rank Matrix Decomposition
   - Hyperspectral Image Interrogation. Random Projections
If time permits - Spectro-Polarimetric Images Taken Through Atmospheric Turbulence - Data forms a 4D Tensor Array.
References Related to Presentation


Some Applications of HSI & LiDAR

- Environmental remote sensing, e.g., monitoring chemical/oil spills - HSI
- Military target discrimination - both HSI & LiDAR
- Astrophysics - HSI
- Remote surveillance for defense & security, e.g., imaging a compound in western Pakistan - both
- Sensing for agriculture & food quality and safety - HSI
- Biomedical optics, medical microscopy, etc. - HSI
- Disaster relief, land/water management
- **LiDAR Archeology:** Angkor Wat in Cambodia, Roman gold mines in Spain - Physics Today, Nov. 2014
NGA Project

- National Geospatial-Intelligence Agency (NGA) Funded Project “Compressive Sensing and Fusion of LiDAR and Hyperspectral Data using Tensor Factorizations”. Boeing and Wake Forest 2011-2013

Randomized matrix and tensor factorization for HSI compression. Low-rank matrix approximations, e.g., approx. tSVD of HSI tensor, then fusion of components with LiDAR data.

Fusion of datasets into a common data representation.

Clustering and classification of fused data, and information-theoretic results, detecting objects in shadows, etc.

Provided a 3D visualization tool for fused LiDAR and HSI.
Fused HSI and LiDAR (using Implicit Geometry Point Cloud Representation.)

Figure 1: Part of the “Gulfport” HSI/LIDAR data supplied by NGA.
Figure 2: Fused HSI/LiDAR. Original data collected by Gred Asner, Stanford Carnegie Airborne Observatory.
“TENSOR-BASED QUALITY PREDICTION FOR BUILDING MODEL RECONSTRUCTION FROM LIDAR DATA”
B. C. Lin1*, R. J. You2 Department of Geomatics, National Cheng Kung University, 1 University Road, Tainan City, Taiwan
Tensor analysis, robust least squares, data fusion
AFOSR Project

- **Air Force (AFOSR) Funded Project**

**Space Object Identification with Spectral Imagers**

Analysis of reflectance.

**Figure**: Reflectance of an object pixel results from additive reflectance of its constitutive elements.
Adding Polarization Channels leads to a 4D Tensor

Simulated HST images with polarization on the right. 4th dimension represents polarization index $p$. Data is $x \times y \times \lambda \times p$. 
Hyperpectral Imaging

- Spectral imagers capture a 3D datacube (tensor) containing:
  - 2D spatial information: x-y
  - 1D spectral information at each spatial location: $\lambda$
- Pixel intensity varies with wavelength bands - provides a spectral trace of intensity values.
- Generates a spatial map of spectral variation.
Spectral (beyond RGB) Imaging at Wavelengths $\lambda$

**Figure 3:** $\lambda$ generally ranges between 400 and 2500 nanometers.
Figure 4: Data is a 3D cube - extract features.
Extract Spectral Signatures (Traces) from HSI to Identify Materials

Figure 5: Illustration - Each pixel represents a vector
Figure 6: Real Data from NGA Project
Another Example of HSI - Hubble Sat.

Figure 7: Space object imaging from AFOSR Project
Low-Rank Matrix Decomposition: Exposing Structure

Our Application: Remote Sensing with Hyperspectral Images
Low-Rank Matrix Decompositions

Figure 8: **Cracking the egg** (Image Credit: Ben Sapp, UPenn).

Figure 9: Low-rank approximation.
Motivation and Advantages

- Compression/Dimensionality Reduction - requires only $mk + nk$ storage rather than $mn$.
- Less numbers - less storage, faster transmission and processing.
- Significant applications from - Truncated SVD - tSVD, PCA, ICA, NMF, NMU, LDA, LLE, Isomap, Diffusion Maps, etc.,
- All involve a low-rank matrix decomposition.
- Often expose data trends and, for us, object features in images, facilitating object detection, identification, and analysis.
Randomized Low-Rank Approximations

- **Classical numerical linear algebra**: orthogonalization by Givens rotations., modified Gram-Schmidt, Householder transformations. - leading to QR, SVD, tSVD, decompositions, etc.

- **Recent approaches**: capture range of $A$, $R(A)$, using randomly chosen vectors with appropriate distributions
  
  - Randomly (intelligently) sample columns and/or rows of $A$ - Drineas, Mahony, Rademacher, Vempala, ...
  
  - **Random projections** onto subspace of $R(A)$ and /or subspace of $R(A^T)$ - Tropp, Martinsson, Halko, Fowler, Candes, Donaho, Tsao, Baraniuk, Sharon, Romberg, ...
Approximate tSVD of Matrix $A$ by Random Projections - Geometric Intuition

- Random Projection: method for projecting high dimensional (and sparse in some basis) data into lower dimensional (and less redundant) representation.

$$\omega_i \sim \mathcal{N}(0, 1)$$

| Figure 10: Probe $A$ with randomly chosen, e.g. Gaussian, vectors. |
Approximate tSVD of Matrix $A$ by Random Projections - Halko et al., 2011 (modified)

**Input**: An $m \times n$ matrix $A$ and a precision measure $\epsilon$.

**Output**: An $m \times k$ matrix $Q$ and rank $k$.

Initialize $Q$ as an empty matrix, $e = 1$ and $k = 0$.

while $e > \epsilon$ do

1. $k = k + 1$.
2. $y_i = A\omega_i$, where $\omega_i$ is a Gaussian random vector.
3. $q_i = (I - QQ^T)y_i$.
4. $q_i = q_i/\|q_i\|$.
5. $Q \leftarrow [Q \ q_i]$.
6. $\Omega \leftarrow [\Omega \ \omega_i]$.
7. Compute error $e = \|A - QQ^TA\|_F/\|A\|_F$.

end
Approximating the rank-k TSVD of $A$ (simplified).

Given $Q$, $m \times k$ matrix whose columns form a basis for $R(A\Omega)$.

1. Form $B = Q^T A$, $k \times n$. $Q^T$ is our projection matrix.
2. Compute the SVD of the small matrix: $B = \tilde{U}\Sigma V^T$.
3. Set $U = Q\tilde{U}$
4. $\tilde{A} = U\Sigma V^T \approx A$

- Columns of $U$ correspond to the first $k$ principal components of HSI data.
- Operation counts, error bounds and extensions, for our HSI applications, can be found in J. Zhang, Erway, Q. Zhang and Ple., JECE 2013).
- Alternatives, project $A$ on the other side, or on both sides, $B = Q^T A P$, forming a smaller double random projection.
Our Motivation

Dimensionality reduction accomplished with random projections at sender and reconstruction at receiving (base) station. Image credit, J. Fowler (MSU) for his Compressive Projection PCA (CPPCA), 2012.

- Our orthogonal random projections are determined from the data. Faster, better approximation than CPPCA.
- First few columns of $U$ provide interrogation of HSI feature space.
- Method extended to double random projection scheme. IEEE TIP paper (Zhang and Ple.).
Data from National Geospatial-Intelligence Agency (NGA).

Hyperspectral data, $320 \times 360 \times 58$, wavelengths $0.4\mu m$ to $1.0\mu m$.

$A$ is $115,200 \times 58$. We take $k = 25$, so $B$ is $25 \times 58$.

HSI/LiDAR (registered) data. Targets placed in scene by NGA.

Approximate tSVD of data matrix $A$: $U\Sigma V^T$.

Figure 11: NGA project data.
Figure 12: First column of $U$
Figure 13: Second column of $U$
Figure 14: Third column of $U$ - Best for Target ID, follow with component fusion with LiDAR data
Figure 15: Seventh column of $U$ - **Notice Cars (white specks)**. Need polarimetric data.
Figure 16: Eighth column of $U$ - noise
Figure 17: True “Desert Radiance” image from ARL

Figure 18: Target detection
Current Work on Deblurring & Feature Extraction

- Spectro-Polarimetric Compressive Sensing from Snapshot HSI-Polarization Imaging through Turbulence

- Using data from a polarimetric coded aperture snapshot spectral imager (CASSPI) camera \textbf{(a 4D tensor problem)} in AFOSR project joint with: D. Brady (ECE, Duke), S. Prasad (Physics, UNM). Also, P. Pauca (WFU), J. Nagy (Emory) and S. Berisha (UPenn).

- Data reconstruction and feature extraction using ADMM.

Preprint available soon at: http://users.wfu.edu/plemmons/
AMOS sensors can collect simultaneously from visible to LWIR.

**AMOS Multiple Sensors. Spectral-polarimetric imaging: HSI, 4 polarization channels. All the PSFs are wavelength and diffraction blur dependent.** (Sanya, CN workshop, January)
Snapshots Spectral & Spectro-Polarimetric Compressive Sensing

Coded-Aperture Snapshot Spectral Imagers: DD-CASSI, SD-CASSI, CASSPI. See, e.g., Dave Brady et al., 2007 - present.

\[ g(x, y) = \int_{\lambda} C_{\lambda}(x, y) f(x, y, \lambda) d\lambda + \epsilon_{\lambda}(x, y). \]

\(C_{\lambda}(x, y)\) is wavelength dependent system function.

- Spatial Light Modulator (SLM)-based CASSPI - snapshot spectro-polarimetric imager forward model.

\[ g(x, y) = \sum_{\mu} \int_{\lambda} C_{\mu}(x, y, \lambda)[h(x, y, \lambda) * f_{\mu}(x, y, \lambda)]d\lambda + \epsilon_{\lambda,\mu}(x, y). \]

\(\mu = \) polarimetric variable. Solve inverse problem for \(f\) using ADMM.
System modulates a 4D tensor array image onto a 2D detector (matrix). Deblur & extract features using ADMM algorithm.
Hyperspectral Imaging (HSI) through Atmosphere -

The image acquired at wavelength $\lambda$ can be represented as

$$g_\lambda(x, y) = h_\lambda(x, y; \phi) \ast f(x, y, \lambda) + \epsilon_\lambda(x, y),$$

where the blurring kernel, with diffractive scaling included, is

$$h_\lambda(x, y; \phi) = \left(\frac{\lambda_0}{\lambda}\right)^2 h_0 \left(\frac{\lambda_0}{\lambda} x, \frac{\lambda_0}{\lambda} y; \phi\right),$$

with $\lambda_0 =$ average wavelength, and

$$h_0(x, y; \phi) = \left|\mathcal{F}^{-1} \left(p e^{i\phi}\right)\right|^2,$$

and where the wavefront phase $\phi = \frac{2\pi}{\lambda} \times OPD$.

- OPD is the optical path difference function, i.e. the optical phase shift in passage through the turbulent atmosphere, which is essentially the same for each HSI wavelength.
Simulate phase function using the von Karman phase spectrum,

\[ P(\kappa_x, \kappa_y) = \sqrt{0.023(D/r_0)^{5/3}}(\kappa_0^2 + \kappa_x^2 + \kappa_y^2)^{-11/6}, \]

where \( r_0 \): Fried parameter, \( D \): telescope aperture diameter, \( \kappa_0 \): low-freq cutoff.

\[ \phi = \sqrt{P(\kappa_x, \kappa_y)} W(\kappa_x, \kappa_y), \]

where \( W \) is a zero-mean, unit-variance, white Gaussian noise array. Set \( D/r_0 = 10 \).

True ODF (left), initial (center), estimated (right)
Overview of Optical Remote Sensing

Low-Rank Matrix Decomposition - Exposing Structure

Our Motivation: Remote Sensing with Hyperspectral Images

(1) Nonnegative Matrix Factorization
   - Work with Nicolas Gillis
   - Importance of Sparsity: M-Matrices Enter the Picture

(2) Randomized Low-Rank Matrix Decomposition
   - Hyperspectral Image Interrogation. Random Projections

Some Current Work: Astrophysics, Spectral-Polarimetric Imaging

Thank You!