Modeling Traffic with Human Variation

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Abstract

In this paper, we consider the effects of human behavior and individual variation on traffic phenomena. We explore several models, and choose to use the Intelligent Driver Model to simulate one- and two-lane traffic. Our simulation is done initially in a one-lane, one-car scenario and then expanded to a multi-car, one-lane scenario. We then develop a two-lane model that allows for lane-changing, and run it on discrete time intervals for short periods of time. We plan to automate the iteration of our two-lane model in the future. All of our calculations are done using Mathematica to execute the models..

1 Introduction

The scientific study of traffic was initiated in 1933 by Bruce Greenshields, when he presented his observations at the 13th Annual Meeting of the Highway Research Board. Greenshields developed his research using a camera which was designed to take pictures of a road at designated time intervals. In 1934, Greenshields graduated from the University of Michigan with his Ph.D., having written his thesis–the Photographic Method of Studying Traffic Behavior. In his thesis, Greenshields described in detail how he obtained his traffic data using his specially designed camera. He especially focused on the settings of his camera which optimized the quality of his data collection. Upon graduation, Greenshields worked as a traffic engineering scientist and taught at many universities [4].

Despite Greenshields effort, traffic flow theory was not a particularly popular area of research until the 1950s, when many engineers from other scientific fields were drawn to research in traffic. It is believed that the rapid rise then in the study of traffic theory was stimulated by General Motors' hiring of a new research laboratory executive, Larry Hafstad, who was originally a physicist. Hafstad aspired to transform GM's laboratory from a development lab to a basic science research lab. Robert Herman was hired for the lab, who worked jointly with Elliott Montroll, and made progress in the development of microscopic (car-following) traffic models despite the lack of technology to support their experiments.[1]

Early contributors to traffic models in the 1950s were Reuschel and Pipes, who introduced a single-lane microscopic model. In addition, Lighthill and Whitham introduced a macroscopic traffic model in the early 1950s.[1] At around the same time, the mathematician John Glen Wardrop began to describe traffic using the language of mathematics and statistics and began the formal development of traffic flow theory. In his 1952 paper "Some Theoretical Aspect of Road Traffic Research," Wardrop utilized mathematical and statistical methods to analyze traffic conditions such as road capacity on open roads and at intersections. In addition, Wardrop also explored traffic planning in his paper. A key statistical idea Wardrop introduced in his paper was to find values of aggregated variables such as Q, the traffic flow rate of the entire road, from individual variables such as q, the flow rate of an individual vehicle. In addition, Wardrop also introduced other theories, such as finding the concentration K of the total traffic from adding up k_i, the individual concentration of each stream.

As computers developed in the 1960s, the focus of traffic theory expanded from simple descriptions of traffic to problem solving such as congestion alleviation. However, the boom in traffic research ended in the late 1960s, because initial techniques had been fully explored and progress had slowed. Then, in the 1990s, researchers interest in traffic was reignited and, during this period, many new models were developed that rendered older, less realistic models obsolete. [3]

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In current research, traffic modeling no longer employs Greenshields original techniques, but instead modern strategies that take advantage of our greatly expanded computational power. As a consequence of its increased accuracy, the field has permeated into many social science areas such as economics, psychology, and sociology. In addition, as vehicles have become ever more widely used around the world, research in traffic flow theory is more relevant than ever.

This paper is organized as follows. In Section 1.1 we will discuss three kinds of traffic models, macroscopic models, microscopic models, and mesoscopic models, and explain why miscroscopic models are the most suitable for our project. Then, in Section 2 we discuss three microscopic models that we considered–the Optimal Velocity Model, the Full Velocity Difference Model, and the Intelligent Driver Model–and explain why we decided to use the Intelligent Driver Model for our computations. Next, in Section 3, we will explain the details of our one-lane Intelligent Driver Model. In Section 4 we explain our two-lane model, considering lane-changing behavior. Then, we present the results of our one-lane simulations in Section 5 for both one and two lane traffic scenarios. Lastly, in Section 6, we will discuss our future research plans.

1.1 Three Kinds of Traffic Models

As described in [2], there are three categories of traffic models. In this section we will explain the advantages and disadvantages of each.

1.1.1 Macroscopic Models

In macroscopic models, cars are considered in the aggregate, generally by studying the density of traffic as a function of position and time. Thus, the perspective of the macroscopic model is to focus on the overall traffic on the road rather than individual cars' behavior. For example, traffic flow $\rho(x, t)$, traffic density $Q(x, t)$, and mean speed $V(x, t)$ are some commonly used variables in macroscopic models and they all focus on the behavior of overall traffic in a scenario.

By focusing on the density of cars as a function of space and time, one generally derives a model which is a partial differential equation. Although partial differential equations can be very hard to solve exactly, they do have well-understood qualitative behavior patterns and numerical solution techniques. Therefore, an advantage of the macroscopic modeling method is that it can allow an easier understanding of large-scale traffic phenomena. It also allows one to draw analogies between traffic flow and other types of physical flows, such as fluids. However, Macroscopic models do not focus on individual drivers, so they do not always provide accurate results, particularly when the individual drivers are engaging in variable behavior. In addition, collecting data for macroscopic models can be difficult, because most data needed for macroscopic models have to be calculated from microscopic data.

1.1.2 Microscopic Models

Microscopic models are suitable for researchers who are interested in analyzing individual drivers' behavior and its influences on overall traffic. In these model, each car's position and velocity is tracked, and the car's acceleration at a given time is predicted as a function of the position and velocity of itself and its neighbors. Then, the researcher has to solve a system of ordinary differential equations which includes the acceleration functions of all cars being considered. Most miscroscopic models are car-following models which means, for each car being considered, the acceleration function considers a car's relation to the the leading car, which is the car directly in front of the car being considered.

Microscopic models are more accurate than macroscopic models, and they are able to analyze individual driver's behavior. However, since there is an acceleration function for each car in the scenario we consider, solving a system of acceleration functions with a large number of cars can be very intensive.

1.1.3 Mesoscopic Models

Mesoscopic models are hybrids of miscroscopic and macroscopic models. In these models, miscroscopic models are employed in important spots such as road junctions. At places that have simple traffic flow, such as a freeway without intersections, though, macroscopic models are used. In mesoscopic models the conversion between miscroscopic and macroscopic variables can be a challenge.

2 Three Continuous Microscopic Models

Since the focus of our research is on heterogeneous driver behavior, microscopic models best serve our purpose. In this section, we discuss three miscroscopic models that we considered at the beginning of our research. In order to better understand the three models, we need to first go over some important variables and parameters that are present in our model.

2.1 Variables

The following is a list of variables that are used in the microscopic models we considered:

- t, the time in seconds since the start of the simulation,
- $s(t)$, the gap between the car being considered and the leading car (that is, the car directly in front of it),
- $v(t)$, velocity of the car being considered,
- $a(t) = \dot{v}(t)$, the acceleration of the car being considered, which is the derivative of its velocity, and
- Δv, the velocity difference between the car being considered and the leading car.

2.2 Parameters

Below is a list of parameters in our models. Some parameters have physically intuitive explanations, while others are more complex.

Notation	Parameter	Explanation
τ	Adaptation Time	Adaptation Time is the time it takes a human, upon observing an obstacle, to decide how to respond to the obstacle. While Adaptation Time is not the same for all humans, we used a reference value representing an average human response time. Later in our work, we allow for this parameter to be random, accounting for human variation.
v_0	Desired Speed	Desired Speed is the desired cruising speed. For a car driving on an infinite freeway with no car in front of it, the car would accelerate until it reaches its desired speed and cruise forever at this speed.
T	Time Gap	The Time Gap describes the time that takes a driver to come to a stop from its cruising speed at maximum braking deceleration.
s_0	Minimum Distance Gap	The Minimum Distance Gap describes the minimum distance between the back bumper of the leading car and the front bumper of the car being considered. This gap is a parameter, which means it won't change in any circumstances; it is important to differentiate the Minimum Distance Gap from the Safe Gap, which depends on current speed. The Minimum Distance Gap is a distance which no driver should ever violate at any time.
a	Maximum Acceleration	The Maximum Acceleration is an acceleration value that the given car will never exceed. This value is equivalent to the initial acceleration at $t = 0$ when a car is accelerating from standstill with no car in front.
b	Comfortable Deceleration	Comfortable deceleration is treated as the maximum deceleration. When a car is slowing down or reaching standstill, the car's deceleration will never exceed b.
δ	Acceleration Exponent	The Acceleration Exponent, δ, determines the qualitative shape of our acceleration function. Increasing δ makes our function react more quickly to changes in Δv, while decreasing δ makes the response more smooth. The particular value does not have any intuitive interpretation, but this parameter can be used to tune the simulation to be more accurate for different situations.

2.3 Optimal Velocity Model (OVM)

The optimal velocity model is a model in which the acceleration function for the given car is as follows:

$$\dot{v} = \frac{v_{opt}(s) - v}{\tau},$$

where v_{opt} is known as the "optimal velocity,"and is given by

$$v_{opt}(s) = \max[0, \min(v_0, \frac{s - s_0}{T})].$$

In this model, the optimal velocity, v_{opt}, changes with respect to time, and each driver decides their optimal velocity using the gap between their car and the car in front of them. From the definition of v_{opt}, we can see that the optimal velocity will never be negative, which means the car being considered will never have backward motion. Thus, even when cars violate the Minimum Distance Gap, they will not go into reverse, which is good because such a decision would be very undesirable in a highway situation.

This, however, is only one metric for how realistic the model is. In other ways, OVM is quite unrealistic. OVM only considers the gap in front of each car, but it does not consider the speed difference Δv between the considered car and the leading car. This means for any cars that are the same distance apart, the car in the back would always react the same regardless of the speed of the front car, which clearly makes no sense when the car in front may be going much slower or faster than the considered car. OVM is a good starting point for developing realistic car-following models of traffic, and each of our upcoming models will be a refinement of OVM to make it more realistic.

2.4 Full Velocity Difference Model (FVDM)

In the next model, the OVM is improved by having the acceleration take the velocity difference into account. The current optimal velocity remains the same for each car, but the car's acceleration is lowered if the car in front is moving more slowly than the considered car.

$$\dot{v} = \frac{v_{opt}(s) - v}{\tau} - \gamma \Delta v$$

$$v_{opt}(s) = \max[0, \min(v_0, \frac{s - s_0}{T})]$$

Here, γ is a small parameter that represents the responsiveness of the driver to velocity differences. In the limit as $\gamma \to 0$, FVDM converges back to the OVM.

FVDM improves on OVM by taking Δv into account. However, simply considering Δv is not enough. FVDM does not related Δv to the gap s. This means that for any car that has a leading car in standstill, the car being considered will always decelerate sharply, regardless of whether the leading car is 10 meters ahead or 10 kilometers ahead. Our final model will solve this problem by considering how Δv and s are related in defining $\dot{v}$.

2.5 Intelligent Driver Model (IDM)

Our final model, the Intelligent Driver Model, is significantly more complicated than the previous two models. The equations for IDM are as follows:

$$\dot{v} = a[1 - (\frac{v}{v_0})^\delta - (\frac{s^*(v, \Delta v)}{s})]$$

$$s^*(v, \Delta v) = s_0 + \max(0, vT + \frac{v \Delta v}{2\sqrt{ab}})$$

IDM does not have the primary issues which make OVM and FVDM unrealistic; it accounts for the relationship between Δv and s and how they both affect $\dot{v}$. According to [2], the IDM is the simplest model that accounts for all the necessary variables in a way that is fairly realistic while not allowing any accidents to occur (because an accident would crash the simulation; allowing for the possibility of accidents in real-world driving behavior is an interesting topic for future investigation). Unfortunately, this makes the model significantly harder to solve. Nevertheless we decided to use IDM for our research and solve the models on a computer using Mathematica. For the numbers of cars and time periods which we considered, we did not find that Mathematica had any difficulty solving this model.

3 IDM Details

3.1 One Lane Traffic

We decided to first build a one-lane mode to familiarize ourselves with programming in Mathematica and to make sure our model was realistic. We chose to employ a circular roadway for all scenarios that we considered, because in a circular roadway each car has a leading car, obviating the need for special considerations for the first car on the track. In our experiments we have also ignored the vehicle's length and regard all vehicles as points.

Below is a chart of the values we used for each parameter. These values are suggested by our reference [2].

Parameter	Notation	Typical Value
Desired Speed	v_0	35 m/s
Time Gap	T	1.0 s
Minimum Gap	s_0	2 m
Acceleration Exponent	δ	4
Acceleration	a	1.0 m/s^2
Comfortable Deceleration	b	1.5 m/s^2

3.1.1 One Lane Traffic with One Car

First, we modeled one car in our one-lane scenario. In this case, the leading car of the car being considered is in fact the car itself and the gap between the "two "cars is always the total distance of the circular road. The results of this experiment, as well as the rest of the experiments described in this section, are displayed in Section 5.

3.1.2 One Lane Traffic with Two Cars

In order to make our model more realistic, we added additional cars. We started by just adding one more car, and subsequently increased the number of cars further. One important thing to keep in mind is that in a one lane model with two cars, the gap between the two cars, $s(t)$, may change as a function of time. Notice that if we define $s(t)$ to be the gap behind car 1 and in front of car 2, then the gap in front of car 1, behind car 2, is simply the total length of the track less $s(t)$. In general, if we have n cars, then we only need to keep track of $n-1$ distance gap functions because the last one can always be obtained by subtracting the sum of the first $n-1$ distance gap functions from the total track length. For the one-lane, two-car model, we ran the experiment using several different sets of initial conditions, to be sure that the response was realistic.

3.1.3 One Lane Traffic with Multiple Cars

In order to complete our one-lane model, we adjusted our model again so that we could choose how many cars were in our one-lane road. Also, in order to make our model more realistic, we let Mathematica randomly assign values to some parameters and initial conditions according to a normal distribution. Below is a list of all the parameters and initial conditions we decided to randomize and their mean and variance:

Parameter	Notation	Mean Value	Variance
Desired Speed	v_0	35 $\frac{\text{m}}{\text{s}}$	12.74
Time Gap	T	1.0 s	0.364
Maximum Acceleration	a	1.0 $\frac{\text{m}}{\text{s}^2}$	0.364
Comfortable Deceleration	b	1.5 $\frac{\text{m}}{\text{s}^2}$	0.546
Initial Gap	$s(0)$	$\frac{\text{Total Distance}}{2\cdot(\text{Number of Cars})}$	$\frac{\text{Total Distance}}{20\cdot(\text{Number of Cars})}$
Initial Velocity	$v(0)$	20	7.28

Our detailed results are found in Section 5, but overall we found that the outputs for our one lane model in a variety of scenarios made sense and reflected normal driver behavior well.

4 Lane Changing

Having fully analyzed the one-lane scenario, we set out to extend our model to two-lane scenarios which allow lane-changing behavior.

4.1 Three Criteria

According to our reference [2], each driver considers three criteria before the driver decides whether to change lanes, and the criteria are as follows:

- Safety Criterion
- Incentive Criterion
- Courtesy Criterion

Safety Criterion: For this criterion, the drivers we are considering decide whether it is safe for them to change lanes. Mathematically, we do so by testing whether drivers would still be able to maintain their safe following distance after they change lanes.

Incentive Criterion: For this criterion, drivers decide whether they would be able to reach a higher acceleration after they change lanes. Mathematically, we check if drivers' acceleration functions would be higher in the other lane.

Courtesy Criterion: For this criterion, drivers consider whether it would impose a significant deceleration force on the driver behind them in the new lane if they were to change lanes.

For each of the three criteria the output is binary–TRUE if the the criterion is passed, and FALSE otherwise. The driver changes lanes if and only if all three criteria output TRUE.

4.2 Our Current Model

Our current model runs in time steps of $\Delta t = 1s$. During each forward step, we solve each lane using a continuous one-lane IDM, and obtain the values for acceleration $a(t)$, velocity $v(t)$, gap $s(t)$, and position $x(t)$ for the next time step. At the end of each time step, we use the values we obtained to determine sequentially whether each car wishes to change lanes. After making any lane-change adjustments, we insert the output from the previous time step into our model as the next time step's initial conditions to obtain values for another time step.

5 Results

5.1 One Lane One Car

At first, we ran our model with an initial condition of $v(0) = 35$. We expected our car's velocity profile to be a horizontal line, which means our car starts cruising at $t = 0$ and maintains the same speed forever.

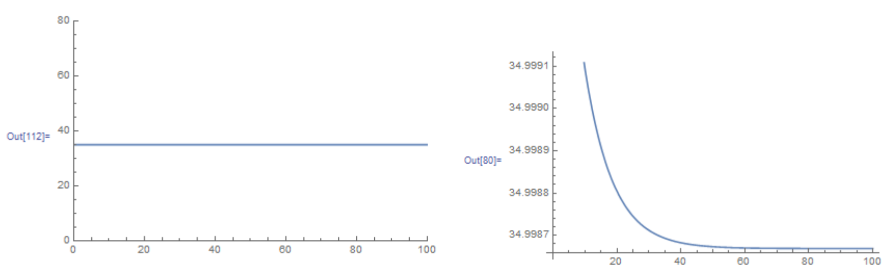

Figure 1: Regular range on the left; zoomed in picture on the right.

At first, from the graph on the left, it seems like our hypothesis is correct. However, it turned out we are not entirely right after we zoomed in on the car's velocity profile.

From the zoomed in picture on the right, it seems like our car decelerates immediately at $t = 0$ and maintains a cruising speed which is slightly slower than $35m/s$. We believes this is the result of our car always detecting itself driving in front of it. Since the desired speed $35m/s$ is the speed for our car when no car is in front of the car we are considering, by having itself always running in front of it, our car can never reach its exact desired speed.

Next we considered a car starting from rest. Below is a figure of the velocity profile of our one-lane-one-car model when we assigned initial condition $v(0) = 0$ to the car. From our hypothesis, we expected the graph to show that our car accelerates from standstill to its desired speed $35m/s$ and cruises at the same speed.

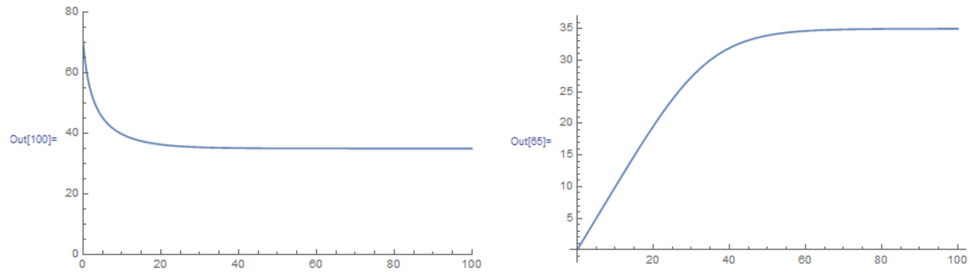

Figure 2: When v(0)=70 on the left; v(0)=0 on the right.

It turns out the graph shows almost exactly what we expected. The car accelerates to its desired speed and cruises close to $35m/s$. However, the car's cruising speed will not be exactly $35m/s$, because it detects itself as a leading car, which makes a very small but non-zero negative contribution to the safe cruising speed. Hence we find that the actual cruising speed for a single car in a $10,000m$ track is $34.9987m/s$.

5.2 One Lane Two Cars

We first ran our experiment with both cars starting from standstill using the initial condition $v_1(0) = v_2(0) = 0$, exactly halfway around the track from each other.

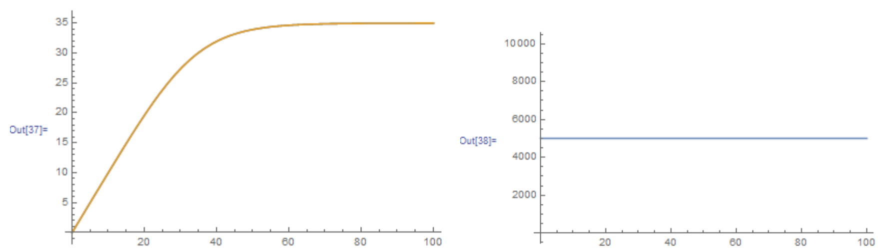

Figure 3: Velocity on the left; Distance between cars on the right.

It turns out that the velocity profile of both cars are the same, which is why we only see a single function on our graph on the left. The two velocity functions for the two cars are overlapping. Both cars accelerate immediately at $t = 0$, and their acceleration reaches 0 after they reach slightly below their desired speed.
From the graph on the right, we can tell the gap between two cars stay constant, which should be expected since the cars are synchronous.

We then tried assigning one of the cars to have initial velocty $v_1(0) = 70$ while keeping the other car's initial velocty at 0. We expected car 1 to decelerate to its desired cruising speed while car 2 accelerates to its desired cruising speed.

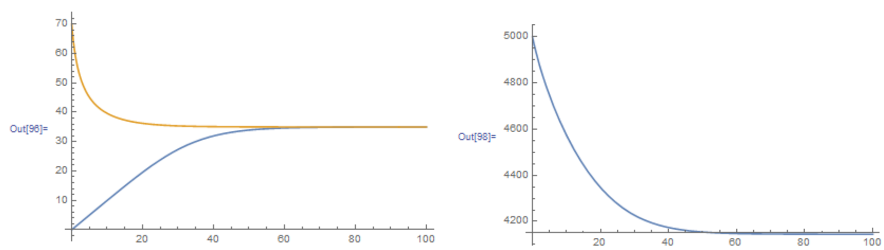

Figure 4: Velocity on the left; Distance between cars on the right.

From the graph on the left, we can see the velocity profile of two cars is exactly the same as what we expected. From the graph on the right, it seems reasonable that the gap behind the slower car and in front of the faster car decreases. Using both graphs we can tell that after both cars reach their desired speed at around $t = 60$, our traffic reaches its equilibrium and the gap between two cars stay constant as soon as both cars reach their desired speed.

5.3 One Lane Multiple Cars

First, we tested our model with three cars. We ran our model for 100 seconds, which is what we did before. However, from the graph on the left we can see that we can no longer make any conclusion to our model since the experiment period is too short.

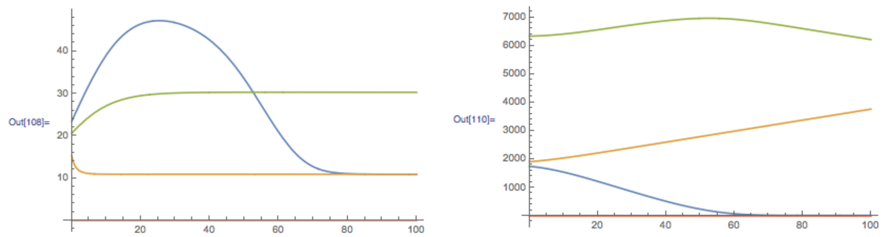

Figure 5: Velocity on the left; Gap between cars on the right.

Then, we extended the period of our experiment to 5000 seconds, and we can see that all three cars eventually end up driving at the same speed and as soon as all three cars reach the same speed the gaps between cars become constant. Most importantly, we noticed that the gap in front of one car is almost the same as the total distance of the circular road, which means the gaps in front of the other two cars must be very small.

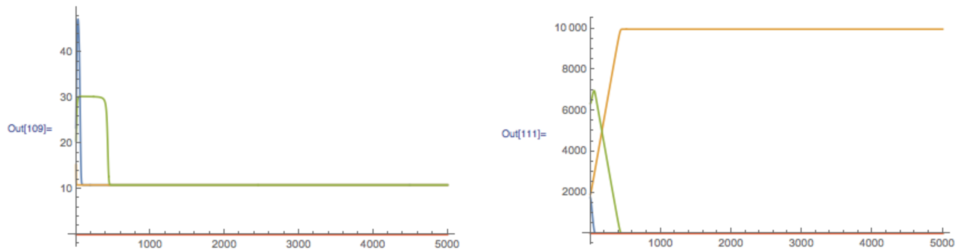

Figure 6: Velocity on the left; Distance between cars on the right.

We zoomed in on the picture above and we found that the two cars which have higher desired speed than the car at the very front are tailgating their leading cars at their own safe following distance. Safe following distance is found by $S_0 + vT$, and because of our randomization on T, the two cars that are tailgating have different safe following distance. Subsequently, we can tell that all three cars end up cruising at the desired speed of the slowest car, the front car, because in a one-lane model no car is able to overtake slower cars that are in front of them.

Then, we increased the number of cars on our road to 20 in order to test if our findings from the 3-car scenarios still hold as the number of cars on the road increases.
As before, when we ran our model for just 100 seconds, the graphs are not very informative.

We ran our model again for a period of 5000 seconds and we see that all cars end up cruising at the desired speed of the slowest car and tailgating behind the car with the slowest desired speed. One other thing we noticed, which was also expected, is that when we have 20 cars in our road, it takes more time for the traffic to reach equilibrium in comparison to the 3-car scenario.

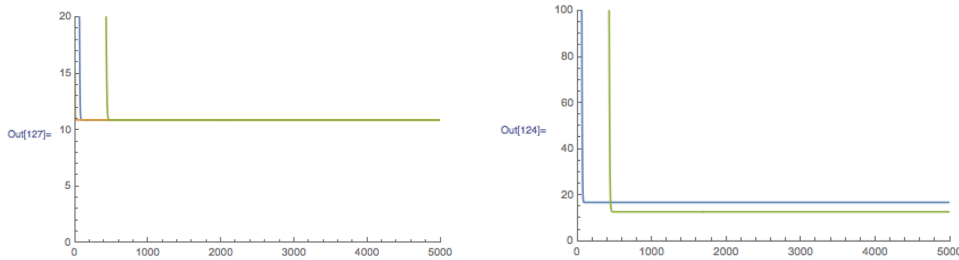

Figure 7: Velocity on the left; Distance between cars on the right.

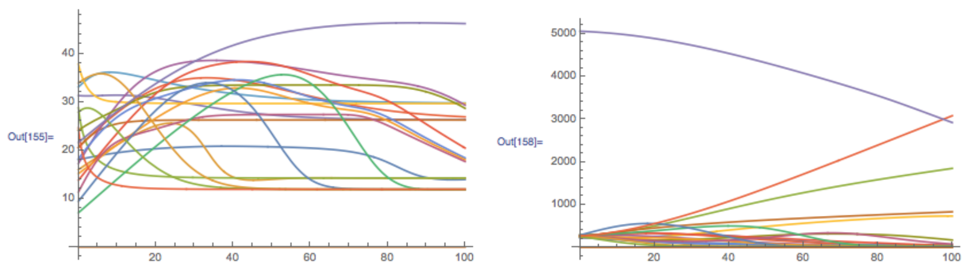

Figure 8: Velocity on the left; Gap between cars on the right.

6 Future Work

There are many more details we want to consider in our model so we will keep revising our current programs until our model is able to discribe most traffic scenarios. In this section, we will discuss some important future plans.

1. We will automate our two-lane model. This means, before we run our model, we want to assign our experiment to run for a certain period, and we let our model run automatically for the period we had previously assigned. Then, we want our model to output a chart with each car's behavior at each time step. The chart should include each car's longitudal position, velocity, acceleration, gap in front of the car, and the lane it is in.
2. Most importantly, we want to assign more cars to our two-lane model. Currently, we have manually put 4 cars into our model, which is highly unrealistic. We will program our model so that we are able to assign the number of cars in our lanes as we wish instead of putting each one in manually.
3. We want to distinguish aggressive drivers from polite drivers. This will be done by allowing some drivers to fail to consider the courtesy criterion when they want to switch lanes. Some research will be done on the most realistic percentage of aggressive drivers on the road.
4. We want to consider human estimation errors in our model. For example, sometimes human underestimate the gap in front them and thus decelerates earlier than they should. In the future, we want to include this in our model and see how this affects the overall traffic.
5. Reaction time can vary for people at different age and people with different driving experience. We want to assign drivers various reaction times based on a probability distribution; we will research an appropriate distribution to use as well.
6. Currently, we assume each driver is only able to see one car in front of them. However, this is not exactly true. Usually, people are able to see multiple cars in front of them, and we call this behavior multi-car-anticipation. In the future, we plan to include multi-car-anticipation in our model and see its effect on the overall traffic.

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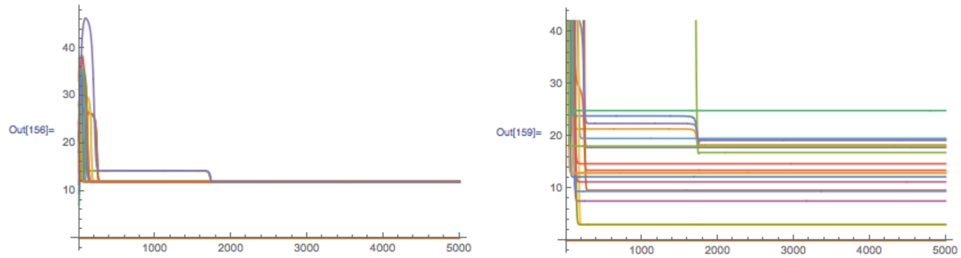

Figure 9: Velocity on the left; Distance between cars on the right.

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Sarah Raynor is an Associate Professor of Mathematics and Statistics at Wake Forest University in central North Carolina. She earned her Bachelors degree in Mathematics and Physics at Yale University in 1998 and her PhD in Mathematics at MIT in 2003. Her specialty area is differential equations, particularly those equations which model wave phenomena such as water and light. She has mentored numerous student projects on varied topics, including sports ranking, fractals, DNA modeling, and, of course, traffic modeling.