



## Fractal images induce fractal pupil dilations and constrictions



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### ABSTRACT

Fractals are self-similar structures or patterns that repeat at increasingly fine magnifications. Research has revealed fractal patterns in many natural and physiological processes. This article investigates pupillary size over time to determine if their oscillations demonstrate a fractal pattern. We predict that pupil size over time will fluctuate in a fractal manner and this may be due to either the fractal neuronal structure or fractal properties of the image viewed. We present evidence that low complexity fractal patterns underlie pupillary oscillations as subjects view spatial fractal patterns. We also present evidence implicating the autonomic nervous system's importance in these patterns. Using the variational method of the box-counting procedure we demonstrate that low complexity fractal patterns are found in changes within pupil size over time in millimeters (mm) and our data suggest that these pupillary oscillation patterns do not depend on the fractal properties of the image viewed.

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### 1. Introduction

Benoit Mandelbrot introduced the term fractal to describe patterns that repeat at increasingly fine magnifications (Mandelbrot, 1982). Fractals can be either exact or statistical (Fairbanks and Taylor, 2011). The patterns of exact fractals repeat precisely. For statistical fractals, only the statistical qualities repeat at finer magnifications with a consequence that they look similar but not exactly the same at different magnifications. Fractals can also be either spatial or temporal (Fairbanks and Taylor, 2011). Spatial fractals are found in physical structures, such as trees or clouds, whereas temporal fractals are fractal patterns found in time series such as some measurement over time (Fig. 1, right-hand images).

Studies have found fractal structures in many natural processes and physiological processes. For example, in natural scenery, trees, mountains, and clouds are fractal (Mandelbrot, 1982). In human anatomy, neurons, bronchial trees, and veins form fractal structures (Caserta et al., 1990). Examples of temporal fractals include heart beat rates, which trace out fractal patterns similar to those in Fig. 1 (Goldberger et al., 2002). Eye movements (saccades) have also been shown to follow fractal trajectories as a function of time (Taylor et al., 2011). It is not known, however, whether pupil diameter fluctuates in a fractal manner.

Fractals are prevalent in nature in part because of the favorable functional properties resulting from the repeating structure. For example, the fractal branching of the bronchial tree generates a large surface area that maximizes oxygen transfer (Mandelbrot, 1982). The fractal properties of nerves, such as cat retinal neurons (Caserta et al., 1990; Kiselev et al., 2003), facilitate connections with neighboring neurons. These examples are all spatial fractals, but there are useful

consequences for temporal fractals as well. The fractal motions of foraging animals and eye movements have both been explained in terms of the fact that the patterns of fractal trajectories cover space efficiently—an important characteristic for animals searching for food and for the eye searching for visual information. More generally, systems that use fractal patterns tend to be more efficient than those that use random patterns, and the pupil may be another example of these systems.

In this study, we examine pupillary dilations and how they are affected when the observer views spatial black and white computer-generated fractal patterns. Fractals are used as the visual stimulus for two reasons. First, fractals are prevalent in nature's scenery and so serve as a generic pattern which humans are exposed to on both daily and evolutionary time scales. Secondly, fractal patterns have been shown to reduce the physiological stress of the observer (Taylor, 2006). The behavior of pupillary dilation might therefore shed light on the mechanism behind this stress-reduction.

Our investigation sets out to answer two questions of pupillary dilation patterns: are these patterns fractal and do they vary systematically with the complexity of the spatial fractal pattern being viewed? This complexity is quantified by a parameter called the fractal dimension  $D$ . A straight line has a  $D$  value of 1 while a completely filled space has a  $D$  value of 2. In contrast, a fractal pattern has a  $D$  value lying between 1 and 2 as they lie between 1 dimension and 2 dimensions.  $D$  quantifies the relative amount of coarse and fine structure within the fractal pattern. For patterns with high  $D$  values, the fine structure contributes more to the fractal mixture of structure than for a low  $D$  fractal, with a consequence

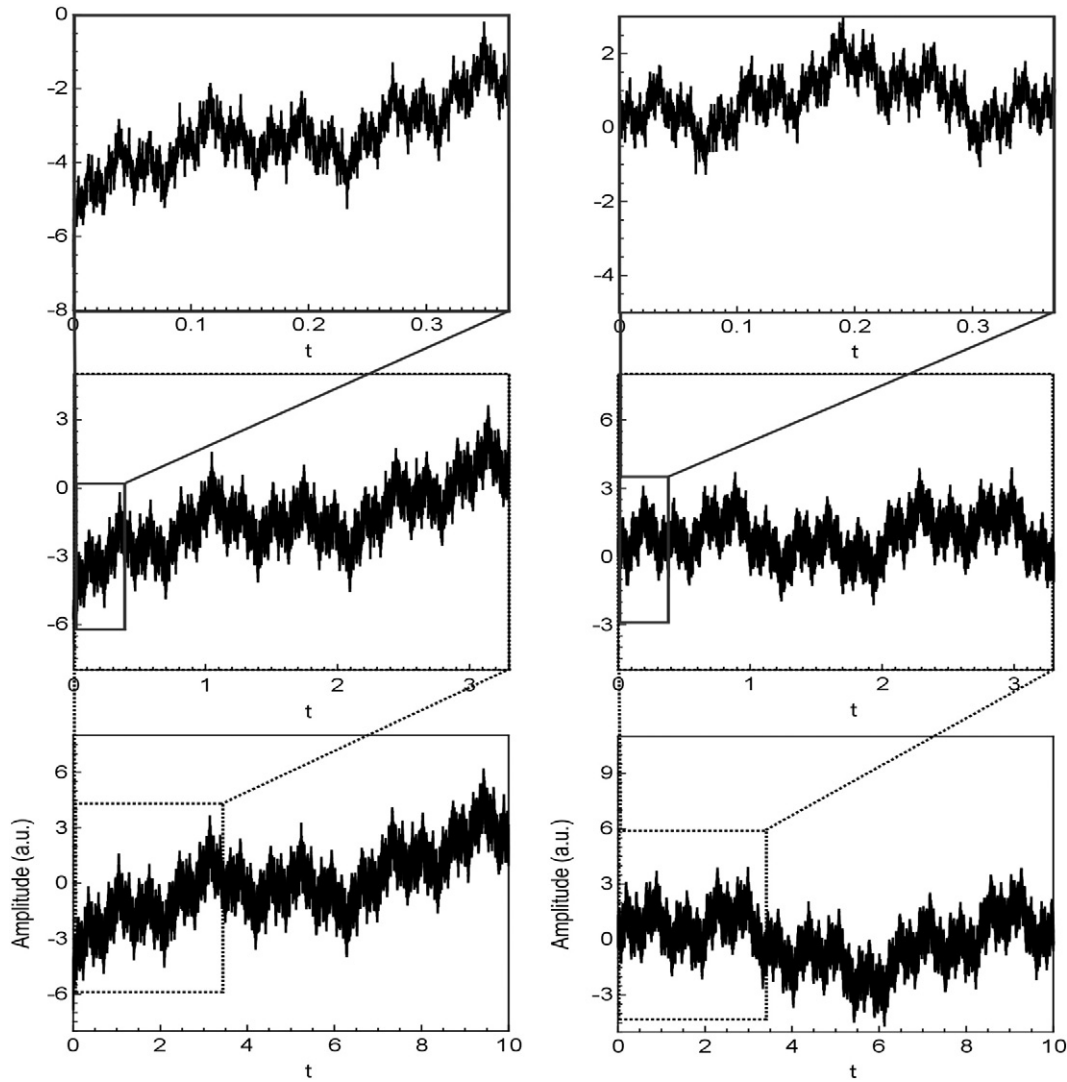


Fig. 1. Examples of computer-generated exact (left) and statistical (right) temporal fractals. The fractals our research will examine are statistical temporal fractals.

that, for example, a  $D = 1.9$  fractal is more visually complex than a  $D = 1.1$  fractal.

### 1.1. Background

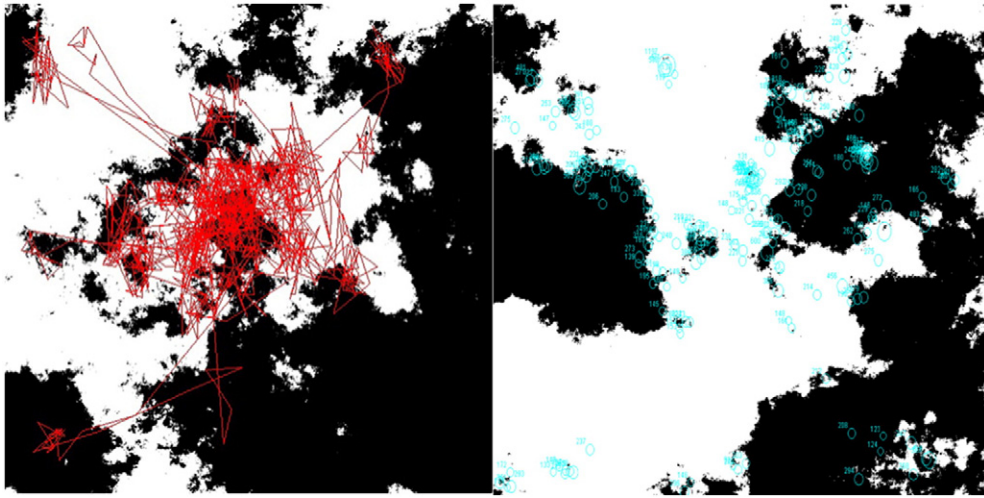
Evidence that pupil size may fluctuate over time stems from Taylor et al.'s (2011) fractal saccade research, which demonstrates that as the eye scans an image it does so in a fractal pattern shown in the left image of Fig. 2. As the eye scans the image, the amount of light to which the retina is exposed will change over time; therefore if the eye scans in a fractal manner the amount of light entering the eye may vary in a fractal pattern leading to fractal pupillary light reflex patterns. It is also possible that the pupil variation occurs for additional reasons beyond light variations of the stimulus. Many systems in the body demonstrate fractal patterns (Mandelbrot, 1982).

When searching for information in a fractal visual stimulus, the eye re-applies a similar pattern to scan the image at all size scales instead of generating new patterns at each scale. This fractal saccadic eye movement might allow the eye to cover area more efficiently than if the motions were random. The saccadic trajectories are quantified by  $D = 1.4$  to 1.5, irrespective of the complexity of the fractal pattern being observed (Fairbanks and Taylor, 2011). Significantly, observation of mid-range  $D$  fractals induces maximum stress reduction (Taylor, 2006).

This is important when we consider that mid-range  $D$  fractals are the most prevalent fractals in natural settings. Fractals with high  $D$  values ( $D = 1.6–1.9$ ) may be too complex to show any therapeutic benefit, while fractals with low  $D$  values ( $D = 1.1–1.2$ ) may be too simple (Taylor, 2006). It may be the case that mid-range  $D$  fractal images produce a different pupillary oscillation pattern compared to those outside the mid- $D$  range. It may also be the case that oscillation patterns remain constant due to the constant complexity of fractal saccadic trajectories regardless of the complexity of image viewed.

A fractal pupillary response may be due to the general nature of healthy physiology, akin to the healthy heart beat literature. A breakdown in fractal heart rate variation is correlated with heart disease (Goldberger et al., 2002). However, heart disease is not the only correlate with fractal heart rate variation breakdown as age is also negatively correlated with fractal heart rate variations (Goldberger et al., 2002). It may be the case that healthy physiology is driven by underlying fractal patterns, and that their breakdown correlates with functional problems as seen in the fractal heart rate research.

We hypothesize that the pupil will oscillate in a fractal manner and that mid-range complexity images may induce systematically different fractal oscillatory patterns than extremely low or high complexity fractal images. We believe that the patterns will either be a result of fractal saccadic scanning patterns or from healthy physiological processes.



**Fig. 2.** Two computer-generated black and white fractal images. A subject's eye gaze trajectory (red lines, from Taylor et al., 2011) is superimposed on the left image, while the right image shows eye fixation data (blue circles). Larger circles indicate longer fixation times.

## 2. Methods

### 2.1. Subjects

All procedures were approved by the Institutional Review Board of Wake Forest University and were performed in accordance with the ethical standards laid down in the 1964 Declaration of Helsinki. Thirty-nine undergraduate subjects were recruited from the undergraduate research pool at Wake Forest University. Subjects participated for optional credit in an introductory psychology course. Thirty subjects' data were analyzed and are reported on below. Seven initial subjects were run as a pilot subjects. One subject's data was removed due to excessively large pupil size (i.e., > 10 mm) and another subject's data was removed due to irregular camera readings. Five of the reported subjects were male and 25 were female. The average age of the subjects reported was 18.6 years.

### 2.2. Methods

Nine computer-generated black and white fractal images and their negatives (18 total images;  $16.6^\circ \times 16.6^\circ$ ) were presented in a random order for 60 s each. These images ranged from  $D$  values of 1.1 to 1.9 in steps of 0.1. Subjects placed their head on a stable headrest which was attached to a table at a predetermined distance from the monitor (58 cm). Prior to presentation of the stimuli an EyeLink camera was calibrated for each subject to ensure accurate measurement. This calibration was then validated for each subject before the stimuli were presented. Upon completion of the experiment subjects were debriefed and were given credit for their participation in the study.

### 2.3. Materials and apparatus

Pupil size was collected using an EyeLink 1000 infrared camera that recorded monocular pupil size (left-eye), eye movement, fixation duration, as well as other ocular movements at 1000 Hertz. The reason our research requires such a rapid sampling period is to allow for magnification of the data over several orders of magnitude for the fractal analysis. Output data are pupil size overtime in mm.

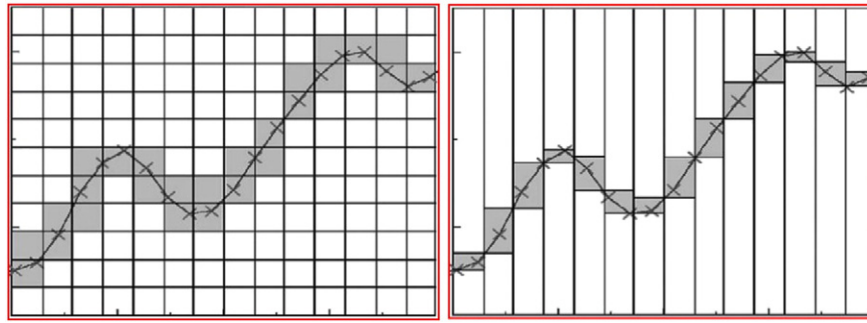
Blinks were removed from the data via an Excel program before the variational box counting analysis. This was done by taking out the zero scores (those representing a closed eye—so no information) and all the data points 100 ms before and after each set of zero's. Partial blinks were removed manually via examining the data to locate near but not

complete occlusion of the pupil and repeating the above process (i.e., removing 100 ms before and after the blink). Then the remaining time series was stitched together (i.e., the deleted section of the data was removed from the series and the remaining data was connected from the beginning of the blink to the end of the blink). There are inherent problems with removing the blinks this way, but it is a better method than using interpolation as removal of the blinks does not add arbitrary data points to the time series. However, stitching the data together in this fashion treats the data as though it is one continuous time series when in fact the time series has sections missing. Blinks make it impossible to resolve this issue in any other more meaningful way given current technology. Removing the blinks also resulted in pupil size time traces of different lengths (which may be observed in Fig. 3). The overall length of each time series was heavily dependent on the number of blinks per 60 s period, with more blinks leading to shorter traces.

### 2.4. Fractal analysis

We employed the variational method of the well-established 'box-counting' method for measuring the  $D$  value of the pupillary fractals (Fairbanks and Taylor, 2011). The box-counting procedure involves superimposing a computer-generated mesh of identical squares (i.e., "boxes") over the temporal trace. The statistical scaling qualities of the pattern were then determined by calculating the proportion of squares occupied by the time series pattern and the proportion that are empty. This process was then repeated for meshes with a range of square sizes. Reducing the square size is equivalent to looking at the pattern at finer magnification. In this way, we could compare the pattern's statistical qualities at different magnifications. Specifically, the number of squares  $N(L)$  that contained part of the time series pattern was counted and this was repeated as the size,  $L$ , of the squares in the mesh was reduced. For fractal behavior,  $N(L)$  scales according to the power law relationship  $N(L) \sim L^{-D}$ , where  $1 < D < 2$ . This power law generates the scale invariant properties that are central to fractal geometry. The  $D$  values, which chart this scale invariance, were extracted from the gradient of a graph of  $\log N(L)$  plotted against  $\log L$ . A regression was run between the two variables,  $\log N(L)$  and  $\log L$ , with a linear relationship confirming the pattern to be fractal. Whereas the slope of the regression quantifies the  $D$  value, its  $R^2$  quantifies the strength of the fractal relationship (i.e., as  $R^2$  increases the data more closely follows the fractal power law relationship) (Fairbanks and Taylor, 2011).

The variational method of the box counting procedure employs columns in lieu of boxes to determine the self-similarity of a pattern and is



**Fig. 3.** An example of sample data using the box counting procedure (left-hand image) and the more accurate variational method (right-hand image) (Fairbanks and Taylor, 2011). The variational method dramatically reduces box overcounting by taking the amount of data that falls within the column into account. If any data falls within a box in the box-counting method the entire box is counted.

a more accurate measure of the relative amount of area containing data versus space void of data. To reduce overcounting this method takes into account the amount of data which falls within the columns as opposed to the box counting procedure which will count an entire box if a single data point falls within the area of a box (Fig. 3). Box sizes are determined by the difference between the area where the time series enters the column and where it exits the column (i.e., gray shaded area in Fig. 3, right hand image).  $N$  is generated by comparing the total box area (i.e., gray area in Fig. 3, right hand image) versus the total amount of the columns containing no data. As in the box counting procedure, this method is then repeated over several orders of magnitude.

The variational procedure has measurement limits referred to as the coarse scale and fine scale “cut-offs” (Fairbanks). The fine scale cut-off is set by the requirement that the finest column size analyzed must contain 100 data points: columns containing fewer than 100 data points will be distorted by the discreteness of the data and should not be included in the analysis as there is no structure within the pattern finer than 100 ms. Given that the data resolution is 1 ms, this sets the fine scale cut-off as  $L = 100$  ms. The coarse scale cut-off is set by the requirement that there must be at least 10 columns spanning the length of the data set: fewer columns than this will lead to a loss in counting statistics. The coarse scale cut-offs varied depending on the length of the trace due to the removal of blinks in the time series. The fitting procedure described above is applied only to the data lying between the coarse and fine cut-offs.

**3. Results**

Analyses were conducted in two phases. First, we used the MatLab variational procedure to confirm that the pupil oscillation patterns were statistical temporal fractals. Following the main analysis of the oscillation patterns we ran several exploratory correlations to examine which variables may affect the complexity of the fractal patterns and the strength of the fractal relationship along with other correlations.

**3.1. Main analysis**

The variational procedure confirmed a fractal oscillation pattern for every image viewed by all 30 subjects analyzed. All 540 fractal images viewed produced fractal pupillary dilation patterns. The average pupil fractal complexity was  $D = 1.277$  (S.D. = 0.04) and the average fractal relationship strength was  $R^2 = 0.9966$  (S.D. = 0.002). The  $R^2$  data should be examined cautiously however, as not every trace consisted of the same number of data points. In general the closer the relationship is to a perfect  $R^2$  of 1 the more the trace follows the fractal power law relationship.

Fig. 3 shows raw data from the highest and lowest pupil  $D$  trials and Fig. 4 shows the same data analyzed using the variational technique. The linear fit lines included in Fig. 4 show that both data traces are fractal at fine scales but that they deviate from fractal behavior at coarse

scales. The differences between fractal oscillation patterns within a given subject were slight: the highest and lowest  $D$  patterns on average nearly overlapped (Fig. 5).

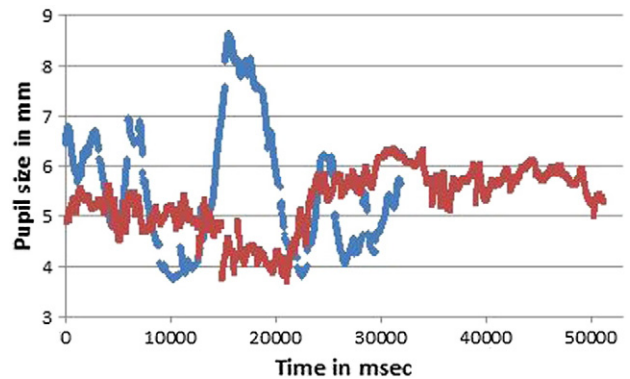
**3.2. Correlational analysis**

If our fractal images elicit a characteristic scan path as in Taylor et al. (2011) then one would expect a systematic change in pupil size that reflects the change in gaze position, not any actual change in pupil size. So, we needed to correct our pupil size data for gaze position. In that the results of our analyses were unchanged, it suggests that the effects we observed are not driven by the changes in gaze position. If the effect did disappear, then it would suggest that it is driven by apparent, rather than actual changes in pupil size, caused by the pupil’s shape (as it appears on the camera retina) changing as a function of gaze position.

There was no significant effect of image  $D$  on pupil  $D$ ,  $r = -.045$ ,  $p = .292$  (Fig. 6). However, a significant negative correlation was found across all trials and subjects between average pupil size and pupil  $D$ ,  $r = -.261$ ,  $p < .001$ . On average, as pupil size became smaller the pupil dilation  $D$  value increased. A significant negative correlation was also found between average pupil size across all trials and subjects and  $R^2$ ,  $r = -.251$ ,  $p < .001$ , meaning that as pupil size decreases the strength of the fractal pattern increased (Fig. 7).

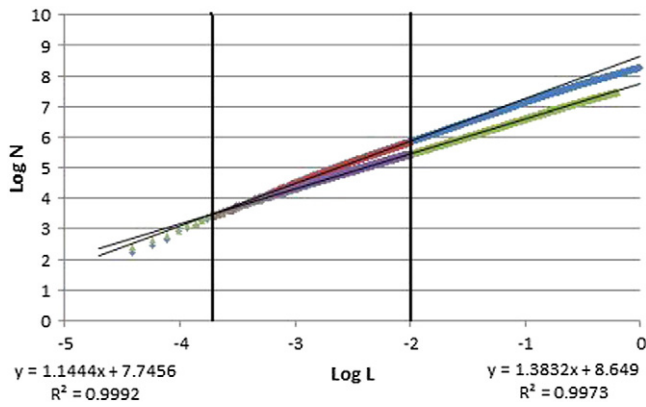
**4. Discussion**

Our main analysis supports our hypotheses that pupillary dilations exhibit fractal behavior as a function of time. The fractal patterns were



**Fig. 4.** Pupil size plotted as a function of time for 2 different subjects observing the same fractal image. The two traces are of the single lowest pupil  $D$  (in blue) across all trials and the single highest pupil  $D$  (in red) across all trials. Note that the traces are of different length. This is caused by differences in blink rate. The length of each time series was dependent on the number of blinks. The highest  $D$  trace demonstrates fine changes in pupil size over time, whereas the lowest  $D$  trace demonstrates coarse changes in pupil size over time.

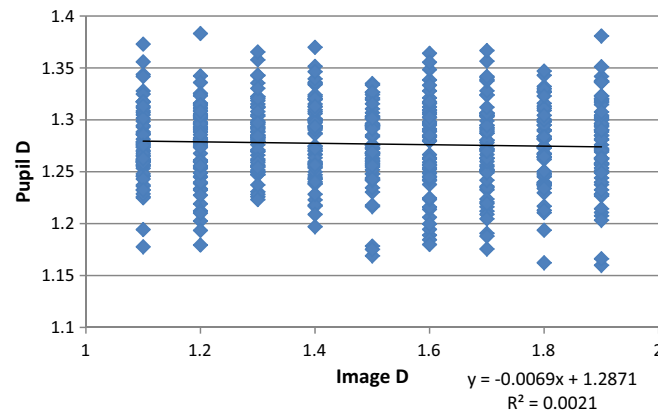




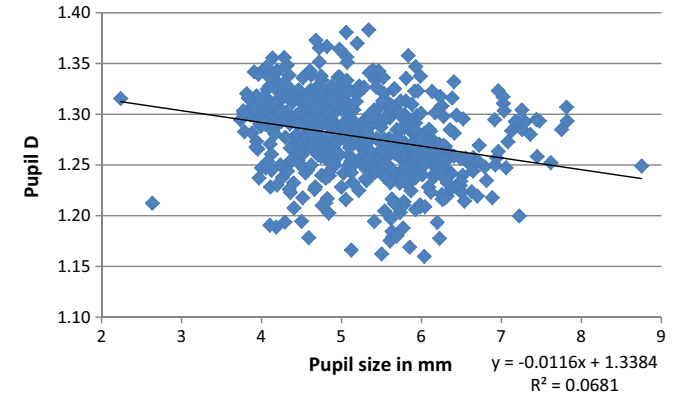
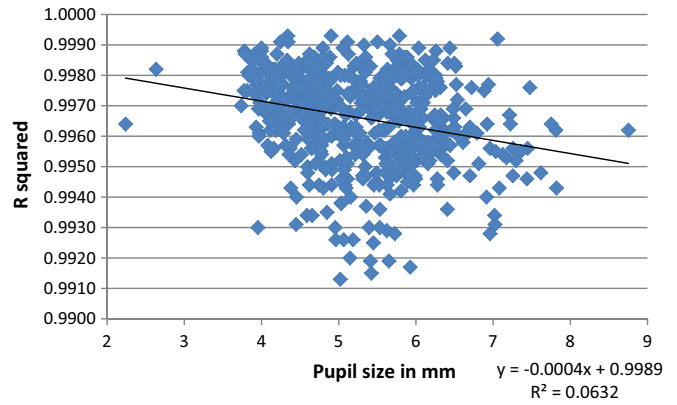
**Fig. 5.** The results of the variational box counting analysis for the 2 sets of data from Fig. 3. The blue and green symbols are all the data points for the highest and lowest *D* oscillation patterns while the red and purple data points are the range over which the patterns were self-similar, respectively. The two vertical lines indicate the coarse (left) and fine (right) scale measurement cut-offs.

very strong, with an average  $R^2$  of .9966. The data also show self-similarity over a range of roughly 1.5 to 2 orders of magnitude, which is fairly typical of natural fractals (Fairbanks and Taylor, 2011). Second, these fractal patterns were relatively constant across stimuli and did not fluctuate in relation to the complexity of the images viewed which did not fully support our original hypothesis. Since this pattern is so strong across all 30 subjects, as evidenced by the  $R^2$ , it may be the case that the underlying physiological process of pupillary oscillations is fractal regardless of the pattern of light falling on the retina. This would further implicate the importance of fractal patterns in healthy physiology as research on healthy hearts has shown a breakdown of this fractal pattern with age and disease such as heart failure (Goldberger et al., 2002).

The correlational evidence presented demonstrated a negative relationship between average pupil size and pupil oscillation complexity (i.e., pupil *D*) and fractal pattern strength (i.e.,  $R^2$ ). We speculate that this may implicate the autonomic nervous system's involvement in these fractal patterns, and particularly, the parasympathetic branch as parasympathetic nervous system activation correlates with pupillary constriction (Milton, 2003). However, it is also possible that the resulting fractal patterns may be driven by inhibition of the sympathetic nervous system. The two systems work in tandem and an increase in one leads to a decrease in the other. We speculate that the parasympathetic nervous system drives the patterns observed as previous research on pupillary dilation patterns has implicated the parasympathetic nervous system's importance in these patterns (Steinhauer et al., 2004).



**Fig. 6.** A scatterplot demonstrating the lack of a relationship between the *D* value of the image and the *D* value of the pupillary oscillation pattern.



**Fig. 7.** Scatterplot of pupil size plotted as a function of pupil *D* (top) and  $R^2$  (bottom).

Pupillary constriction is controlled by the parasympathetic nervous system via the Edinger–Westphal complex. These signals travel from the Edinger–Westphal complex down the third cranial nerve to the ciliary ganglion cells, and then to the pupillary sphincter muscle. Pupillary dilation is triggered by inhibition of the Edinger–Westphal nucleus (Milton, 2003). Research has suggested that the parasympathetic nervous system is far more involved in pupillary oscillation patterns as when the parasympathetic system has been inhibited the pupil does not constrict in reaction to light (Steinhauer et al., 2004). Coupled with the fact that pupil size decreases with parasympathetic activation, our results suggest that the parasympathetic branch of the autonomic nervous system may be driving the complexity and strength of these fractal patterns. Future research will investigate this relationship to determine whether fractal oscillation patterns are indeed parasympathetically controlled.

Previous research has suggested that abnormally slow pupillary oscillations may be the result of parasympathetic nervous system dysfunction (Martyn and Ewing, 1986). In a recent study, abnormal pupillary light reflex (PLR) (i.e., the pupil's natural reaction to constrict in the presence of illumination or dilate in the absence of illumination) may be predictive of Alzheimer's disease (AD), as AD patients showed significant differences in redilation and constriction patterns compared to healthy patients and patients on AD medication (Fotiou et al., 2000). AD patients showed reduced latency periods of the PLR as well as a limited range of constriction compared to healthy patients and AD patients on medication. This again may implicate the parasympathetic nervous system dysfunction with disease as the parasympathetic branch is responsible for pupillary constriction. Moreover, heart rate patterns tend to become abnormal and non-fractal with disease and correlate with heart failure risk, which in turn correlates with autonomic nervous system abnormalities (Goldberger et al., 2002). The case may be that certain types of ocular disease are related to parasympathetic dysfunction. It may also be the case that cardiovascular disease correlates with autonomic nervous system dysfunction, which in turn predicts a

breakdown in the fractal nature of pupillary oscillations. Age and disease may play a role in the breakdown of fractal patterns as well. As the heart rate literature demonstrates, fractal heart rate variation breaks down with age and non-fractal heart rate variation predicts heart failure risk. It is possible that with age pupillary oscillations become non-fractal. It is unclear, however, if non-fractal pupillary oscillation patterns relate to any sort of disease. Future research should aim to determine whether or not ocular fractal patterns are found in the elderly or those with ocular diseases.

The lack of any relationship between image  $D$  and pupil  $D$  is surprising. Previous research has typically found a relationship between the fractal density of the stimuli and the physiological response. The overall difference between fractal pupil oscillation patterns was also quite small (Figs. 4 & 5). Taylor et al.'s (2011) work may help to shed light onto why there was no apparent relationship between the fractal characteristics of the images and the physiological response observed. In their investigation, they discovered that saccadic scanning patterns while viewing statistical fractal paintings remained constant at roughly  $D$  of 1.5 regardless of the fractal density of the painting viewed. It is possible that our subjects' saccade patterns remained constant across images. This in turn may have led the input into the retina to be a constant temporal fractal. Therefore, if the input is a constant temporal fractal then the output should be a constant temporal fractal.

The benefit of these observed patterns within pupillary oscillation patterns remains unclear. Previous research has shown that the absence of fractal patterns within other organs may predict adverse health outcomes (Goldberger et al., 2002) and that abnormal pupillary fluctuations may predict dementia (Fotiou et al., 2000). However, this does not directly implicate non-fractal pupillary oscillation patterns with disease. We posit that these patterns may be a result of the fractal structure of neurons (Smith et al., 1989) and whether or not non-fractal pupillary oscillation patterns predict any sort of disease is a natural next step for this line of research. This may be an inexpensive and non-invasive way to determine whether or not a person is at risk for disease. Testing such a population may suggest the functional significance of these patterns.

We interpret our data set with caution because we had to stitch together pupil oscillatory data with blinks removed. This was done to insure that we had close to 1 min of total data from which to extract our fractal time series. This method was considered to be cleaner than trying to interpolate pupil size during the blink (which is sometimes done in pupillography) since this would have introduced arbitrary points into the time-series. While this stitching does distort the continuity of the time-series, because the blinks occur at random intervals, any resulting distortions are non-systematic. Moreover, blink time compared to total time viewing the images was small, thus keeping the continuity distortions to a minimum.

The box-counting procedure must also be viewed with caution as it may produce false positives for self-similar repeating patterns. However, we are confident that this is not the case with our data as the self-similar patterns were observed over nearly two orders of magnitude. We were also quite conservative when calculating our cut-offs under which we looked for self-similar patterns to ensure that we did not include data in our analysis that was too fine to represent actual fluctuations in pupil size. Overall we found correlational evidence between pupil  $D$  and pupil size; however, it remains unclear what this relationship means. We speculate that our data implicate the autonomic

nervous system. However it is difficult to make many assumptions without additional correlates of the autonomic nervous system. It is possible that ocular disease may arise in the absence of fractal pupillary oscillation patterns similar to fractal heart rate patterns (Goldberger et al., 2002). Further research may help to shed light on this point within a clinical population.

## 5. Conclusion

Prior to the current study no research had investigated whether pupillary oscillation patterns are fractal. Our study demonstrated that over all subjects and over all trials pupillary oscillations were low complexity fractals, and that the fractal pattern was quite strong. Since pupil size decreases with parasympathetic activation, these fractal oscillations may be driven by the parasympathetic branch of the autonomic nervous system or an inhibition of the sympathetic branch as both complexity of the oscillation patterns and strength of the fractal nature of these oscillation patterns were negatively correlated with average pupil size. Because all of the images used in this study were perfectly self-similar mathematical fractals, making assumptions of whether or not the effect is driven by temporally fractal stimulation of the pupil or if the effect is internally driven is difficult. However, based on the results of this study we suggest that two possibilities remain: scanning patterns of the images led to fractal stimulation of the retina or, similar to health heart beats, the pupil's natural oscillatory pattern is fractal.

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