Day 10 homework - Assigned 2/5 and due on 2/14.

Starred problems below are extra-credit for undergraduates and required for graduate students.

4. Let G be a group of order 8 whose elements are $\{1, 2, 3, 4, 5, 6, 7, 8\}$ with the Cayley table

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	3	4	1	6	7	8	5
3	3	4	1	2	7	8	5	6
4	4	1	2	3	8	5	6	7
5	5	8	7	6	3	2	1	4
6	6	5	8	7	4	3	2	1
7	7	6	5	8	1	4	3	2
8	8	7	6	5	2	1	4	3

(a) Compute $Z(G)$.

(b) Compute $C(6)$, the centralizer of the element 6.

5. (a) Let p be a prime number. Show that $Z(GL(2, Z_p))$ is the set $\left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} : a \in U(p) \right\}$ of scalar multiples of the identity.

(b) Compute the centralizer of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ in $GL(2, Z_7)$. What is the order of the centralizer?

6. $*$ Let G be a group and let a be an element in G. A *conjugate* of a is an element of the form gag^{-1} for some $g \in G$. For $x, y \in G$, define $x \sim y$ if $xax^{-1} = yay^{-1}$.

(a) Show that $\sim$ is an equivalence relation.

(b) Show that $x \sim y$ if and only if $x^{-1}y \in C(a)$, the centralizer of a.