Day 12 homework - Assigned 2/12 and due on 2/21

Starred problems below are extra-credit for undergraduates and required for graduate students.

4. Suppose that G is a group an $a \in G$. If $|a^4| = 3$, what are the possibilities for $|a|$?

5. Let G be a cyclic group of order n, d be a divisor of n and H the unique subgroup of G of order d. Given an element $g \in G$, prove that $g \in H$ if and only if $g^d = e$. [This fact is useful in number theory.]

6. $*$ Let G be a finite group and $a \in G$. Prove that a has odd order if and only if for all integers $k \geq 1$ there is some element $b_k \in G$ so that $b_k^{2^k} = a$.