

Day 18 homework - Assigned 2/26 and due 3/6

Starred problems below are extra-credit for undergraduates and required for graduate students.

3. Suppose that the “cups” magic trick is being done. Initially the cups are arranged as ?, 2, 3, 4, 5, 6, ? and the ball of kleenex is put under the leftmost cup. A student volunteer makes a swap of two cups 15 times. The resulting arrangement is the following: 5, 6, ?, 3, 4, ?, 2. Is the ball of kleenex between the 6 and the 3, or between the 4 and the 2?
4. Let  $n > 1$  be an odd integer. Define  $\phi : U(2n) \rightarrow U(n)$  by  $\phi(x) = x \bmod n$ . Prove that  $\phi$  is an isomorphism.
5. \* If  $n$  is a positive integer, show that there is a subgroup  $G$  of  $A_{n+2}$  that is isomorphic to  $S_n$ .