Day 21 homework - Assigned 3/4 and due 3/27

Starred problems below are extra-credit for undergraduates and required for graduate students.

4. Let $G = Z_{12}$ be the additive group mod 12. How many automorphisms $\phi : G \to G$ have the property that $\phi(2) = 10$? Justify your answer.

5. Let G be a group. Let $\phi : G \to G$ be the function given by $\phi(x) = x^{-1}$. Show that ϕ is an automorphism of G if and only if G is abelian.

6. Let G and $\overline{G}$ be groups and suppose that $\phi : G \to \overline{G}$ is an isomorphism.

(a) Show that if ψ is an automorphism of G, then $\phi \circ \psi \circ \phi^{-1}$ is an automorphism of $\overline{G}$.

(b) Define $\rho : \text{Aut}(G) \to \text{Aut}(\overline{G})$ by $\rho(\psi) = \phi \circ \psi \circ \phi^{-1}$. Prove that ρ is an isomorphism. (This proves that if $G \approx \overline{G}$, then $\text{Aut}(G) \approx \text{Aut}(\overline{G})$.)